FORCED VIBRATIONS DUE TO MECHANICAL LOADS IN THERMOVISCOELASTIC HALFSPACES

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In the present article the Kelvin-Voigt model of linear viscoelasticity which describes the viscoelastic nature of a material is used to investigate the forced vibrations due to mechanical loads acting on the boundary of a thermoviscoelastic continuum. The Laplace and Hankel transform technique has been employed to solve the boundary value problem in the transform domain, in the context of various theories of generalized thermoelasticity. The inverse transform integrals are evaluated by using Romberg integration in order to obtain the results in the physical domain. The temperature and stresses so obtained in the physical domain are computed numerically and presented graphically in different situations for a copper material. The comparison of results for different theories of generalized thermoviscoelasticity is also presented at appropriate stages of this work.

Key words: mechanical loads, thermal relaxation, mechanical relaxation time, Romberg integration, thermoviscoelasticity.

1. Introduction

The theory of thermoelasticity deals with the effects of mechanical and thermal disturbances on an elastic body. There are two defects in the Uncoupled Thermoelasticity Theory (UCT). First, the mechanical state of an elastic body has no effect on the temperature is not consistent with true physical experiments. Second, the heat equation being parabolic predicts an infinite speed of heat propagation, a physically unrealistic phenomenon. The theory of coupling of thermal and strain fields gives rise to Coupled Thermoelasticity (CT) and was first postulated by Duhamel (1837), shortly after the theory of elasticity. He derived the equations for the distribution of strains in an elastic medium subjected to temperature gradient and introduced the dilatation term in the heat conduction equation, but the equation was not based on a thermodynamical grounds. Neumann (1855), made an attempt on thermodynamical justification of Duhamel's theory. The work of Biot (1956), gave a satisfactory derivation of heat conduction equation, which included the dilatation term based on thermodynamics of irreversible processes. This development removed the first defect of uncoupled thermoelasticity. However, this theory shares the second defect of infinite speed of heat wave propagation. During the last three decades, non-classical theories have been developed to remove the paradox of infinite velocity of heat transportation. Lord and Shulman (1967), incorporated a heat flux-rate term into Fourier's law to formulate a generalized theory that admits finite speed for thermal signals. Also Green and Lindsay (1972), by including the temperature rate, violated the

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classical Fourier's law of heat conduction when the body under consideration has a center of symmetry. This theory also predicts a finite speed of heat propagation. According to these theories (hereinafter called LS and GL theories, respectively), heat propagation is to be viewed as a wave phenomenon rather than a diffusion one. A wave like thermal disturbance is referred to as "second sound" by Chandrasekharaiah (1986). Some researchers such as Ackerman *et al*. (1966), Guyer and Krumhansl (1966), and Ackerman and Overtone (1969), proved experimentally for solid Helium that thermal waves (second sound) propagating with a finite, though quite large, speed also exist.

The most recent and relevant theoretical developments on this subject are due to Green and Nagdhi (1991; 1993; 1992), which provide sufficient basic modifications in the constitutive equations that permit treatment of a much wider class of heat flow problems. Li and Dhaliwal (1996), solved a boundary value problem of an isotropic elastic half space with its plane boundary either held rigidly fixed or stress free and subjected to a sudden temperature increase. The approximate small time solutions, for displacement, temperature, and stress fields have been obtained by employing the Laplace transform technique in the context of thermoelasticity developed by Green and Nagdhi (1993), Chandrasekharaiah (1996), studied one-dimensional waves in a homogeneous isotropic half-space due to a sudden input of temperature and stress on the boundary by employing the Laplace transform method in the context of thermoelasticity without energy dissipation. The exact closed form solution for displacement, temperature, strain and stress fields has been obtained and analyzed in the light of their counterparts in earlier works. Harinath (1975), considered the problems of surface point and line loads over a homogeneous isotropic generalized thermoelastic half space. Nayfeh and Nasser (1972) used the Cagniard and De Hoop method (De-Hoop, 1959) to develop the displacements and temperature fields in a homogeneous isotropic generalized thermoelastic halfspace subjected to an instantaneously applied heat source on the free surface. Sharma (1986) used the Cagniard (1962) method to study the transient behaviour of thermoelastic waves in a transversely isotropic solid half-space subjected to an instantaneous line load that is applied on its free surface. Sharma *et al*. (2000) investigated the disturbance due to normal point load and thermal source acting on the free surface of the half space by applying the Hankel transform technique in the context of various theories of generalized thermoelasticity. Sharma and Chauhan (2001) studied the disturbance in a halfspace due to mechanical loads and heat sources. Sharma and Sharma (2001) worked on the transient thermoelastic waves by employing Cagniard (1962) method of seismic wave propagation.

The effect of internal friction on the propagation of plane waves in an elastic medium may also be considered owing to the fact that dissipation accompanies vibrations in solid media due to the conversion of elastic energy to heat energy (Ewing *et al*., 1957). Several mathematical models have been used by authors (Ewing *et al*., 1957; Hunter, 1960; Lord and Schulman, 1967; Flugge, 1967), to accommodate the energy dissipation in vibrating solids where it is observed that internal friction produces attenuation and dispersion and hence the effect of the viscoelastic nature of the material medium in the process of wave propagation cannot be neglected. The viscoelastic nature of a medium has special significance in wave propagation in a solid medium. Acharya and Mondal (2002), investigated the propagation of Rayleigh surface waves in a Voigt (1887), type viscoelastic solid under the linear theory of non-local elasticity. As pointed out by Freudenthal (1954), most of the solids when subjected to dynamic loading, exhibit viscous effects. The Kelvin-Voigt model is one of the macroscopic mechanical models often used to describe the visco-elastic behaviour of a material. This model represents the delayed elastic response subjected to stress when the deformation is time dependent but recoverable. The dynamical interaction of thermal and mechanical fields in solids has great practical applications in modern aeronautics, astronautics, nuclear reactors, and high-energy particle accelerators, for example. Mukhopadhyay (2000) studied the thermal relaxation effects and compared the various theories of generalized thermoelasticity for thermoviscoelastic interactions in an infinite viscoelastic solid of Kelvin-Voigt type with a spherical cavity. Mukhopadhyay and Bera (1989) investigated the effect of distributed instantaneous continuous heat sources in an infinite conducting magneto-thermoviscoelastic solid with relaxation time.

