EFFECT OF HEAT AND MASS TRANSFER ON FREE CONVECTION FLOW OVER A CONE WITH UNIFORM SUCTION OR INJECTION IN MICROPOLAR FLUIDS

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A regular perturbation is presented to study the effect of heat and mass transfer on free convection flow with a uniform suction and injection over a cone in a micropolar fluid. The velocity, temperature, concentration and microrotation profiles were computed for various values of suction/injection, Schmidt number and micropolar parameters. The governing equations are first cast into a dimensionless form by a nonsimilar transformation and the resulting equations are then solved numerically by using the Runge-Kutta numerical integration, the procedure in conjunction with the shooting technique. The results indicate that as the micropolar parameter Δ increases the wall couple stress, the shear stress, the Nusselt number and Sherwood number decrease with it. While the Shmidt number increases the shear stress, the wall couple stress and Nusselt number decrease, the opposite is true for the Sherwood number. The results are shown in figures and tables followed by a quantitative discussion.

Key words: free convection, micropolar fluids, heat and mass transfer, suction/injection.

1. Introduction

Many transport processes exist in geophysics, aeronautics, engineering and industrial applications in which the transfer of heat and mass occurs simultaneously. The heat and mass transfer in natural convection along a vertical surface has been studied (Jer-Huan Jang and Wei-Mon Yan, 2004; Schenk *et al*., 1976; Bottemanne, 1972; Chen and Yuh, 1980; Hasan and Mujumdar, 1985). Gebhart and Pera (1971) studied laminar flows which arise in fluids due to the interaction of the force of gravity and density differences caused by the simultaneous action of diffusion of thermal energy and chemical energy. Mollendorf and Gebhart (1974) presented a similarity solution for double diffuse free convection in the axisymmetric case. El-Hakiem *et al*. (2000) studied natural convection with combined thermal and mass diffusion boundary effects in micropolar fluids. Yan (1996) presented a numerical study of mixed convection heat and mass transfer in horizontal rectangular ducts. Soundalgekar (1976) carried out an analysis of the mass transfer effect on the free convective of a viscous fluid past an infinite vertical porous plate. A boundary layer analysis was presented by Mansour *et al*. (1991) for combined heat and mass transfer characteristics of a micropolar fluid flowing past a vertical cylinder. Mansour *et al*. (2000) studied heat and mass transfer in magnetohydrodynamic flow of a micropolar fluid in a circular cylinder with a uniform heat and mass flux.
The effect of suction or injection on a free convection boundary layer flow over a cone was studied by Watanabe (1991).

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In the present work we study the effect of heat and mass transfer boundary layer flow with a uniform suction or injection over a cone in a micropolar fluid. We have reduced the two-dimensional continuity, momentum, angular momentum and energy equations to a system of ordinary differential equations and obtained a numerical solution for the flow, heat and mass transfer characteristics. The governing system of equations is first transformed into a dimensionless form, the resulting equations are then solved by using the Runge-Kutta numerical integration, the procedure in conjunction with the shooting technique. Numerical solutions are obtained for different values of the Schmidt number, suction and injection and micropolar parameters

2. Analysis

Consider a steady free convection boundary layer flow of a micropolar fluid over a cone with uniform suction or injection. The coordinate system is such that x measures the distance along the surface of the body from the appex, $x=0$ being the leading edge, and y measures the distance normally outward. The physical model under consideration and the coordinates chosen are depicted in Fig.1 (note that the injection velocity is perpendicular to the x axis). The body is held at constant temperature T_w greater than the ambient temperature T_∞. The governing conservation equations for the laminar boundary layer flow can be written as:

Mass:

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0. \tag{2.1}$$

Momentum:

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\left(\upsilon+\frac{K}{\rho}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{K}{\rho}\frac{\partial \bar{N}}{\partial \bar{y}}+g^*\left[\beta\left(\bar{T}-\bar{T}_\infty\right)+\beta^*\left(\bar{C}-\bar{C}_\infty\right)\right]\cos\left(\frac{\Omega}{2}\right). \tag{2.2}$$

Angular momentum:

$$\bar{u}\frac{\partial \bar{N}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{N}}{\partial \bar{y}}=\frac{\gamma}{\rho j}\frac{\partial^2 \bar{N}}{\partial \bar{y}^2}-\frac{K}{\rho j}\left[2\bar{N}+\frac{\partial \bar{u}}{\partial \bar{y}}\right]. \tag{2.3}$$

Energy:

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}=\frac{k}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}. \tag{2.4}$$

Concentration:

$$\bar{u}\frac{\partial \bar{C}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}}=\frac{D}{\rho C_p}\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}. \tag{2.5}$$

In the above equations u and v are the components of fluid velocity in the x and y directions respectively, N – the component of microrotation, and T – the component of temperature and C is the fluid concentration.

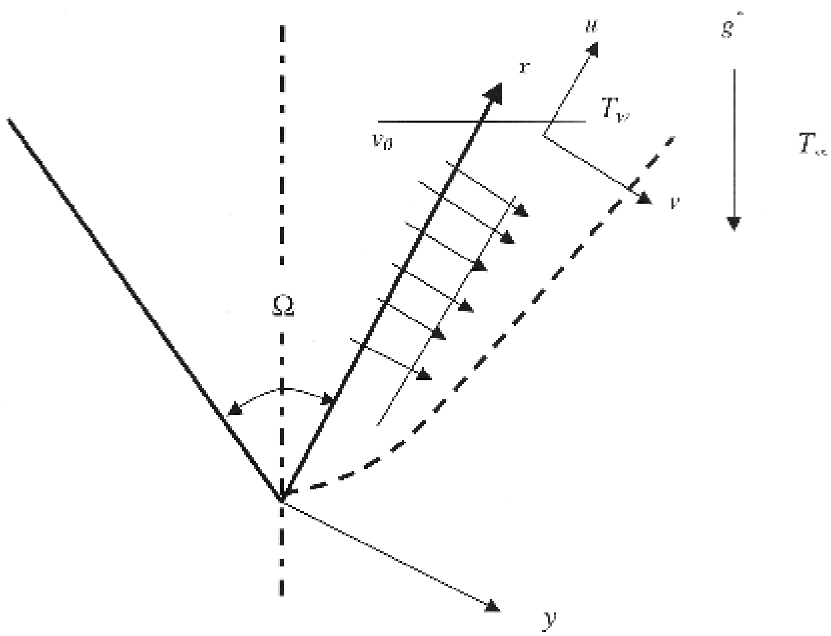

Fig.1.Physical model and co-ordinate system.

The boundary conditions of the problem are

$$\bar{y}=0: \quad \bar{u}=0, \quad \bar{v}=\bar{v}_0(const), \quad \bar{T}=\bar{T}_w, \quad \bar{C}=\bar{C}_w, \quad g(0)=-nf''(0),$$
$$\bar{y}\to\infty: \quad \bar{u}=0, \quad \bar{T}=\bar{T}_\infty, \quad \bar{C}=\bar{C}_\infty, \quad g(\infty)=0. \tag{2.6}$$

Where

$$\bar{x}=\frac{1}{L^2}\int_0^x R^2 dx\,, \qquad \bar{y}=\frac{R}{L}y\,, \qquad \bar{u}=u\,, \qquad \bar{N}=N\,,$$
$$\bar{v}=\frac{R}{L}\left(v+\frac{1}{R}\frac{dR}{dx}yu\right), \qquad \bar{T}=T\,, \qquad \bar{C}=C\,. \tag{2.7}$$

The coefficient of volume expansion β is replaced by $1/\bar{T}_\infty$ and β^* is replaced by $1/\bar{C}_\infty$. Next the following transformations are introduced to obtain the equations in terms of the generalized stream, temperature, concentration and microrotation functions.

