DEFORMATION DUE TO MOVING LOADS IN THERMOELASTIC BODY WITH VOIDS

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The voids effect of loads which are moving at a constant velocity along one of the coordinate axis in a generalized thermoelastic half-space is studied. The analytical expressions of the displacements, stresses, temperature distribution and change in the volume fraction field for two different theories, i.e., Lord-Shulman (L-S), Green-Lindsay (G-L) are obtained by the use of the Fourier transform technique. The integral transform has been inverted by using a numerical technique and numerical results are illustrated graphically for a magnesium crystal-like material for the insulated boundary and temperature gradient boundary

Key words: thermoelastic, concentrated force, generalized thermoelasticity, thermal relaxation parameters, Fourier transform.

1. Introduction

The theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory is useful for investigating various types of geological and biological materials for which the elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the void volume is included among the kinematics variables and in the limiting case of vanishing this volume it reduces to the classical theory of elasticity.

A nonlinear theory of elastic material with voids was developed by Nunziato and Cowin (1979). Later Cowin and Nunziato (1983) developed a theory of linear elastic materials with voids. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of beams and small amplitudes of acoustic waves. Puri and Cowin (1985) studied the behaviour of plane waves in a linear elastic materials with voids. The domain of influence theorem in the linear theory of elastic materials with voids was discussed by Dhaliwal and Wang (1994). Scarpetta (1995) studied well posedness theorems for linear elastic materials with voids. Birsan (2000) established existence and uniqueness of the weak solution in the linear theory of elastic shells with voids.

Rusu (1987) studied the existence and uniqueness in thermoelastic materials with voids. Saccomandi (1992) presented some remarks about the thermoelastic theory of materials with voids. Ciarletta and Scalia (1993) discussed the non-linear theory of non simple thermoelastic materials with voids. Ciarletta and Scarpetta (1995) discussed some results on thermoelasticity for dielectric materials with voids. Dhaliwal and Wang (1995) developed a heat flux dependent theory of thermoelasticity with voids. Marin (1997a; b; 1998) studied uniqueness results and domain of influence in thermoelastic bodies with voids. Marin and Salca (1998) obtained the relation of Knopoff-de Hoop type in thermoelasticity of dipolar bodies with voids.

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Chirita and Scalia (2001) studied the spatial and temporal behaviour in linear thermoelasticity of materials with voids. Pompei and Scalia (2002) studied the asymptotic spatial behaviour in linear thermoelasticity of materials with voids.

The problem of determining the response of an elastic system subjected to a moving load has received considerable attention. Work in this area has been mostly motivated by the need to analyze the vibrations of such structures as bridges and rail/road tracks caused by moving vehicles. The design of highways, airport runways as well as the foundation problems in soil mechanics, particularly when the earth mass is supporting a heavy structure having a moving load over its free plane surface, lead to the investigation of the dynamic stress distribution associated with the problem. An important problem concerning such diverse fields as wave propagation, contact mechanics and tribology is the rapid motion of a line mechanical and/or thermal load over the surface of a half-space. Indeed, this is the case when (a) ground motion and stresses are produced by the surface blast waves due to explosives or by supersonic aircraft (b) high velocity rockets sleds moving on guide rails. Such dynamical mechanical/thermal loading may produce severe deformation and temperature rise in a thin zone near the half-space surface and thereby causes excessive wear and even cracking near the contact zone.

In many cases, the above described problem can be modeled as a plane-strain steady state situation, involving an elastic half-plane under a concentrated line mechanical/thermal loading which moves over the half-plane surface of constant speed.