In the present article, the Kelvin-Voigt model of linear viscoelasticity which describes the viscoelastic nature of the material is used to investigate the forced vibrations due to mechanical loads acting on the boundary of a generalized thermoviscoelastic continuum by applying the Laplace and Hankel transform technique. The results in the physical domain are attained by inverting the integral transforms with the help of a numerical technique (Sharma and Chauhan, 2001). The results obtained theoretically have been computed numerically and are presented graphically for a copper material. A complete and comprehensive analysis and comparison of results in various theories are presented.

2. Formulation of the problem

We consider a homogenous isotropic thermoviscoelastic half space initially undisturbed and at uniform temperature T_0. The Kelvin-Voigt model of linear viscoelasticity which describes the viscoelastic nature of the material has been employed to study the problem. We take the origin of cylindrical coordinate system (r, θ, z) as any point on the surface $z = 0$ and z-axis pointing vertically downward into the medium so that the half-space occupies the region $z \geq 0$. It is assumed that a normal point force is acting at a point on the surface $z = 0$ of the medium and hence all the quantities are independent of the θ co-ordinate. The basic governing equations of motion and heat conduction, in the context of the generalized theory of thermoelasticity, in the absence of body forces and heat sources are given by

$$\mu\nabla^2\boldsymbol{u} + (\lambda+\mu)\nabla\nabla\cdot\boldsymbol{u} - \beta\nabla\left(T + t_1\delta_{2k}\dot{T}\right) = \rho\ddot{\boldsymbol{u}}, \tag{2.1}$$

$$K\nabla^2 T = \rho C_e\left(\dot{T} + t_0\ddot{T}\right) + \beta T_0\left(\frac{\partial}{\partial t} + t_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\nabla\cdot\boldsymbol{u} \tag{2.2}$$

where

$$\lambda = \lambda_e\left(1+\alpha_0\frac{\partial}{\partial t}\right), \quad \mu = \mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right), \quad \beta = (3\lambda+2\mu)\alpha_T = \beta_e\left(1+\beta_0\frac{\partial}{\partial t}\right),$$
$$\beta_e = (3\lambda_e + 2\mu_e)\alpha_T, \quad \beta_0 = (3\lambda_e\alpha_0 + 2\mu_e\alpha_1)\alpha_T/\beta_e. \tag{2.3}$$

Here $\boldsymbol{u}(r, z, t) = (u, 0, w)$ is the displacement vector; $T(r, z, t)$ is the temperature change; λ_e, μ_e are the Lame's parameters; ρ, C_e and α_T are the density, specific heat at constant strain and co-efficient of linear expansion respectively; K is the thermal conductivity; α_0, α_1 are viscoelastic relaxation times and t_0, t_1 are thermal relaxation times; δ_{1k} is the Kronecker's delta in which $k = 1$ for Lord-Shulman (LS) theory and $k = 2$ in the case of Green-Lindsay (GL) theory. The thermal relaxation time parameters t_0 and t_1 satisfy the inequalities (Green, 1972)

$$t_0 \geq t_1 \geq 0, \tag{2.4}$$

in the case of GL theory only. However, it has been proved by Strunin (2001) that the inequalities (2.4) are not necessary to be satisfied.

2.1. Initial and regularity conditions

The initial and regularity conditions are given by

$$u(r, z, 0) = 0 = \dot{u}(r, z, 0), \quad w(r, z, 0) = 0 = \dot{w}(r, z, 0),$$

$$T(r,z,0)=0=\dot{T}(r,z,0), \quad \text{for} \quad z \ge 0, \quad r \ge 0,$$
$$u(r,z,t)=0 \quad w(r,z,t)=0, \quad T(r,z,t)=0 \quad \text{for} \quad t>0, \quad z \to \infty. \tag{2.5}$$

2.2. Boundary conditions

The surface $z=0$ of the thermoviscoelastic solid is subjected to the action of an instantaneous normal point load at the origin and assumed either thermally insulated or isothermal. Therefore, the corresponding boundary conditions are given as

$$\tau_{zz} = \frac{-P\delta(r)f(t)}{2\pi r}, \quad \tau_{rz}=0, \quad \frac{\partial T}{\partial z}+hT=0 \tag{2.6}$$

where $\delta(r)$ denotes the Dirac delta function; $f(t)$ is a well behaved function of time and h is the coefficient of surface heat transfer. Here $h \to 0$ corresponds to thermally insulated boundary and $h \to \infty$ refers to isothermal surface of the half space.

We define the quantities

$$r' = \frac{\omega^* r}{c_1}, \quad z' = \frac{\omega^* z}{c_1}, \quad t' = \omega^* t, \quad u_i' = \frac{\rho\,\omega^*\, c_1 u_i}{\beta_e\, T_0}, \quad T' = \frac{T}{T_0},$$
$$t_0' = \omega^* t_0, \quad t_1' = \omega^* t_1, \quad \tau_{ij}' = \frac{\tau_{ij}}{\beta_e T_0}, \quad \alpha_1' = \omega^* \alpha_1, \quad \alpha_0' = \omega^* \alpha_0,$$
$$\delta^2 = \frac{c_2^2}{c_1^2}, \quad P^* = \frac{\omega^*\, P}{\beta_e T_0 c_1}, \quad \omega^* = \frac{C_e(\lambda_e+2\mu_e)}{K}, \quad \in = \frac{\beta^2 T_0}{\rho\, C_e(\lambda_e+2\mu_e)}, \tag{2.7}$$
$$c_1^2 = \frac{\lambda_e+2\mu_e}{\rho}, \quad c_2^2 = \frac{\mu_e}{\rho}, \quad \beta_0' = \omega\beta_0' = \omega^*\beta_0, \quad h' = hc_1/\omega^*.$$

Using quantities (2.7) in Eqs (2.1) to (2.3) and suppressing dashes, we obtain

$$\left(1+\alpha_1\frac{\partial}{\partial t}\right)\delta^2\nabla^2\boldsymbol{u}+\left[1-\delta^2+\left\{\alpha_0\left(1-2\delta^2\right)+\alpha_1\delta^2\right\}\frac{\partial}{\partial t}\right]\nabla\nabla\cdot\boldsymbol{u}+$$
$$-\left(1+\beta_0\frac{\partial}{\partial t}\right)\nabla\left(T+t_1\delta_{2k}\dot{T}\right)=\ddot{\boldsymbol{u}}, \tag{2.8}$$

$$\nabla^2 T-\left(\dot{T}+t_0\ddot{T}\right)=\in\left(1+\beta_0\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}+t_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\nabla\cdot\boldsymbol{u}. \tag{2.9}$$