A comment on the boundary condition used for the microrotation term will be made here. When $n=0$, we obtain from the boundary condition stated in Eq.(2.6) for the microrotation, $g(0)=0$. This represents the case of concetrated particle flows in which the microlements close to the wall are not able to rotate (Jena and Mathur, 1982). The case corresponding to $n=1/2$ results in the vanishing of the antisymmetric part of the stress tensor and represents weak concentrations (Ahmadi, 1976). Ahmadi suggested that the particle spin is equal to the fluid vorticity at the boundary for fine particle suspensions. As suggested by Peddieson (1972), Thus, for $n = 0$, particles are not free to rotate near the surface. Whereas, as $n = 0.5$ the micrortation term gets augmented and induces flow enhancement.

$$\eta = \frac{\bar{y}}{\bar{x}}\left(\frac{m+3}{6}\right)^{\frac{1}{2}}\left(\frac{\mathrm{Gr}_T}{4}\right)^{\frac{1}{4}}, \tag{2.8a}$$

$$\psi(\bar{x},\eta) = 4\upsilon\left(\frac{6}{m+3}\right)^{\frac{1}{2}}\left(\frac{\mathrm{Gr}_T}{4}\right)^{\frac{1}{4}} f(\bar{x},\eta), \tag{2.8b}$$

$$\phi(\bar{x},\eta) = \frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty}, \tag{2.8c}$$

$$\theta(\bar{x},\eta) = \frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty}, \tag{2.8d}$$

$$\bar{N}(\bar{x},\eta) = \frac{U_c}{x}\left(\frac{m+3}{6}\right)^{\frac{1}{2}}\left(\frac{\mathrm{Gr}_T}{4}\right)^{\frac{1}{4}} g(\bar{x},\eta) \tag{2.8e}$$

where
$$\mathrm{Gr}_T = \frac{g^*\left(\bar{T}_w-\bar{T}_\infty\right)}{\upsilon^2\bar{T}_\infty}\bar{x}^3 \tag{2.9a}$$

and
$$\mathrm{Gr}_C = \frac{g^*\left(\bar{C}_w-\bar{C}_\infty\right)}{\upsilon^2\bar{C}_\infty}\bar{x}^3 \tag{2.9b}$$

where the parameter m depends solely on the cone angle Ω. The relation between m and Ω was shown by Whitehead and Canetti (1950), and the tabulated values m and Ω were given by Hess and Faulkner (1956).

The component of the velocity can be expressed as

$$\bar{u} = U_c\frac{\partial f}{\partial \eta}, \tag{2.10a}$$

$$\bar{v} = -4\upsilon\left(\frac{6}{m+3}\right)^{\frac{1}{2}}\left(\frac{\mathrm{Gr}}{4}\right)^{\frac{1}{4}}\left(\frac{1}{4}\frac{1}{\mathrm{Gr}_T}\frac{d\mathrm{Gr}_T}{d\bar{x}}f + \frac{\partial f}{\partial \bar{x}} + \frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial \bar{x}}\right) \tag{2.10b}$$

where $U_c = \frac{4\upsilon\left(\mathrm{Gr}_T/4\right)^{\frac{1}{2}}}{\bar{x}}$ is the reference conductive velocity.

$$(1+\Delta)f''' + \Delta g' - \left(\frac{6}{m+3}\right)\left[3ff'' - 2f'^2 + (\phi + A\theta)\cos\left(\frac{\Omega}{2}\right)\right] = \\ = \left(\frac{24}{m+3}\right)\left(f'\frac{\partial f'}{\partial \bar{x}} - f''\frac{\partial f}{\partial \bar{x}}\right)\bar{x}, \tag{2.11}$$

$$\lambda g'' - \Delta B\left(\frac{6}{m+3}\right)(2g + f'') + \left(\frac{6}{m+3}\right)\{3g'f - f'g\} = \\ = \left(\frac{24}{m+6}\right)\left(f'\frac{\partial g}{\partial \bar{x}} - g'\frac{\partial f}{\partial \bar{x}}\right)\bar{x}, \tag{2.12}$$

$$\frac{1}{\mathrm{Pr}}\phi'' + \left(\frac{18}{m+3}\right)f\phi' = \left(\frac{24}{m+3}\right)\left(f'\frac{\partial \phi}{\partial \bar{x}} - \phi'\frac{\partial f}{\partial \bar{x}}\right)\bar{x}, \tag{2.13}$$

$$\frac{1}{\mathrm{Sc}}\theta'' + \left(\frac{18}{m+3}\right)f\theta' = \left(\frac{24}{m+3}\right)\left(f'\frac{\partial \theta}{\partial \bar{x}} - \theta'\frac{\partial f}{\partial \bar{x}}\right)\bar{x}. \tag{2.14}$$

We define the following dimensionless parameters as

$$A = \frac{\beta^*(C_w - C_\infty)}{\beta(T_w - T_\infty)} = \frac{\mathrm{Gr}_c}{\mathrm{Gr}_T}, \qquad \Delta = \frac{K}{\rho\upsilon}, \\ \lambda = \frac{\gamma}{\rho j}, \qquad B = \frac{4\upsilon\bar{x}}{jU_c}, \qquad \mathrm{Pr} = \frac{\upsilon}{k\rho C_p}, \qquad \mathrm{Sc} = \frac{\upsilon}{D\rho C_p}. \tag{2.15}$$

The boundary conditions

$$\eta = 0 :\ f'(0) = 0, \qquad \frac{1}{4}\frac{d\mathrm{Gr}_T}{d\bar{x}}\frac{\bar{x}}{\mathrm{Gr}_T}f + \bar{x}\frac{\partial f}{\partial \bar{x}} = s, \qquad g(0) = -nf''(0), \qquad \phi(0) = 1, \qquad \theta(0) = 1, \\ \eta \to \infty \qquad f'(\infty) = 0, \qquad g(\infty) = 0, \qquad \phi(0) = 0, \qquad \theta(\infty) = 0 \tag{2.16}$$

where s is the parameter of suction or injection which is defined by

$$s = -\left(\frac{m+3}{6}\right)^{1/2}\frac{\bar{v}_0}{4}\frac{\bar{x}}{\nu}\left(\frac{\mathrm{Gr}_T}{4}\right)^{-1/4} \tag{2.17}$$

where $\bar{v}_0$ is the velocity of suction or injection, respectively. We see from the above equation the case of suction corresponds to $s > 0$ and the case of injection to $s < 0$. We approximate the partial differential Eqs (2.11)-(2.14) and (2.16) to a system of ordinary differential equations replacing the $\bar{x}$ derivative by a finite difference. For convenience, we put $x^* = k\bar{x}^{-1/4}(\equiv s)$, then Eqs (2.11)-(2.14) and (2.16) become

$$(1+\Delta)f''' + \Delta g' + \left(\frac{6}{m+3}\right)\left[3ff'' - 2f'^2 + (\phi + A\theta)\cos\left(\frac{\Omega}{2}\right)\right\} = \\ = \left(\frac{6}{m+3}\right)\left(f'\frac{\partial f'}{\partial x^*} - f''\frac{\partial f}{\partial x^*}\right)x^*, \tag{2.18}$$

$$\lambda g'' - \Delta B x^*\left(\frac{6}{m+3}\right)(2g + f'') - \left(\frac{6}{m+3}\right)\{3g'f - f'g\} = \\ = \left(\frac{6}{m+6}\right)\left(f'\frac{\partial g}{\partial x^*} - g'\frac{\partial f}{\partial x^*}\right)x^*, \tag{2.19}$$

$$\frac{1}{\Pr}\phi'' + \left(\frac{18}{m+3}\right)f\phi' = \left(\frac{6}{m+3}\right)\left(f'\frac{\partial\phi}{\partial x^*} - \phi'\frac{\partial f}{\partial x^*}\right)x^*, \tag{2.20}$$

$$\frac{1}{\mathrm{Sc}}\theta'' + \left(\frac{18}{m+3}\right)f\theta' = \left(\frac{6}{m+3}\right)\left(f'\frac{\partial\theta}{\partial x^*} - \theta'\frac{\partial f}{\partial x^*}\right)x^*. \tag{2.21}$$

$$\eta = 0 : f'(0) = 0, \quad \frac{1}{4}\frac{d\mathrm{Gr}_T}{dx^*}\frac{x^*}{\mathrm{Gr}_T}f + x^*\frac{\partial \mathrm{f}}{\partial x^*} = x^*, \quad g(0) = -nf''(0), \quad \phi(0) = 1, \quad \theta(0) = 1, \\ \eta \to \infty \quad f'(\infty) = 0, \quad g(\infty) = 0, \quad \phi(\infty) = 0, \quad \theta(\infty) = 0. \tag{2.22}$$

Expansions for the stream, microrotation, temperature and concentration functions $f(x,\eta)$, $g(x,\eta)$, $\phi(x,\eta)$ and $\theta(x,\eta)$ are postulated as

$$f(\xi,\chi) = f_0(\eta) + x^* f_1(\eta) + x^{*2} f_2(\eta) + \dots, \tag{2.23a}$$

$$g(\chi,\eta) = g_0(\eta) + x^* g_1(\eta) + x^{*2} g_2(\eta) + \dots, \tag{2.23b}$$

$$\phi(\chi,\eta) = \phi_0(\eta) + x^*\phi_1(\eta) + x^{*2}\phi_2(\eta) + \dots, \tag{2.23c}$$

$$\theta(\chi,\eta) = \theta_0(\eta) + x^*\theta_1(\eta) + x^{*2}\theta_2(\eta) + \dots. \tag{2.23d}$$