To relate the present study and previous work, we first note that, in the absence of thermal effects, our study turns into the well known problem of steady-state elastodynamic motion of a line force along half-plane surface considered by Cole and Huth (1958) and Georgiadis and Barber (1993). Various authors have studied the thermal effect in the last three decades. Barber and Martin-Moran (1982), Barber (1984), Azarkhin and Barber (1985), Hills and Barber (1985) use principles of Carslaw and Jaeger (1959) applied to thermoelasticity to develop stresses and displacements. Barber (1984) successfully used these ideas to obtain exact solutions for the tangential stress and displacements on the surface of half-space for a given moving heat source. Barber (1984) obtained the expressions of displacements and stresses due to heat source moving over the surface of a thermoelastic half plane. Bryant (1988) developed a method for obtaining fundamental thermal and thermoelastic solutions for two-dimensional distributions moving over the surface of an elastic half-space. Steady state response of a thermoelastic half-space due to the rapid motion of thermal/mechanical surface loads has been discussed by Brock and Rodgers (1997). Brock and Georgiadis (1997) obtained the surface displacement and temperature due to a line mechanical/heat source that moves at a constant velocity over the surface of a thermoelastic half-space. The problem of transient disturbances in a thermoelastic half-space due to moving the internal heat source has been studied by Chakravorty and Chakravorty (1998).

The present investigation is to determine the component of displacements, stresses, temperature distribution and change in the volume fraction field in a homogenous, isotropic, thermoelastic half-space with voids due to moving mechanical and thermal sources. The steady state assumption employed here has its own justification in the dynamic analysis of moving sources (e.g. Fung, 1965; Eringen and Suhubi, 1975; Brock, 1994; 1995) and may yield reliable results when the mechanical/thermal load in question has, as here, been applied and moving for a long time.

2. Basic equations

Following Lord and Shulman (1967), Green and Lindsay (1972) and Cowin and Nunziato (1983) the field equations and constitutive relations in a thermoelastic solid with voids without body forces, heat sources and extrinsic equilibrated body force can be written as

$$(\lambda+2\mu)\,\mathrm{grad}(\mathrm{div}\,\boldsymbol{u})-\mu\,\mathrm{curl}\,\mathrm{curl}\,\boldsymbol{u}+b\nabla\phi-\beta\,\mathrm{grad}\left(T+\delta_{2k}\tau_1\dot{T}\right)=\rho\ddot{\boldsymbol{u}}, \tag{2.1}$$

$$\alpha\nabla^2\phi-b(\mathrm{div}\boldsymbol{u})-\xi_1\phi-\omega_0\dot{\phi}+MT=\rho\chi\ddot{\phi} \tag{2.2}$$

$$K\nabla^2 T - \beta T_0\left(\nabla\cdot\dot{\mathbf{u}} + \tau_0\delta_{1k}\nabla\cdot\ddot{\mathbf{u}}\right) - MT_0\dot{\phi} = \rho c_e\left(\dot{T} + \tau_0\ddot{T}\right), \tag{2.3}$$

and

$$t_{ij} = \lambda\, u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + b\phi\delta_{ij} - \beta\left(T + \delta_{2k}\tau_1\dot{T}\right)\delta_{ij}, \qquad (i, j = x, y, z) \tag{2.4}$$

where the list of symbols is given in Appendix A.

3. Formulation and solution of the problem

We consider a homogenous, isotropic, thermally conducting, elastic half-space with voids in the undeformed state at uniform temperature T_0 under plane strain conditions. The rectangular Cartesian co-ordinate system (x, y, z) having the origin on the plane surface $z = 0$ with the z-axis normal to the medium is introduced. A concentrated normal point force or thermal source, is assumed to be moving on the thermoelastic half-space with a constant velocity U in the negative x-direction. After the load has been moving for some time and transient effects have died away, the displacements will appear stationary in a coordinate system moving with the load.

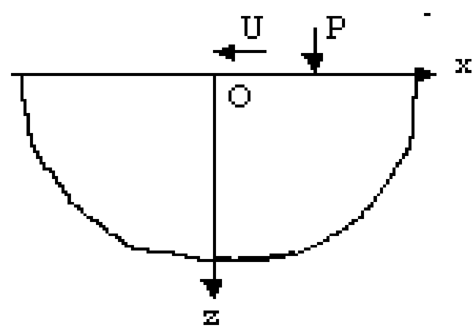

Fig.1. A mechanical/thermal source P moving with a constant velocity U over the surface of the thermooelastic half-space.