The non-dimensional form of boundary conditions (2.6) on the surface $z=0$ is given as

$$\tau_{zz} = -P^*\frac{\delta(r)f(t)}{2\pi r}, \quad \tau_{rz}=0, \quad \frac{\partial T}{\partial z}+hT=0. \tag{2.10}$$

3. Solution of the problem

We introduce the potential functions ϕ and φ through the relations

$$u=\frac{\partial\phi}{\partial r}+\frac{\partial\psi}{\partial z}, \qquad v=0, \qquad w=\frac{\partial\phi}{\partial z}-\frac{\partial\psi}{\partial r}-\frac{\psi}{r}. \tag{3.1}$$

Using Eq.(3.1) in Eqs (2.8) and (2.9), we obtain

$$\left(1+\alpha_1\frac{\partial}{\partial t}\right)\left(\nabla^2\Psi-\frac{\Psi}{r^2}\right)-\frac{\ddot{\Psi}}{\delta^2}=0, \tag{3.2}$$

$$\left(1+\delta_0\frac{\partial}{\partial t}\right)\nabla^2\phi-\ddot{\phi}=\left(1+\beta_0\frac{\partial}{\partial t}\right)\left(T+t_1\delta_{2k}\dot{T}\right), \tag{3.3}$$

$$\nabla^2 T-\left(\dot{T}+t_0\ddot{T}\right)=\in\left(1+\beta_0\frac{\partial}{\partial t}\right)\nabla^2\left(\dot{\phi}+t_0\delta_{1k}\ddot{\phi}\right) \tag{3.4}$$

where

$$\delta_0=\alpha_0+2\delta^2(\alpha_1-\alpha_0), \qquad \nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}.$$

We define the Laplace transform by

$$\bar{f}(r,z,p)=\int_0^\infty f(r,z,t)e^{-pt}dt$$

and the Hankel transform as

$$\hat{f}(q,z,p)=\int_0^\infty \bar{f}(r,z,p)rJ_n(qr)dr$$

where $J_n(x)$ is a Bessel function of first kind and of order n. Here $n=0$ for the functions ϕ, T and $n=1$ for ψ. Applying the Laplace transform followed by Hankel transform to Eqs (3.2) to (3.4), we obtain

$$D^2\hat{\psi}-\left(q^2+\frac{p}{\alpha_1^*\delta^2}\right)\hat{\psi}=0, \tag{3.5}$$

$$D^2\hat{\phi}-\left(q^2+\frac{p}{\delta_0^*}\right)\hat{\phi}=\frac{p\beta_0^*\tau_1\hat{T}}{\delta_0^*}, \tag{3.6}$$

$$\left(D^2-q^2-\tau_0 p^2\right)\hat{T}-\in p^3\beta_0^*\tau_0'\left(D^2-q^2\right)\hat{\phi}=0 \tag{3.7}$$

where

$$\alpha_1^* = p^{-1} + \alpha_1, \quad \tau_1 = p^{-1} + t_1\delta_{2k}, \quad \delta_0^* = p^{-1} + \delta_0,$$
$$\tau_0 = p^{-1} + t_0, \quad \beta_0^* = p^{-1} + \beta_0, \quad \tau_0' = p^{-1} + t_0\delta_{1k}. \tag{3.8}$$

Equations (3.6) and (3.7) give

$$\left(D^2 - q^2 - m_1^2\right)\left(D^2 - q^2 - m_2^2\right)\left(\hat{\phi}, \hat{T}\right) = 0 \tag{3.9}$$

where

$$m_1^2 + m_2^2 = \frac{p}{\delta_0^*} + p^2\tau_0 + \frac{\in p^4 \beta_0^{*2}\tau_0'\tau_1}{\delta_0^*},$$
$$m_1^2 m_2^2 = p^3\tau_0 / \delta_0^*. \tag{3.10}$$

The non-vanishing solution of Eqs (3.5) and (3.9) which satisfies the radiating condition, viz. the disturbance is assumed to be confined to the surface $z = 0$, is obtained as

$$\hat{\psi} = C\exp(-\xi_3 z), \tag{3.11}$$

$$\hat{\phi} = A\exp(-\xi_1 z) + B\exp(-\xi_2 z), \tag{3.12}$$

$$\hat{T} = \left(\frac{\delta_0^*}{p\beta_0^*\tau_1}\right)\left[Q_1 A\exp(-\xi_1 z) + Q_2 B\exp(-\xi_2 z)\right] \tag{3.13}$$

where $\mathrm{Re}(\xi_i) \geq 0$, $i = 1, 2, 3$ and

$$Q_1 = -\frac{p}{\delta_0^*} + m_1{}^2, \quad Q_2 = -\frac{p}{\delta_0^*} + m_2^2, \tag{3.14}$$

$$\xi_i^2 = q^2 + m_i^2, \quad (i = 1, 2), \quad \xi_3^2 = q^2 + \frac{p}{\alpha_1^*\delta^2}. \tag{3.15}$$

We take $f(t) = \delta(t)$ and apply the Laplace transform followed by the Hankel transform to boundary conditions (2.10). Then upon using the expressions for $\hat{\phi}, \hat{\psi}$ and $\hat{T}$ from Eqs (3.11) to (3.13) after lengthy but straightforward calculation, the displacements, temperature change and stresses in the transformed domain are obtained as

$$\hat{u} = -\left[q\left(M_1 e^{-\xi_1 z} + M_2 e^{-\xi_2 z}\right) + M_3\xi_3 e^{-\xi_3 z}\right] / \Delta, \tag{3.16}$$

$$\hat{w} = -\left[M_1\xi_1 e^{-\xi_1 z} + M_2\xi_2 e^{-\xi_2 z} + M_3 q e^{-\xi_3 z}\right] \Big/ \Delta\,, \tag{3.17}$$

$$\hat{\tau}_{zz} = p\alpha_1^*\left[F\left(M_1 e^{-\xi_1 z} + M_2 e^{-\xi_2 z}\right) + M_3 F_1 e^{-\xi_3 z}\right] \Big/ \Delta\,, \tag{3.18}$$

$$\hat{\tau}_{rz} = \delta^2 p\alpha_1^*\left[2q\left(\xi_1 M_1 e^{-\xi_1 z} + \xi_2 M_2 e^{-\xi_2 z}\right) + \left(q^2 + \xi_3^2\right) M_3 e^{-\xi_3 z}\right] \Big/ \Delta\,, \tag{3.19}$$