Substituting (2.23a), (2.23b), (2.23c) and (2.23d) into Eqs (2.18)-(2.21) and (2.22) and comparing the term of equal power of yields the following set of ordinary differential equations governing the momentum, angular momentum and energy fields

$$(1+\Delta)f_0''' + \Delta g_0' + \frac{6}{m+3}\left[3f_0 f_0'' - 2f_0'^2 + (\phi_0 + A\theta_0)\cos\left(\frac{\Omega}{2}\right)\right] = 0, \tag{2.24a}$$

$$\lambda g_0'' + \frac{6}{m+3}[3f_0 g_0' - f_0' g_0] = 0, \tag{2.24b}$$

$$\frac{1}{\mathrm{Pr}}\phi_0'' + \frac{6}{m+3}\left(3 f_0 \phi_0'\right) = 0\,, \tag{2.24c}$$

$$\frac{1}{\mathrm{Sc}}\theta_0'' + \frac{6}{m+3}\left(3 f_0 \theta_0'\right) = 0\,, \tag{2.24d}$$

with the boundary conditions

$$\begin{aligned}
&\eta = 0 \quad : \quad f_0(0) = 0, \quad f_0'(0) = 0, \quad g_0(0) = -n f_0''(0), \quad \phi_0(0) = 1, \quad \theta_0(0) = 1, \\
&\eta \to \infty \quad f_0'(\infty) = 0, \quad g_0(\infty) = 0, \quad \phi_0(\infty) = 0\,, \quad \theta_0(\infty) = 0.
\end{aligned} \tag{2.25}$$

$$(1+\Delta) f_1''' + \Delta g_1' + \frac{6}{m+3}\left[3 f_0 f_1'' + 4 f_1 f_0'' - 5 f_0' f_1' + (\phi_1 + A\theta_1)\cos\left(\frac{\Omega}{2}\right)\right] = 0\,, \tag{2.26a}$$

$$\lambda g_1'' + \frac{6}{m+3}\left[3 f_0 g_1' + 4 f_1 g_0' - 2 f_0' g_1 - f_1' g_0\right] = 0\,, \tag{2.26b}$$

$$\frac{1}{\mathrm{Pr}}\phi_1'' + \frac{6}{m+3}\left(3 f_0 \phi_1' + 4 f_1 \phi_0' - f_0' \phi_1\right) = 0\,, \tag{2.26c}$$

$$\frac{1}{\mathrm{Sc}}\theta_1'' + \frac{6}{m+3}\left(3 f_0 \theta_1' + 4 f_1 \theta_0' - f_0' \theta_1\right) = 0\,, \tag{2.26d}$$

with the boundary conditions

$$\begin{aligned}
&\eta = 0 \quad : \quad f_1(0) = 1, \quad f_1'(0) = 0, \quad g_1(0) = -n f_1''(0), \quad \phi_1(0) = 0, \quad \theta_1(0) = 0, \\
&\eta \to \infty \quad f_1'(\infty) = 0, \quad g_1(\infty) = 0, \quad \phi_1(\infty) = 0\,, \quad \theta_1(\infty) = 0.
\end{aligned} \tag{2.27}$$

$$(1+\Delta) f_2''' + \frac{6}{m+3}\left[3 f_0 f_2'' + 5 f_2 f_0'' - 6 f_0' f_2' + 4 f_1 f_1'' - 3 f_1'^2 + (\phi_2 + A\theta_2)\cos\left(\frac{\Omega}{2}\right)\right] + \Delta g' = 0 \tag{2.28a}$$

$$\lambda g_2'' + \frac{6}{m+3}\left[3 f_0 g_2' + 5 f_2 g_0' - 3 f_0' g_2 - f_2' g_0 + 4 f_1 g_1' - 2 f_1' g_1 - \Delta B\left(f_0'' + 2 g_0\right)\right] = 0\,, \tag{2.28b}$$

$$\frac{1}{\mathrm{Pr}}\phi_2'' + \frac{6}{m+3}\left(3 f_0 \phi_2' + 5 f_2 \phi_0' - 2 f_0' \phi_2 + 4 f_1 \phi' \theta_1' - f_1' \phi_1\right) = 0\,, \tag{2.28c}$$

$$\frac{1}{\mathrm{Sc}}\theta_2'' + \frac{6}{m+3}\left(3 f_0 \theta_2' + 5 f_2 \theta_0' - 2 f_0' \theta_2 + 4 f_1 \theta_1' - f_1' \theta_1\right) = 0\,, \tag{2.28d}$$

with the boundary conditions

$$\eta = 0 \quad : \quad f_2(0) = 0, \quad f_2'(0) = 0, \quad g_2(0) = -nf_2''(0), \quad \phi_2(0) = 0, \quad \theta_2(0) = 0,$$
$$\eta \to \infty \quad f_2'(\infty) = 0, \quad g_2(\infty) = 0, \quad \phi_2(\infty) = 0, \quad \theta_2(\infty) = 0. \tag{2.29}$$

The most important results are the local wall shear stress τ_w, local rate of heat transfer q_w, local rate of mass transfer m_w and local wall couple stress M_w which may be written as

$$\tau_w = \left[(\mu + K) \frac{\partial \bar{u}}{\partial \bar{y}} + K\bar{N} \right]_{\bar{y}=0} = \frac{4\upsilon^2 \rho}{\bar{x}^2} \left(\frac{m+3}{6} \right)^{1/2} \left(\frac{\text{Gr}}{4} \right)^{3/4} [1 + \Delta(1-n)] f''(0), \tag{2.30}$$

$$q_w = -k \left(\frac{\partial \bar{T}}{\partial \bar{y}} \right)_{\bar{y}=0} = -\frac{k}{\bar{x}} (\bar{T}_w - \bar{T}_\infty) \left(\frac{m+3}{6} \right)^{1/2} \left(\frac{\text{Gr}}{4} \right)^{1/4} \phi'(0), \tag{2.31}$$

$$m_w = -D \left(\frac{\partial \bar{C}}{\partial \bar{y}} \right)_{\bar{y}=0} = -\frac{D}{\bar{x}} (\bar{C}_w - \bar{C}_\infty) \left(\frac{m+3}{6} \right)^{1/2} \left(\frac{\text{Gr}}{4} \right)^{1/4} \theta'(0), \tag{2.32}$$

$$M_w = \gamma \left(\frac{\partial \bar{N}}{\partial \bar{y}} \right)_{\bar{y}=0} = \frac{4\gamma\upsilon}{\bar{x}^3} \left(\frac{m+3}{6} \right) \left(\frac{\text{Gr}}{4} \right) g'(0). \tag{2.33}$$

We define the Nusselt number Nu, Sherwood number Sh and heat transfer rate h as

$$\text{Nu} = \frac{q_w \bar{x}}{k(\bar{T}_w - \bar{T}_\infty)}, \tag{2.34}$$

$$\text{Sh} = \frac{m_w \bar{x}}{D(\bar{C}_w - \bar{C}_\infty)}, \tag{2.35}$$

$$h = \frac{q_w}{\bar{T}_w - \bar{T}_\infty}. \tag{2.36}$$

3. Results and discussion

The system of Eqs (2.24), (2.26), (2.28) with the boundary conditions (2.25), (2.27), (2.29) respectively was solved numerically by the fourth order Runge-Kutta integration scheme. Calculations carried out for the indicated values of the Prandtl number Pr, Schmidt number Sc, the micropolar parameter Δ, suction/injection parameter s and the relative effect of mass and thermal diffusion A, are summarized with $\text{Pr} = 0.73$, $B = 0.1$, $m = 0.11556$ and $\lambda = 0.5$.

Tables 1-4 show the results for the constant wall temperature and uniform suction or injection over a cone which show the surface values of velocity, temperature, concentration and the microrotation gradient components, which are proportional to the friction factor, Nusselt number, Sherwood number and the wall couple stress respectively.

These data will facilitate the computation of the friction factor, surface heat and mass transfer rate and the surface couple stress for various values of suction/injection parameter.

Table 1. Values of $f''(0)$, $-\phi'(0)$, $-\theta'(0)$ and $-g'(0)$ with $A=-0.1$ and $n=0.0$.

s	Sc	Δ	$f''(0)$	$-\phi'(0)$	$-\theta'(0)$	$-g'(0)\times 10^2$
-0.2	0.73	0.0	0.60576	0.24895	0.24895	0.00000
		0.5	0.52107	0.22937	0.22937	0.19588
		1.5	0.41509	0.20272	0.20272	0.50032
		5.0	0.26292	0.15680	0.15680	1.20572
	4.0	0.0	0.59720	0.24729	0.38334	0.00000
		0.5	0.51374	0.22700	0.49124	0.20236
		1.5	0.40892	0.19937	0.65510	0.51704
		5.0	0.25803	0.15147	1.03805	1.24804
-0.1	0.73	0.0	0.70729	0.42135	0.42135	0.00000
		0.5	0.56995	0.38924	0.38924	0.04897
		1.5	0.42667	0.34670	0.34670	0.12508
		5.0	0.25048	0.27138	0.27138	0.30143
	4.0	0.0	0.71272	0.42703	0.34185	0.00000
		0.5	0.57443	0.39426	0.29710	0.05059
		1.5	0.42994	0.35088	0.24989	0.12926
		5.0	0.25204	0.27414	0.20275	0.31201
0.0	0.73	0.0	0.77645	0.66125	0.66125	0.00000
		0.5	0.59469	0.62308	0.62308	0.00000
		1.5	0.42238	0.57293	0.57293	0.00000
		5.0	0.23066	0.48397	0.48397	0.00000
	4.0	0.0	0.79665	0.67211	1.41365	0.00000
		0.5	0.61081	0.63328	1.30700	0.00000
		1.5	0.43438	0.58233	1.17857	0.00000
		5.0	0.23769	0.49217	0.97522	0.00000
0.1	0.73	0.0	0.81325	0.96864	0.96864	0.00000
		0.5	0.59528	0.93088	0.93088	0.04897
		1.5	0.40222	0.88141	0.88141	0.12508
		5.0	0.20345	0.79457	0.79457	0.30143
	4.0	0.0	0.84898	0.98254	3.59875	0.00000
		0.5	0.62288	0.94405	3.52093	0.05059
		1.5	0.42223	0.89371	3.44114	0.12926
		5.0	0.21497	0.80556	3.35546	0.31201
0.2	0.73	0.0	0.81768	1.34353	1.34353	0.00000
		0.5	0.57172	1.31264	1.31264	0.19588
		1.5	0.36620	1.27214	1.27214	0.50032
		5.0	0.16884	1.20317	1.20317	1.20572
	4.0	0.0	0.86972	1.35832	6.89714	0.00000
		0.5	0.61064	1.32658	6.93890	0.20236
		1.5	0.39348	1.28502	7.03759	0.51704
		5.0	0.18389	1.21431	7.34346	1.24804

Table 2. Values of $f''(0)$, $-\phi'(0)$, $-\theta'(0)$ and $-g'(0)$ with $A=1.0$ and $n=0.0$.

s	Sc	Δ	$f''(0)$	$-\phi'(0)$	$-\theta'(0)$	$-g'(0)\times 10^2$
-0.2	0.73	0.0	1.17624	0.37063	0.37063	0.00000
		0.5	0.98570	0.34031	0.34031	0.29212
		1.5	0.76662	0.29977	0.29977	0.74712
		5.0	0.47314	0.22966	0.22966	1.82196
	4.0	0.0	1.20102	0.36377	0.43001	0.00000
		0.5	1.00752	0.33800	0.49913	0.24264
		1.5	0.78648	0.30346	0.61359	0.62000
		5.0	0.49158	0.24391	0.90577	1.50140
-0.1	0.73	0.0	1.31453	0.56136	0.56136	0.00000
		0.5	1.04884	0.52028	0.52028	0.07303
		1.5	0.77754	0.46620	0.46620	0.18678
		5.0	0.45180	0.37167	0.37167	0.45549
	4.0	0.0	1.24975	0.51850	0.51839	0.00000
		0.5	0.99575	0.48209	0.45147	0.06066
		1.5	0.73818	0.43379	0.37754	0.15500
		5.0	0.43085	0.34851	0.28701	0.37535
0.0	0.73	0.0	1.41330	0.80737	0.80737	0.00000
		0.5	1.08249	0.76082	0.76082	0.00000
		1.5	0.76911	0.69995	0.69995	0.00000
		5.0	0.42153	0.59365	0.59365	0.00000
	4.0	0.0	1.24243	0.73501	1.58874	0.00000
		0.5	0.94593	0.69322	1.46695	0.00000
		1.5	0.66744	0.63803	1.32076	0.00000
		5.0	0.36185	0.54018	1.09117	0.00000
0.1	0.73	0.0	1.47256	1.10865	1.10865	0.00000
		0.5	1.08666	1.06192	1.06192	0.07303
		1.5	0.74133	1.00103	1.00103	0.18678
		5.0	0.38234	0.89561	0.89561	0.45549
	4.0	0.0	1.17905	1.01330	3.64107	0.00000
		0.5	0.85806	0.97140	3.54555	0.06066
		1.5	0.57426	0.91619	3.44325	0.15500
		5.0	0.28457	0.81894	3.31826	0.37535
0.2	0.73	0.0	1.49230	1.46521	1.46521	0.00000
		0.5	1.06134	1.42359	1.42359	0.29212
		1.5	0.69420	1.36943	1.36943	0.74712
		5.0	0.33422	1.27756	1.27756	1.82196
	4.0	0.0	1.05962	1.35336	6.67538	0.00000
		0.5	0.73213	1.31663	6.68729	0.24264
		1.5	0.45863	1.26826	6.74500	0.62000
		5.0	0.19901	1.18477	6.96828	1.50140

Table 3. Values of $f''(0)$, $-\phi'(0)$, $-\theta'(0)$ and $-g'(0)$ with $A=-0.1$ and $n=0.5$.

s	Sc	Δ	$f''(0)$	$-\phi'(0)$	$-\theta'(0)$	$g'(0)$
-0.2	0.73	0.5	0.52300	0.24224	0.24224	0.09079
		1.5	0.43369	0.21671	0.21671	0.01797
		5.0	0.30301	0.16333	0.16333	0.00542
	4.0	0.5	0.51981	0.23841	0.44429	0.02502
		1.5	0.42862	0.21343	0.14579	0.02210
		5.0	0.30089	0.16952	0.08910	0.01556
-0.1	0.73	0.5	0.60797	0.40459	0.40459	0.16318
		1.5	0.48880	0.37874	0.37874	0.09785
		5.0	0.32275	0.32024	0.32024	0.06543
	4.0	0.5	0.61421	0.40932	0.31508	0.14239
		1.5	0.49421	0.38302	0.17781	0.10108
		5.0	0.32655	0.32553	0.00475	0.04576
0.0	0.73	0.5	0.65259	0.63993	0.63993	0.38500
		1.5	0.51114	0.60616	0.60616	0.28130
		5.0	0.31559	0.53301	0.53301	0.15000
	4.0	0.5	0.67071	0.65014	1.34627	0.40106
		1.5	0.52631	0.61560	1.25289	0.29357
		5.0	0.32583	0.54133	1.07944	0.15702
0.1	0.73	0.5	0.65687	0.94827	0.94827	0.75626
		1.5	0.50072	0.89896	0.89896	0.56833
		5.0	0.28151	0.80163	0.80163	0.30914
	4.0	0.5	0.68932	0.96086	3.53785	0.78028
		1.5	0.52493	0.91118	3.37103	0.59956
		5.0	0.29872	0.81691	3.13496	0.30877
0.2	0.73	0.5	0.62080	1.32960	1.32960	1.27695
		1.5	0.45754	1.25714	1.25714	0.95892
		5.0	0.22053	1.12610	1.12610	0.54285
	4.0	0.5	0.67003	1.34149	6.88982	1.28005
		1.5	0.49007	1.26976	6.53223	1.01906
		5.0	0.24523	1.15227	6.17132	0.50100

Table 4. Values of $f''(0)$, $-\phi'(0)$, $-\theta'(0)$ and $-g'(0)$ with $A = 0.1$ and $n = 0.5$.