$$\hat{T} = \frac{\delta_0^*}{p\beta_0^*\tau_1}\left[M_1 Q_1 e^{-\xi_1 z} + M_2 Q_2 e^{-\xi_2 z}\right] \Big/ \Delta \tag{3.20}$$

where
$$M_1 = \frac{P_0}{p\alpha_1^*} Q_2\left(q^2 + \xi_3^2\right)\left(h - \xi_2\right),$$

$$M_2 = -\frac{P_0}{p\alpha_1^*} Q_1\left(q^2 + \xi_3^2\right)\left(h - \xi_1\right),$$

$$M_3 = -2q\left(M_1\xi_1 + M_2\xi_2\right) \Big/ \left(q^2 + \xi_3^2\right), \tag{3.21}$$

$$F = \frac{p}{\alpha_1^*} + 2\delta^2 q^2\,, \quad F_1 = 2q\xi_3\delta^2\,, \quad P_0 = \frac{P^*}{2\pi}\,, \tag{3.22}$$

$$\Delta = Q_1\left[\left(q^2 + \xi_3^2\right)F - 2q\xi_2 F_1\right]\left(h - \xi_1\right) - Q_2\left[\left(q^2 + \xi_3^2\right)F - 2q\xi_1 F_1\right]\left(h - \xi_2\right). \tag{3.23}$$

The results for the LS and GL theories can be obtained by setting $k = 1$ and $k = 2$ respectively, in the values of τ_0, τ_0', τ_1 and those for coupled thermoelasticity (CT) can be obtained by taking $t_0 = 0 = t_1$ in the foregoing analysis. Here $h \to 0$ corresponds to the thermally insulated boundary of the halfspace and $h \to \infty$ refers to the isothermal one. The results for uncoupled thermoviscoelasticity can be obtained by setting the thermal coupling parameter $\in = 0$ and thermal relaxation times $t_0 = 0 = t_1$ in the above analysis. The results for non-viscous thermoelastic continuum can be deduced from the above-obtained results by taking $\alpha_0 = \alpha_1 = 0$ in the appropriate relations and functions.

4. Inversion of the transforms

Due to the existence of damping, dependence of roots ξ_i, $(i = 1, 2, 3)$ on the integral transform parameters p and q is complicated; hence the inversion of the integral transform is difficult because the isolation of p is impossible. These difficulties, however, are reduced if we use some approximate or numerical methods. Therefore, in order to obtain the solution of the problem in the physical domain, we must invert the transform in Eqs (3.16) to (3.20) with a numerical technique. These expressions can formally be expressed as functions of z, q and p of the form $\hat{f}(q, z, p)$. First, we invert the Hankel transform, which gives the Laplace transform expression for the function $\bar{f}(r, z, p)$ as

$$\bar{f}(r, z, p) = \int_0^\infty q\hat{f}(q, z, p) J_n(qr)\, dq \tag{4.1}$$

where $n = 1$ in the case of $\bar{u}(q, z, p)$ and $n = 0$ for $\bar{w}(q, z, p)$ as well as $\bar{T}(q, z, p)$. For a fixed value of *q*, *r* and *z*, the function under the integral in Eq.(4.1) can be considered as a Laplace transform $\bar{g}(p)$ of same function $g(t)$. The inversion for the Laplace Transform is given by

$$g(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \bar{g}(p) e^{pt} dp . \tag{4.2}$$

The integral (4.2) can be evaluated by using the numerical technique outlined and used by Sharma and Chauhan (2001). After evaluating the integral (4.2) the next step in the inversion process is to evaluate the integral (4.1). This was done by using Romberg integration with adaptive step-size. This method uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step-size tends to zero, the details of which can be found in the Press *et al*. (1992).

5. Numerical results and discussion

In order to illustrate and compare the theoretical results obtained in the previous sections in the context of the LS, CT, UCT, GL, CT (NV) and UCT (NV) theories of thermoelasticity, we now present some numerical results. The material chosen for the purpose of numerical computations is copper. The physical data for such a material is given as Mukhopadhyay (2000)

$$\lambda_e = 8.2\times 10^{10}\ N/m^2 , \quad \mu_e = 4.2\times 10^{10}\ N/m^2 , \qquad \rho = 8.950\times 10^3\ kgm^{-3} ,$$

$$\alpha_T = 1.0\times 10^{-8}/{}^0K , \quad \in = 0.05 , \quad K = 4.746\times 10^2\ joule/s/m/{}^\circ K , \quad \omega^* = 1.11\times 10^{11}\ s^{-1} ,$$

$$t_0 = 6.131\times 10^{-13}\ s , \qquad t_1 = 8.7565\times 10^{-13}\ s , \qquad \alpha_0 = \alpha_1 = 6.8831\times 10^{-13}\ s .$$

The non dimensional temperature change and stresses are computed from Eqs (3.18) to (3.20) at three different instants of time, viz. $t = 0.1$, 0.25 and 0.5 on the surface $z = 0$ by taking non-dimensional values of thermal and mechanical relaxation times $t_0 = 0.097$, $t_1 = 0.07$, $\alpha_0 = \alpha_1 = 0.08$. Due to the closeness of results and to avoid clustering of different curves, the variations of temperature change and stresses are presented graphically at $t = 0.25$ only in the case of viscous and non-viscous; Lord-Shulman (LS), coupled theory of thermoelasticity (CT), uncoupled theory of thermoelasticity UCT, Green-Lindsay (GL), theories of thermoelasticity. Also the variations of temperature change and stresses are plotted at three different instants of time, viz. $t = 0.1$, 0.25 and 0.5 in the case of coupled thermoelasticity (CT) and uncoupled theory of thermoelasticity (UCT).

From Fig.1, it is noted that the temperature change in the context of LS, GL and CT theories for a viscous half space, though of quite small magnitude, increases instantaneously in the domain $0 \le r \le 0.1$, decreases monotonically in $0.1 \le r \le 0.75$ and ultimately becomes close to zero in an oscillatory manner afterwards. This phenomenon is attributed to compression and expansion of the molecules of the solid due to the application of the load. Initially, the internal friction in viscous medium due to application of an instantaneous normal point load with thermally insulated boundary increases which results in the increase of temperature change and after that there is a rapid decay in temperature change due to a decrease of internal friction in the viscous medium. However, in the case of a viscous solid the temperature change in UCT theory remains almost close to zero because of very small changes in internal friction of the medium. The temperature change for CT (NV) and UCT (NV) shows the opposite trends to that of LS, GL and CT viscous theories. In the case of UCT theory, both for viscous and non-viscous solids, the variations of temperature change are observed to be negligibly small throughout in the case of the insulated boundary under the action of an instantaneous normal point load. The curves in the case of CT (NV) and UCT (NV) here are merged together because of weak thermomechanical coupling effects and that one corresponding to UCT for a viscous solid almost coincides with the radial axis. It is found from Fig.2 that the variations of vertical stress in the context of LS, CT, UCT, GL, CT (NV) and UCT (NV) theories, though the curves are not very clearly distinguishable due to a significantly small effect of relaxation time, first increase monotonically in the domain $0 \le r \le 0.8$, decrease steadily in $0.8 \le r \le 1.35$, and then close to zero in an oscillating manner afterwards. However, the magnitude of vertical stress in the case of CT (NV) and UCT (NV) theories for non-viscous solid half spaces is smaller than that in the case of a viscous one in the context of CT, UCT, LS and GL theories of thermoelasticity. This shows that the effect of mechanical relaxation time parameters on the vertical stress is quite significant whereas thermal relaxation times have a negligibly small effect on this function. The behaviour of this function in the case of isothermal boundary conditions is also observed to be similar.

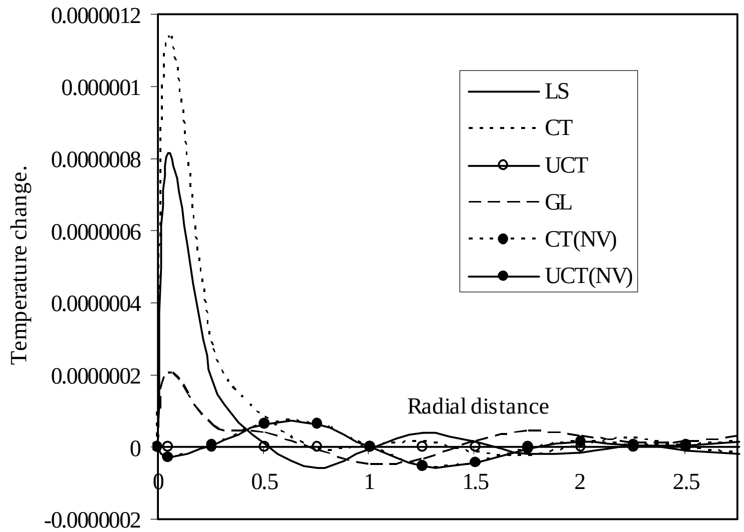

Fig.1. Temperature change with radial distance in the case of insulated boundary $(t = 0.25)$.

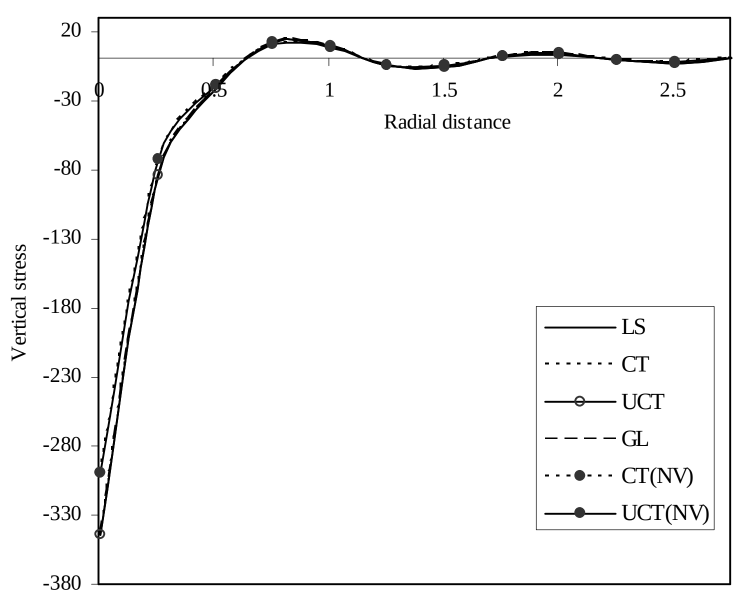

Fig.2. Vertical stress with radial distance in the case of insulated and isothermal boundary $(t = 0.25)$.

It is observed in Fig.3 that the variations of shear stress development in the case of thermally insulated boundary are found to increase rapidly in $0 \le r \le 0.1$, decrease monotonically for $0.1 \le r \le 0.75$ and ultimately tend to zero afterwards in the context of CT (NV), UCT (NV) and UCT theories of thermoelasticity. However, this function in the case of LS, GL and CT theories shows the opposite trends for a viscous solid. The magnitude of shear stress variations has quite small values in the case of a viscous solid as compared to that of a non-viscous one. The effect of thermal relaxation times is quite pertinent on the shear stress. Moreover, the shear stress development is very small as compared to the vertical stress, which is consistent with the boundary conditions. The variations of temperature change, vertical stress and shear stress are respectively plotted in Figs 4, 5 and 6 at different values of time, namely: $t = 0.1$, 0.25 and 0.5 in the context of CT theory of thermoelasticity only. It is noticed from Fig.4 that the magnitudes of variations of temperature change initially and decrease with the passage of time. This is attributed to the instantaneous increase of internal friction at the time of application of the load and then to a decrease in the value of internal friction with the passage of time. This phenomenon results in a decreasing value of the conversion of mechanical energy into thermal one. Moreover, the absolute variation of temperature change is quite small as compared to the vertical stress development as can be seen from Figs 4 and 5. This signifies that only a very small amount of mechanical energy is converted into thermal energy in this case. Figure 5 shows that in the case of viscous and non-viscous solids, the absolute variations of vertical stress in the context of CT and UCT theories of thermoelasticity at three considered instants of time follow similar trends. However, the magnitude of variations of vertical stress at small time is higher than that at large time in the vicinity of the load. It is seen from Fig.6 that in the case of viscous and non-viscous solids the shear stress development has opposite trends at all considered instants of time and dies out in an oscillating fashion as we move away from the vicinity of the load along the radial direction for thermally insulated (CT) and isothermal (CT, UCT) boundaries of the halfspace. The amplitude of vibrations gets suppressed due to the viscous effect of the medium and goes on decreasing with the passage of time. It is observed in Fig.7 that the variations of vertical and shear stresses in the UCT theory, both in viscous and non-viscous solids, follow similar trends and behavior at the considered instants of time as in the case of CT theory in Figs 5 and 6, respectively, with the exception that the shear stress does not follow the opposite trend here.