s	Sc	Δ	$f''(0)$	$-\phi'(0)$	$-\theta'(0)$	$g'(0)$
-0.2	0.73	0.5	1.01179	0.35007	0.35007	0.16673
		1.5	0.83523	0.32584	0.32584	0.13478
		5.0	0.52009	0.23983	0.23983	0.04485
	4.0	0.5	1.02861	0.35045	0.53664	0.07206
		1.5	0.78192	0.33042	0.39151	0.04567
		5.0	0.42749	0.27041	0.12563	0.01525
-0.1	0.73	0.5	1.11952	0.53840	0.53840	0.37085
		1.5	0.89839	0.50615	0.50615	0.27588
		5.0	0.56088	0.42357	0.42357	0.12087
	4.0	0.5	1.05571	0.50119	0.49212	0.31092
		1.5	0.82267	0.47430	0.38056	0.21753
		5.0	0.49673	0.40785	0.17038	0.10062
0.0	0.73	0.5	1.18788	0.78139	0.78139	0.85571
		1.5	0.93072	0.74046	0.74046	0.62555
		5.0	0.57710	0.65339	0.65339	0.33572
	4.0	0.5	1.03394	0.71417	1.51347	0.69210
		1.5	0.80166	0.67904	1.40838	0.50092
		5.0	0.48978	0.60019	1.21342	0.26463
0.1	0.73	0.5	1.21689	1.07903	1.07903	1.62131
		1.5	0.93223	1.02878	1.02878	1.18380
		5.0	0.56875	0.92930	0.92930	0.68941
	4.0	0.5	0.96329	0.98938	3.60068	1.21561
		1.5	0.71889	0.94463	3.47496	0.89585
		5.0	0.40664	0.84743	3.25474	0.50727
0.2	0.73	0.5	1.20654	1.43131	1.43131	2.66765
		1.5	0.90291	1.37110	1.37110	1.95062
		5.0	0.53583	1.25129	1.25129	1.18195
	4.0	0.5	0.84376	1.32683	6.75377	1.88144
		1.5	0.57437	1.27108	6.58031	1.40232
		5.0	0.24731	1.14956	6.29434	0.82854

The results indicate that as the micropolar parameter Δ increases the wall couple stress, the shear stress, Nusselt number and Sherwood number decrease with it. We notice that as the Shmidt number increases the shear stress, wall couple stress and Nusselt number decrease, the opposite is the case for the Sherwood number. We also note that both the friction factor, Nusselt number, Sherwood number and the wall couple stress increase as the buoyancy parameter A increases.

The results are given for velocity, temperature, concentration, microrotation distributions, skin friction, heat and mass transfer for various values of suction/injection and micropolar parameters.

Figures 2-5 show the effects of suction/injection, the micropolar parameter and Schmidt number on the distribution of velocity, temperature and concentration within the boundary layer, here we have chosen $\Delta = 1.5, 5.0$, $A = -0.1$, $\mathrm{Sc} = 0.73, 4.0$.

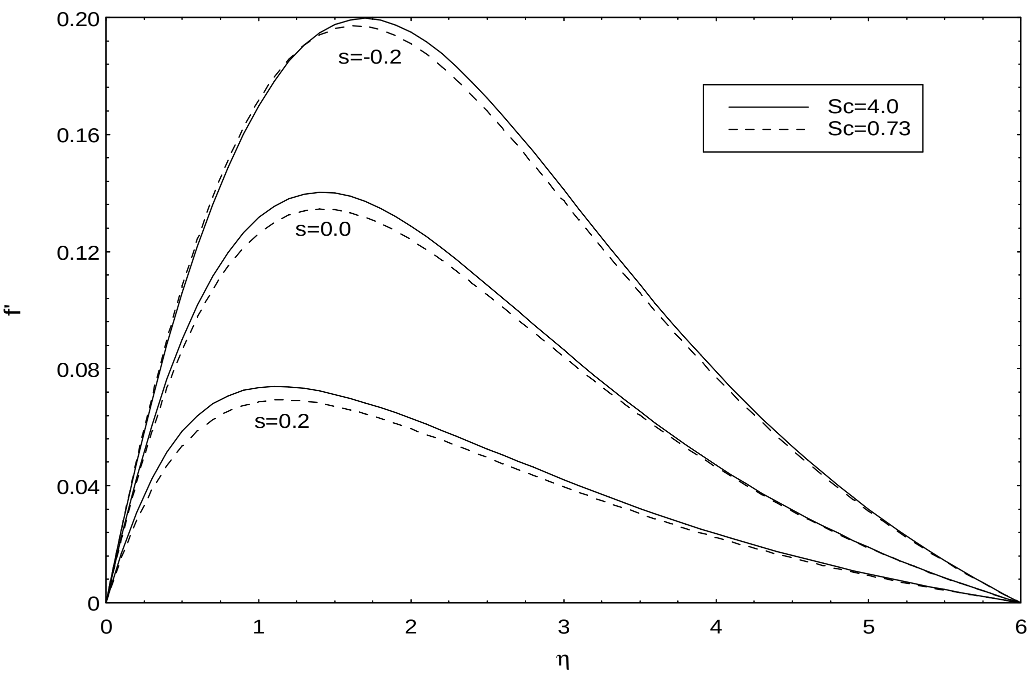

Fig.2. Velocity distribution with $A = -0.1$ and $\Pr = 0.73$.

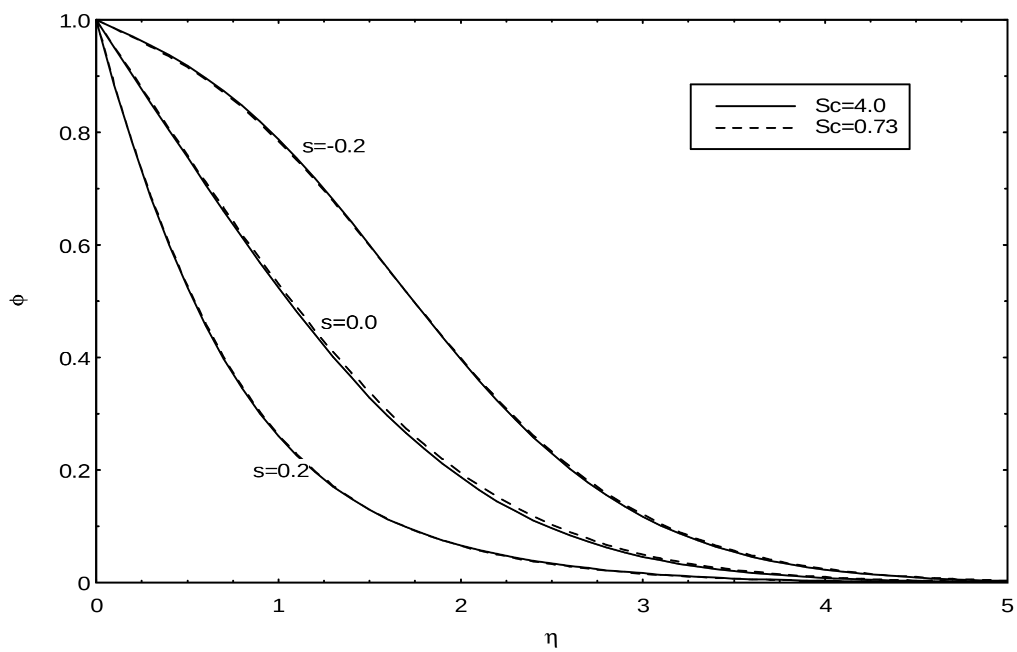

Fig.3. Temperature distribution with $A = -0.1$ and $\Pr = 0.73$.