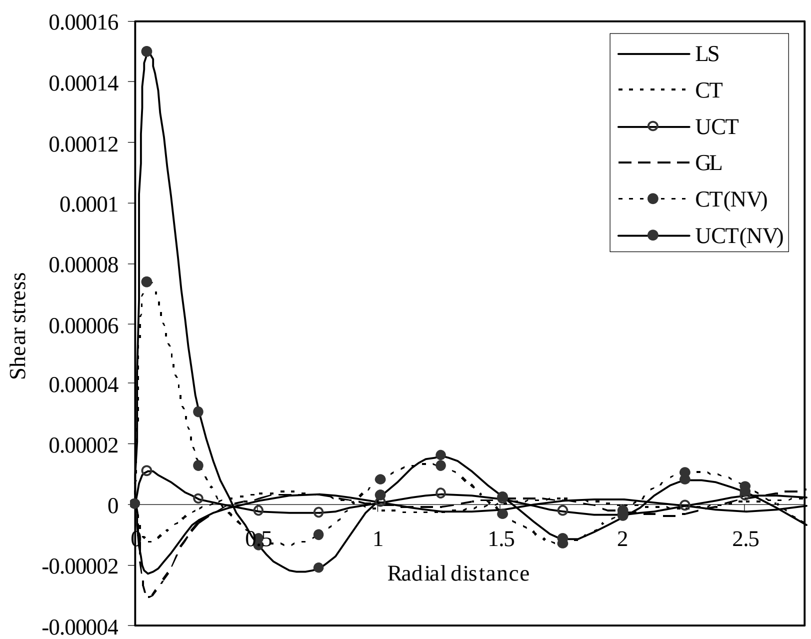

Fig.3. Shear stress with radial distance in the case of insulated boundary $(t = 0.25)$.

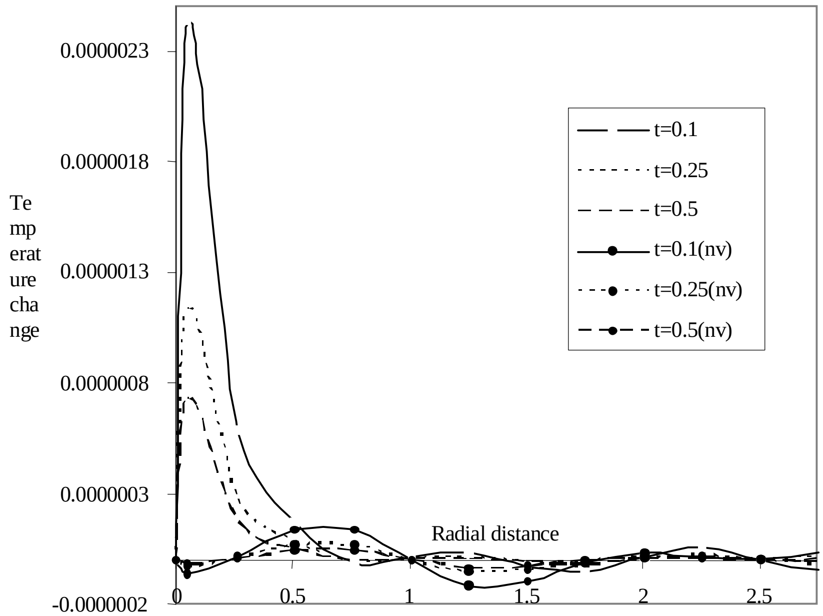

Fig.4. Temperature change with radial distance in the case of insulated boundary (CT).

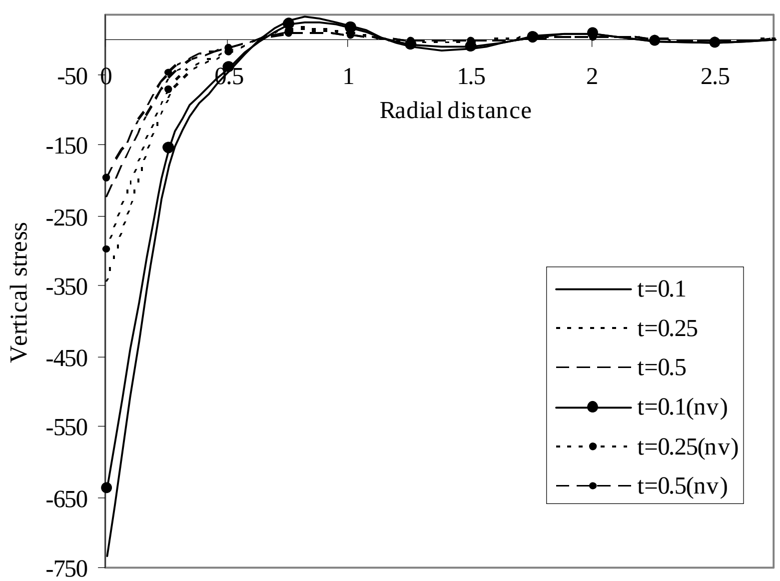

Fig.5. Vertical stress with radial distance in the case of insulated boundary (CT, UCT).

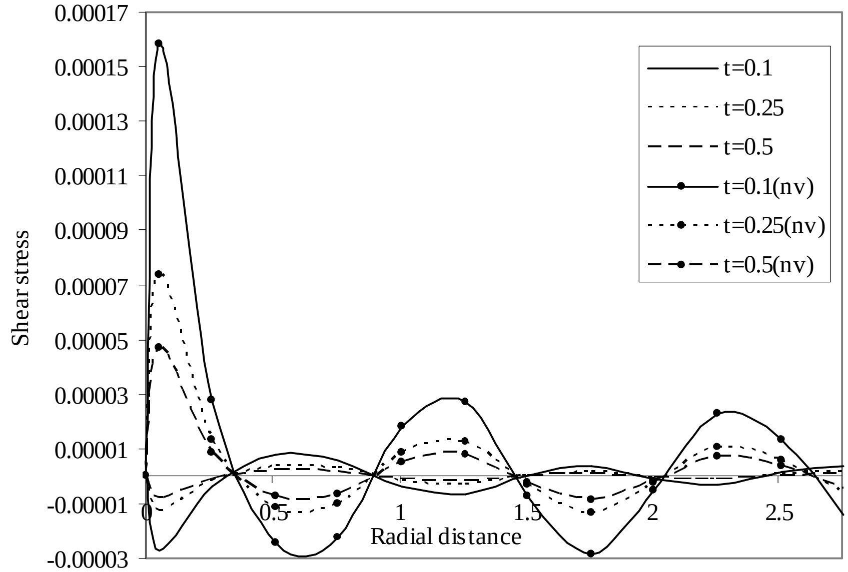

Fig.6. Shear stress with radial distance in the case of insulated (CT) and isothermal (CT, UCT) boundary.

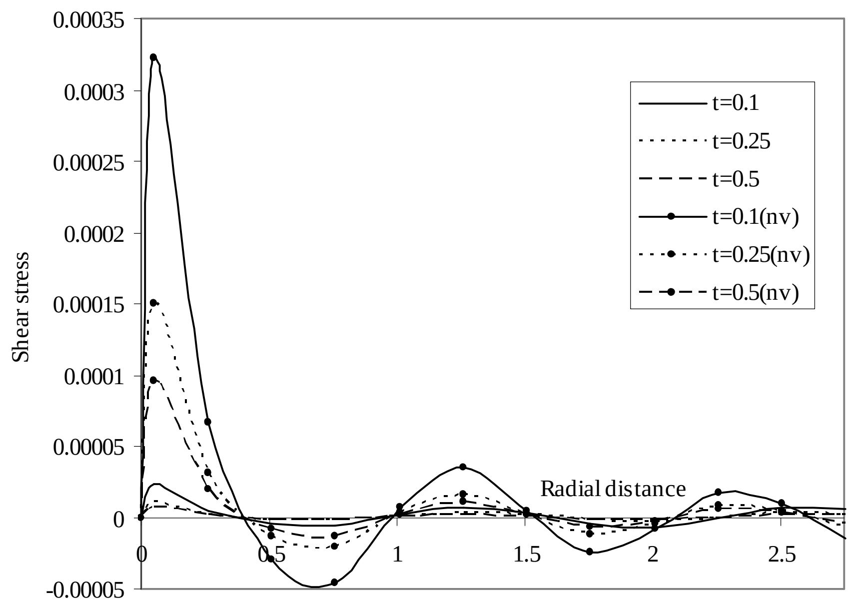

Fig.7. Shear stress with radial distance in the case of insulated boundary (UCT).

It is noticed from Fig.8 that the variations of temperature change in the context of LS, GL, CT and CT(NV) theories, first increase instantaneously in the domain $0 \le r \le 0.1$, decrease rapidly in $0.1 \le r \le 0.3$, slowly and steadily in the domain $0.3 \le r \le 2$, before the temperature creeps along the isothermal boundary under the action of an instantaneous normal point load. The temperature change in the case of UCT and UCT (NV) theories for viscous and non-viscous solids remains zero throughout the isothermal boundary for an instantaneous normal point load. This is also in agreement with the boundary conditions. It is observed from Figs 9 and 10 that the variations of shear stress function in CT (NV), UCT (NV) theories for a viscous/non-viscous medium and GL for a viscous solid first increase rapidly in $0 \leq r \leq 0.1$, decrease monotonically in the domain $0.1 \leq r \leq 0.75$ and then approach zero slowly and steadily afterwards at the isothermal boundary of the solid. Shear stress in the case of GL theory for a viscous medium almost remains unchanged for all values of the radial distance. The variations of shear stress function in the context of LS, CT and UCT theories for viscous solids show the opposite trend to that of non-viscous media. It is noticed from Fig.10 that the magnitude of variations of temperature change at small time is higher than that at large time. From Fig.11 it is observed that when the instantaneous normal point load is applied at the isothermal boundary, the trend and behaviour of variations of vertical stress in CT and UCT theories at different considered values of time are similar to that of vertical stress in Fig.2. It is also noticed that in viscous solids the vertical stress has a small value as compared to that in the non-viscous one.

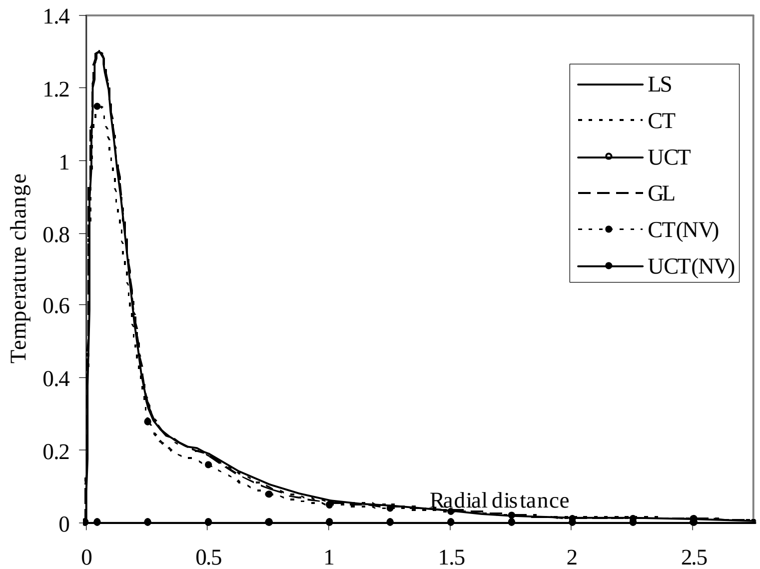

Fig.8. Temperature change with radial distance for isothermal boundary $(t = 0.25)$.

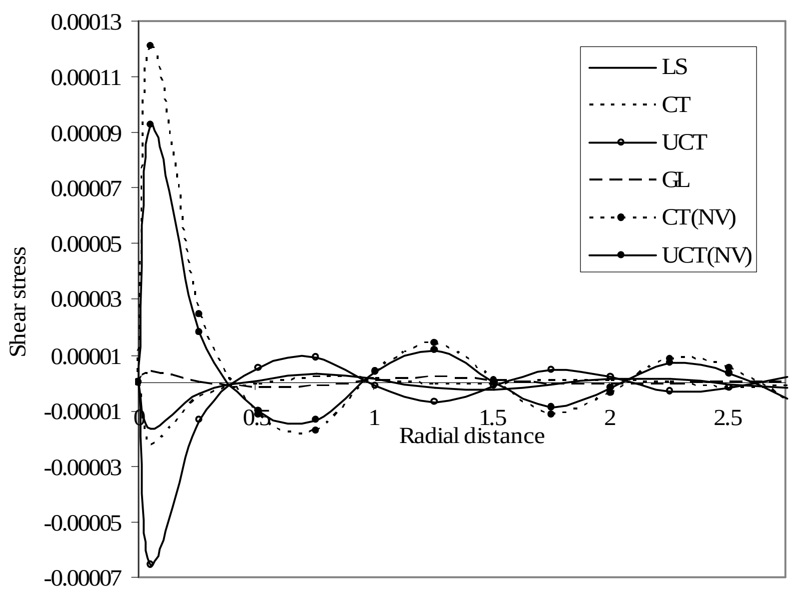

Fig.9. Shear stress with radial distance for isothermal boundary $(t = 0.25)$.