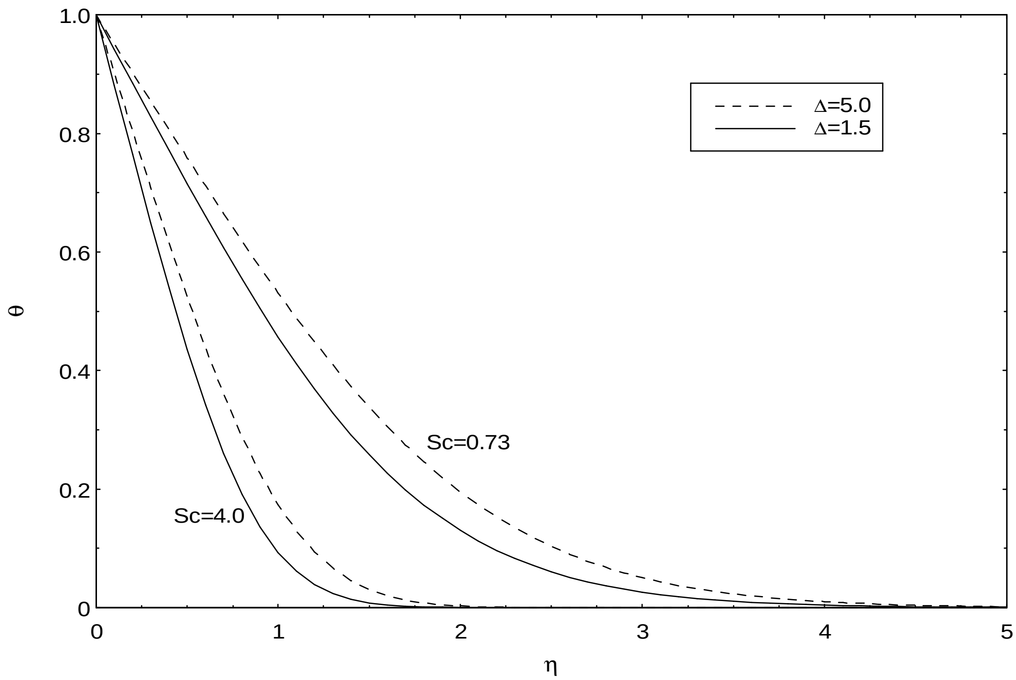

Fig.4. Concentration distribution with $A = -0.1$ and $\Pr = 0.73$.

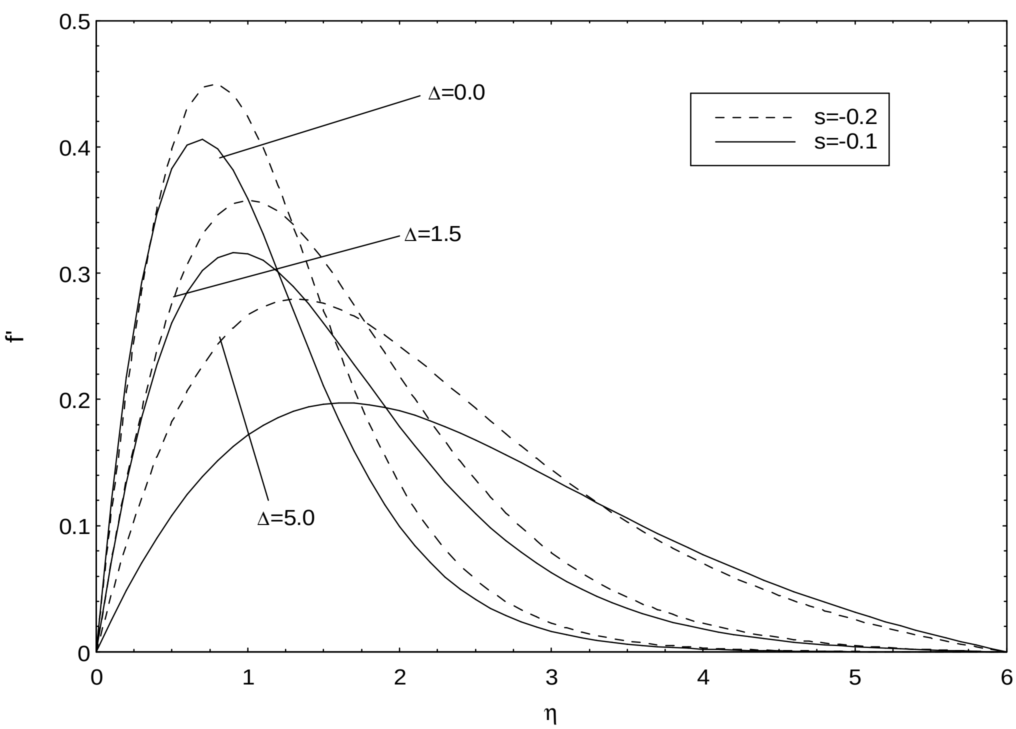

Fig.5. Velocity distribution with $A = 1.0$, $Sc = 0.73$ and $\Pr = 0.73$.

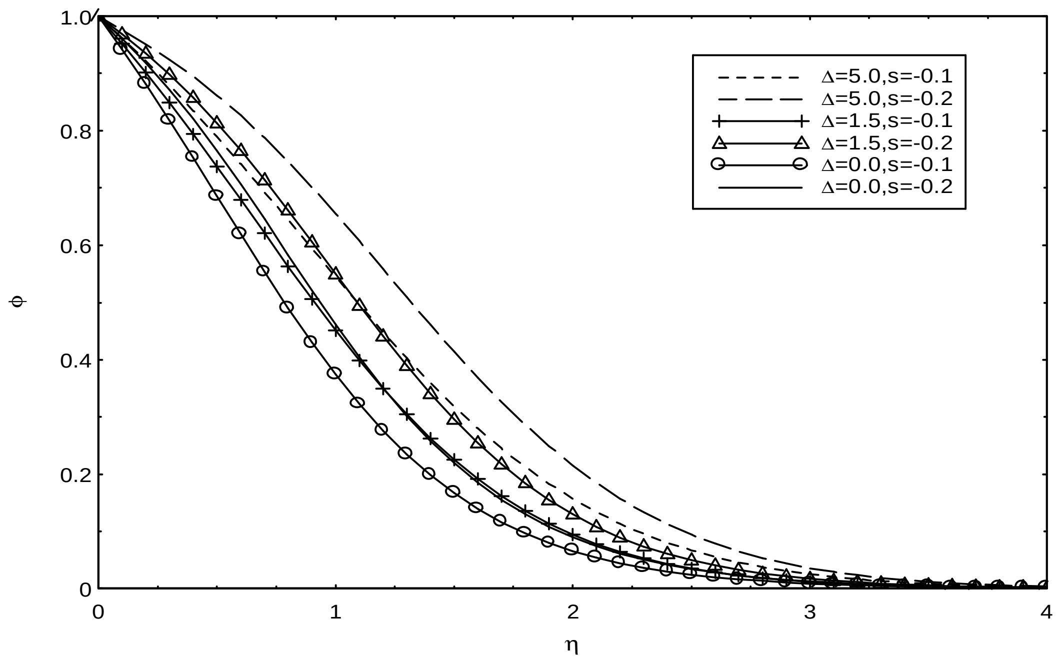

Fig.6. Temperature distribution with $A = 1.0$, $\mathrm{Sc} = 0.73$ and $\mathrm{Pr} = 0.73$.

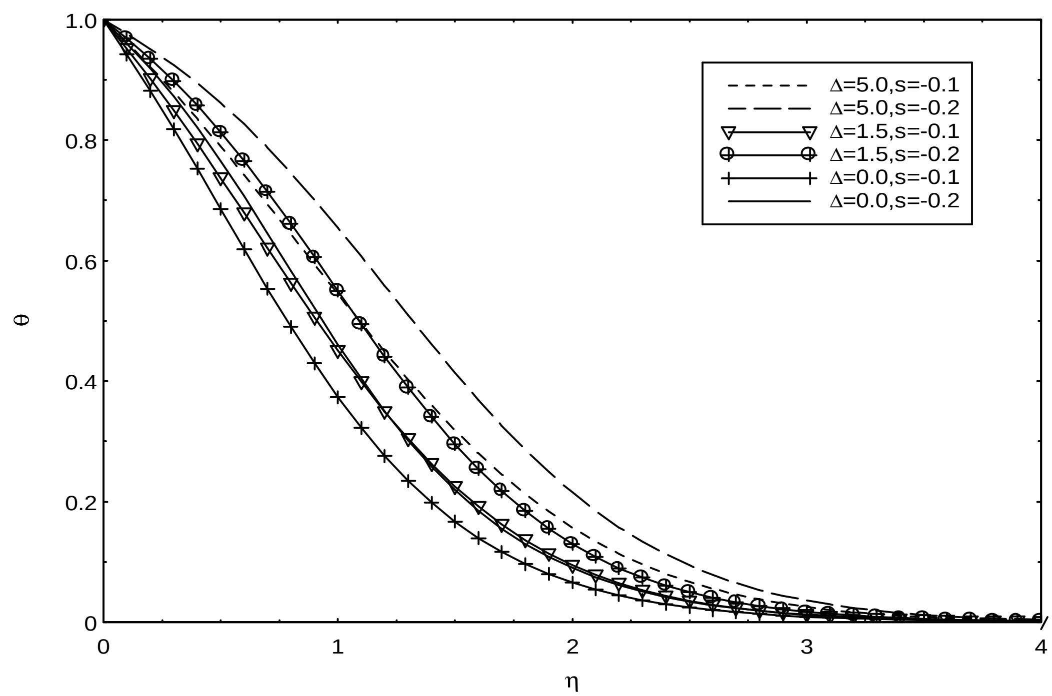

Fig.7. Concentration distribution with $A = 1.0$, $\mathrm{Sc} = 0.73$ and $\mathrm{Pr} = 0.73$.

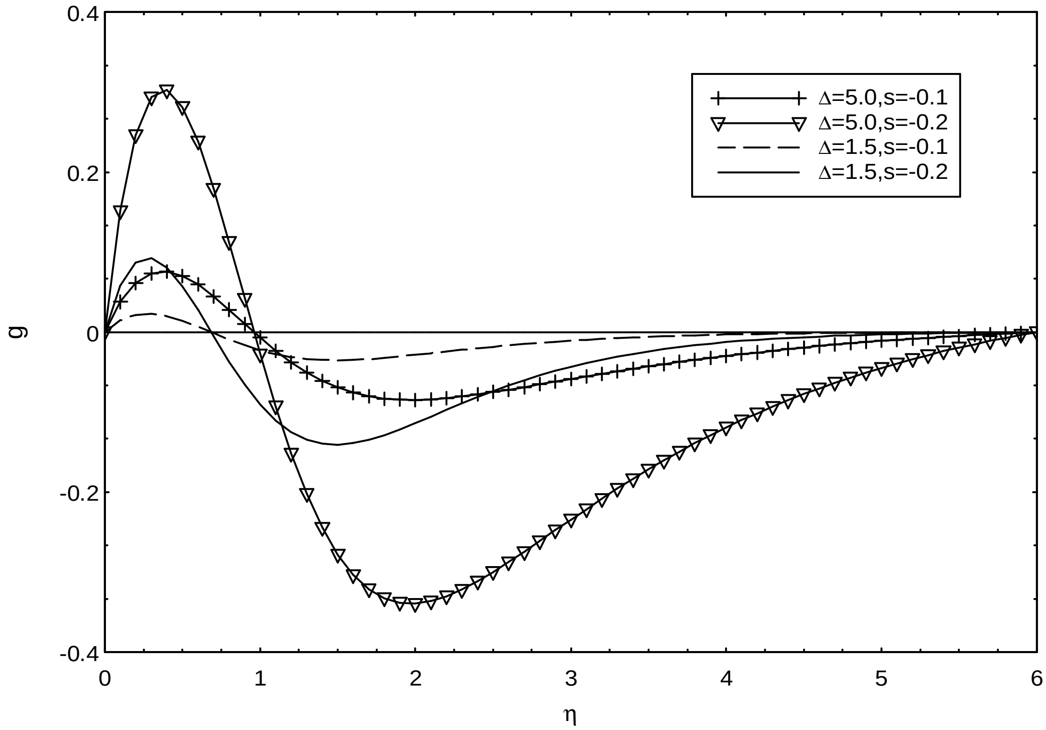

Fig.8. Microrotation distribution with $A = 1.0$, $\mathrm{Sc} = 0.73$ and $\mathrm{Pr} = 0.73$.

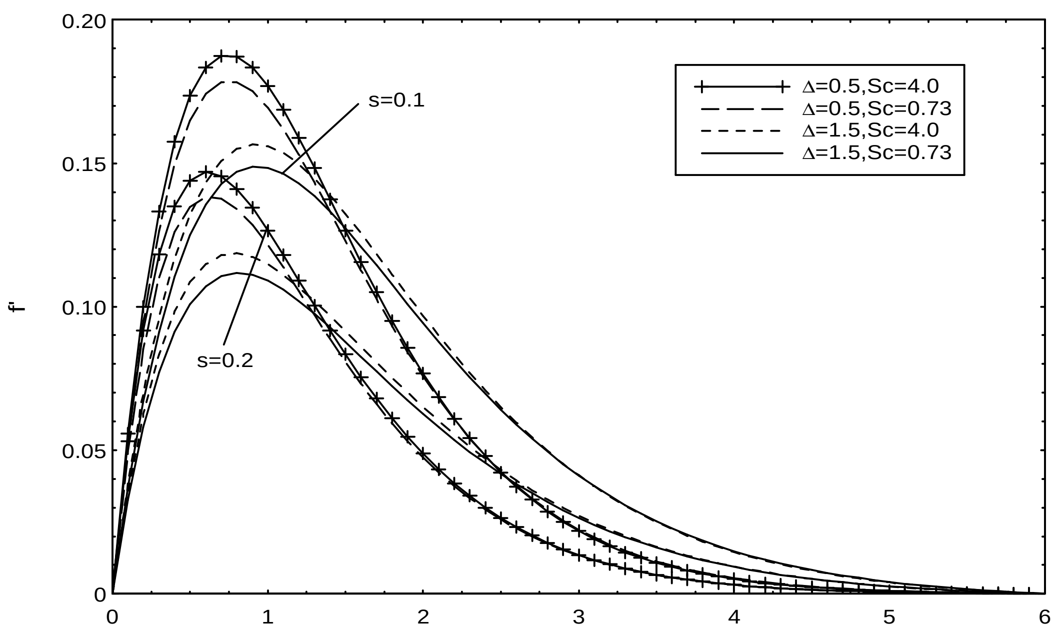

Fig.9. Velocity distribution with $A = -0.1$ and $\mathrm{Pr} = 0.73$.

Figures 6-9 illustrate the effect of suction on velocity, temperature, concentration and microrotation fields within the boundary for various values of micropolar parameter Δ, $\mathrm{Sc} = 0.73$ and $A = 1.0$.

Figures 10-12 illustrate the effect of injection on velocity, temperature, concentration and microrotation fields within the boundary for various values of the micropolar parameter Δ and the Schmidt number with $A = -0.1$. The velocity, temperature and concentration profiles get larger while the velocity, temperature and concentration boundary layer thickness increase with decreasing suction or injection parameter *s*, which corresponds to either decreasing suction or injection. We noticed that as the micropolar parameter Δ increases the velocity distribution becomes more linear, the temperature and concentration distribution become more uniform. We observe the velocity and the temperature decreases with the increase of the micropolar parameter Δ, while the concentration increase with it. It is clear that the velocity increases with the increasing values of suction/injection parameter s but the temperature and concentration decrease. We also observe that the velocity increases with the distance η from the cone surface, takes its maximum value inside the boundary layer and tends asymptotically to zero for higher values of $\eta(\eta \to \infty)$ while the microrotation changes sign from negative to positive values within the boundary layer.

The significance of the present work lies in its application in heat transfer augmentation or reduction processes. In micropolar applications involving either suction flow or injection flow cases, one can choose the proper value of the parameter *s* to obtain an appropriate augmentation and (or) reduction in the heat transfer rate.

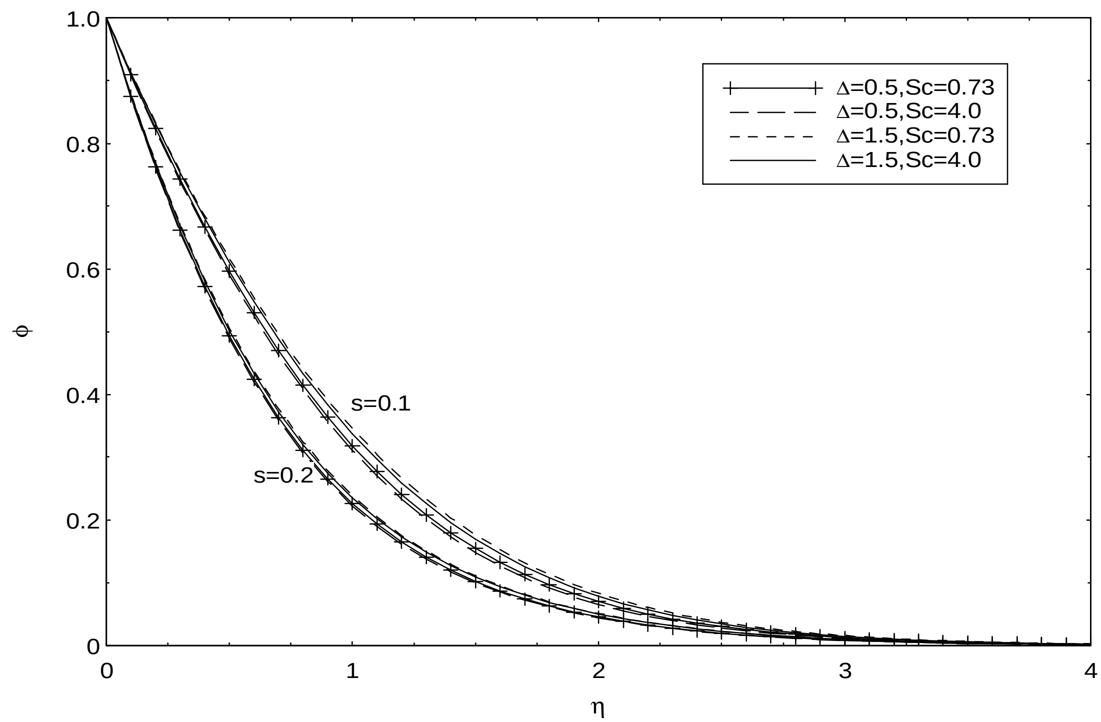

Fig.10. Temperature distribution with $A = -0.1$ and $\mathrm{Pr} = 0.73$.