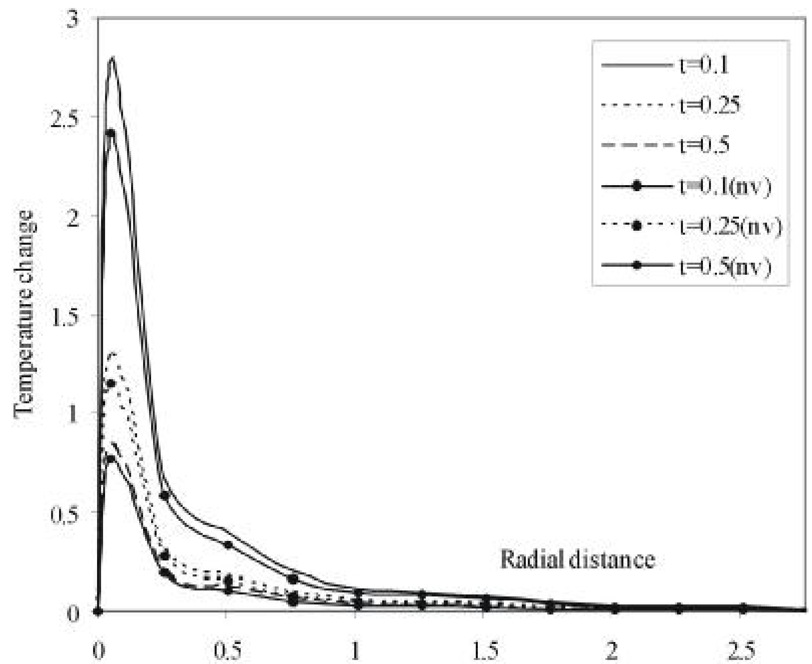

Fig.10. Temperature change with radial distance for an isothermal boundary (CT).

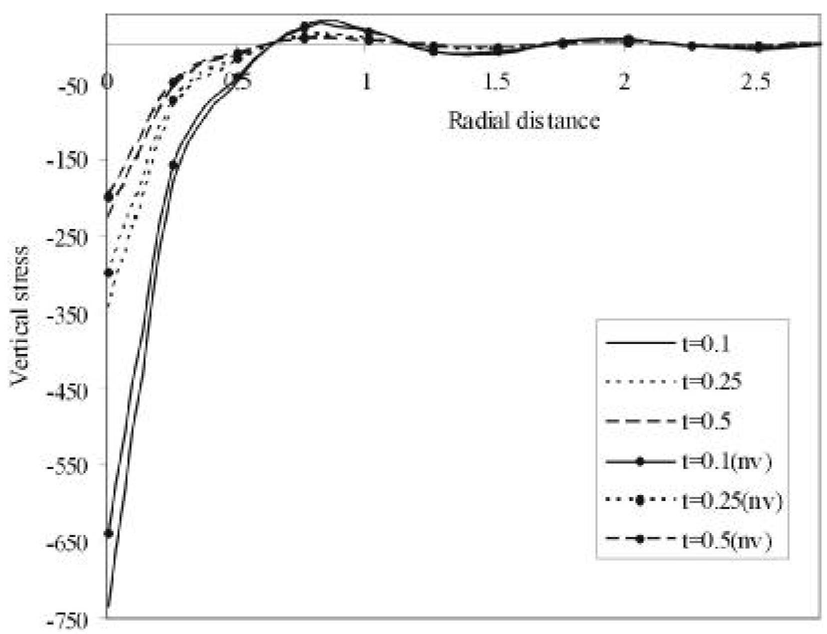

Fig.11. Vertical stress with radial distance for an isothermal boundary (CT, UCT).

In conclusion, all the considered functions are noticed to vanish at certain values of the radial distance approximately near $r = 2.5$, which shows the existence of wave fronts. The impact of thermal relaxation time on various functions is found to be negligibly small, but it is quite significant at small time on the shear stress development and temperature change. This shows that 'second sound' effects are short lived and the thermal wave travels with a finite, though quite large, speed in such solids. The effect of mechanical relaxation time on various considered functions is noticed to be quite significant and due to this resistive phenomenon the amplitude of vibrations decreases and some part of mechanical energy, though small, gets converted to thermal energy due to internal friction among the material particles. In the case of insulated boundary of the solid halfspace, most of the energy is observed to be carried in the form of vertical stress wave and a meager amount propagates in the form of shear stress and thermal waves, which is quite in agreement with the boundary conditions. However, in the case of an isothermal boundary, the significant amount of energy is carried in the form of a thermal wave in addition to vertical stress wave which carries major portion of the energy. The shear stress again carries a negligible small amount of energy in the case of isothermal boundary of the solid. This happens because the boundary of the solid halfspace is free to exchange heat with the surrounding in the isothermal case whereas it is not exposed thermally to the neighbouring environment in the case of thermally insulated boundaries.

Nomenclature

C_e – specific heat at constant strain

$c_1^2 = \dfrac{\lambda_e + 2\mu_e}{\rho}$ – longitudinal wave velocity

$c_2^2 = \dfrac{\mu_e}{\rho}$ – shear wave velocity

h – surface heat transfer coefficient

K – coefficient of thermal conductivity

T_0 – equilibrium temperature prior to the appearance of disturbance

$T(r, z, t)$ – change in temperature of the medium at any time

t_0, t_1 – thermal relaxation times

$u(r, z, t) = (u, 0, w)$ – displacement vectors

α_T – coefficient of linear thermal expansion

α_0, α_1 – mechanical relaxation parameters

$\delta(r)$ – Dirac delta function

δ_{1k} – Kronecker's delta

μ_e, λ_e – Lame's elastic constants

ρ – density

$\phi,\ \psi$ – potential functions

ω – angular frequency

$\omega^* = C_e(\lambda_e + 2\mu_e)/K$ – characteristic frequency of the medium

$\in$ – thermoelastic coupling constant

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