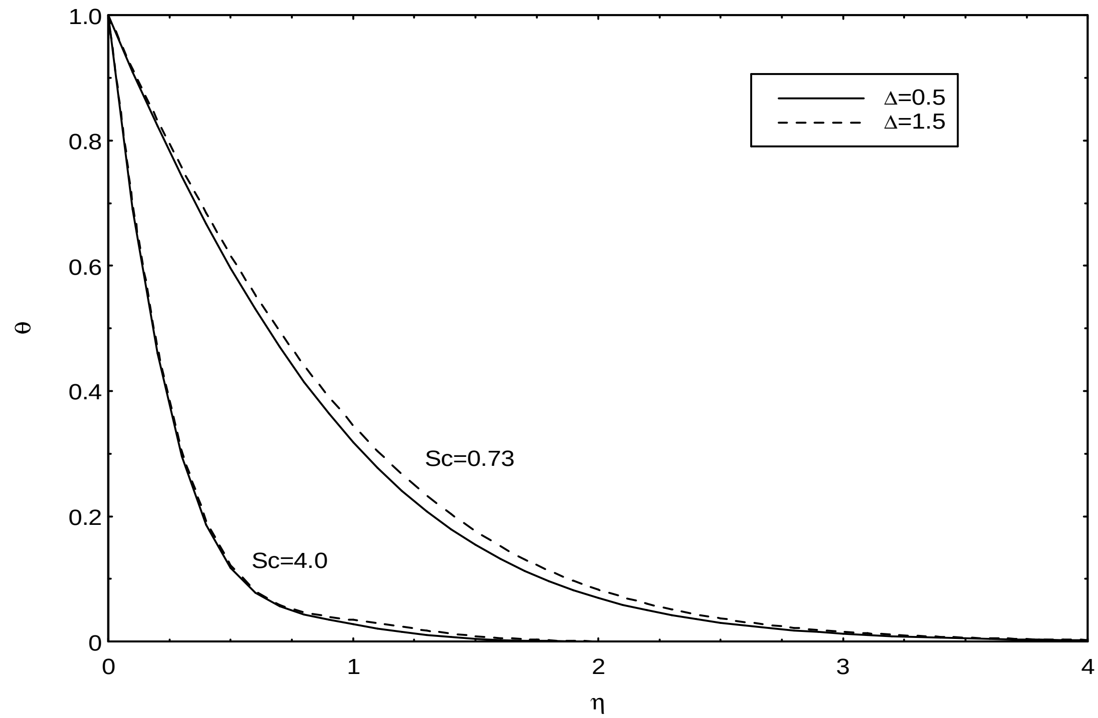

Fig.11. Concentration distribution with $A = -0.1$ and $\Pr = 0.73$.

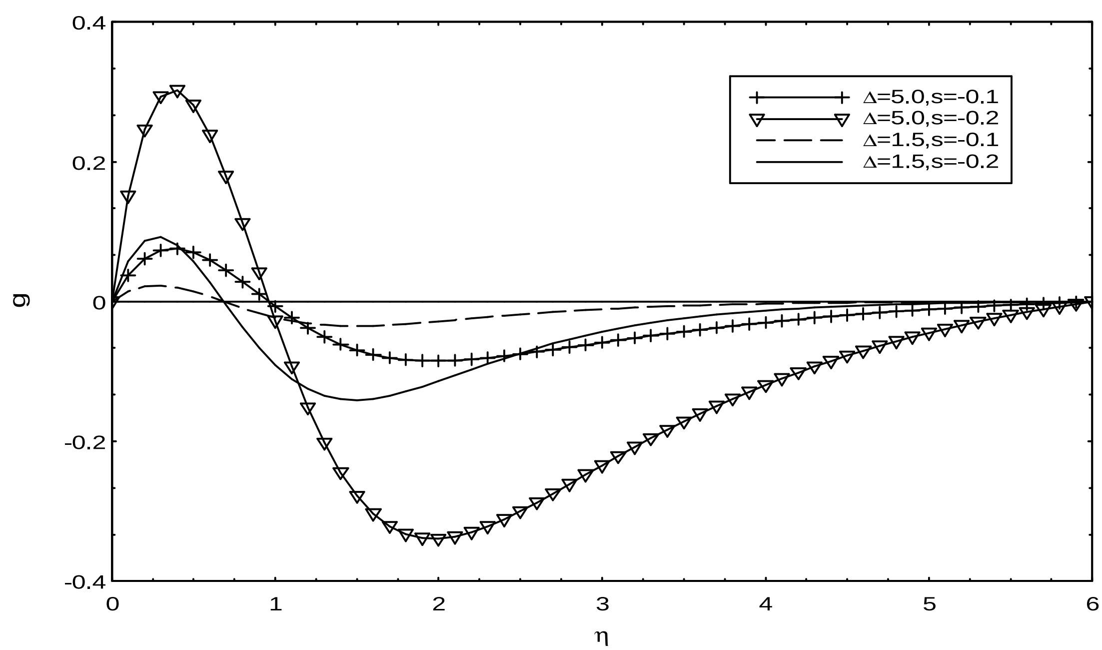

Fig.12. Microrotation distribution with $A = -0.1$ and $\Pr = 0.73$.

4. Concluding remarks

In this work, we have studied the effects of heat and mass transfer on free convection boundary layer flow with a uniform suction or injection over a cone in micropolar fluids. The governing equations are first cast into a dimensionless form by a nonsimilar transformation and the resulting equations are then solved numerically by using the Runge-Kutta numerical integration. This procedure in conjunction with the shooting technique. The results indicate that as the micropolar parameter Δ increases the wall couple stress, the shear stress, Nusselt number and Sherwood number decrease with it. While the Shmidt number increases the shear stress, wall couple stress and Nusselt number decrease, the opposite is the case for the Sherwood number. Results given in tables or shown in figures indicate that the micropolar fluids show drag and heat transfer rate reduction characteristics.

Nomenclature

A – buoyancy ratio, $A = \mathrm{Gr}_C / \mathrm{Gr}_T$
B – dimensionless parameter
C – concentration
C_p – specific heat
D – chemical molecular diffusiviy
f – dimensionless velocity
g – dimensionless microrotation
g^* – acceleration due to gravity
Gr_C – local Grashof number for mass diffusion
Gr_T – local Grashof number
h – heat transfer coefficient
j – microinertia per unit mass
k – thermal conductivity of fluid
K – vertex viscosity
m – wedge angle parameter
m_w – local wall mass flux
M_w – local couple stress
N – angular velocity
Nu – local Nusselt number, $\mathrm{Nu} = hx/k$
Pr – Prandtl number
q_w – local wall heat flux
Sh – local Sherwood number
Sc – the Schmidt number
T – temperature
u – velocity component in x-direction
u_∞ – free stream velocity
U_c – reference convective velocity
v – velocity component in y-direction
x – horizontal co-ordinate
$\bar{x}, x^*$ – dimensionless co-ordinates
y – vertical co-ordinate
β – thermal expansion coefficient
γ – spin-gradient viscosity
η – dimensionless co-ordinate

θ – dimensionless concentration
λ, Δ – dimensionless material parameter
μ – dynamic viscosity
ρ – density of the fluid
τ_w – local shear stess
υ – kinematic viscosity
ϕ – dimensionless temperature
ψ – stream function

Subscripts

w – at the wall
∞ – condition far away from the surface

Superscript

$'$ – differentiation with respect to η

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Received: February 26, 2004

Revised: February 8, 2005