We consider the displacements $u(x, y, z, t)$, $w(x, y, z, t)$ and change in the volume fraction field $\phi(x, y, z, t)$ which are assumed to be of the form

$$u(x, y, z, t) = u^*\left(x^*, z^*\right),$$

$$w(x, y, z, t) = w^*\left(x^*, z^*\right),$$

$$\phi(x, y, z, t) = \phi^*\left(x^*, z^*\right)$$

where as per Fung (1968), a Galilean transformation

$$x^* = x + Ut, \qquad z^* = z, \qquad t^* = t,$$

is introduced; then the boundary conditions would be independent of t^*.

The initial conditions are

$$u(x, y, z, 0) = u_s(x, z), \quad \frac{\partial u}{\partial t} = U \frac{\partial u_s}{\partial x}(x, z),$$
$$w(x, y, z, 0) = w_s(x, z), \quad \frac{\partial w}{\partial t} = U \frac{\partial w_s}{\partial x}(x, z), \tag{3.1}$$
$$\phi(x, y, z, 0) = \phi_s(x, z), \quad \frac{\partial \phi}{\partial t} = U \frac{\partial \phi_s}{\partial x}(x, z)$$

where $u_s(x, z)$, $w_s(x, z)$, $\phi_s(x, z)$ are solutions of the static problem.

For the two dimensional problem, we assume $\boldsymbol{u} = (u, 0, w)$ in Eqs (2.1)-(2.4). We define the dimensionless quantities

$$x' = \frac{\omega_1^*}{c_1} x^*, \quad z' = \frac{\omega_1^*}{c_1} z^*, \quad t' = \omega_1^* t^*, \quad u' = \frac{\omega_1^*}{c_1} u, \quad w' = \frac{\omega_1^*}{c_1} w, \quad T' = \frac{T}{T_0},$$
$$\phi' = \frac{\omega_1^{*2}\chi}{c_1^2}\phi, \quad \in_1 = \frac{\beta c_1^2}{K\omega_1^*}, \quad \tau_0' = \omega_1^* \tau_0, \quad \tau_1' = \omega_1^* \tau_1, \tag{3.2}$$

$$t_{zz}' = \frac{t_{zz}}{\beta T_0}, \quad t_{zx}' = \frac{t_{zx}}{\beta T_0}, \quad h' = \frac{h c_1}{\omega_1^*}, \quad P' = \frac{P}{\beta T_0} \tag{3.3}$$

where

$$c_1 = \left(\frac{\lambda + 2\mu}{\rho}\right)^{\frac{1}{2}} \quad \text{and} \quad \omega_1^* = \frac{\rho c_e c_1^2}{K}$$

in Eqs (2.1)-(2.3), and applying the Fourier transform defined by

$$\tilde{f}(\xi, z) = \int_{-\infty}^{\infty} f(x, z) e^{i\xi x} dx, \tag{3.4}$$

on the resulting expressions, we obtain

$$\frac{d^2\tilde{u}}{dz^2} = R_{11}\tilde{u} + R_{13}\tilde{T} + R_{14}\tilde{\phi} + R_{16}\frac{d\tilde{w}}{dz}, \tag{3.5}$$

$$\frac{d^2\tilde{w}}{dz^2} = R_{11}\tilde{w} + R_{16}\frac{d\tilde{u}}{dz} - \frac{R_{13}}{i\xi}\frac{d\tilde{T}}{dz} + R_{28}\frac{d\tilde{\phi}}{dz}, \tag{3.6}$$

$$\frac{d^2\tilde{T}}{dz^2} = R_{31}\tilde{u} + R_{33}\tilde{T} + R_{34}\tilde{\phi} + R_{36}\frac{d\tilde{w}}{dz}, \tag{3.7}$$

$$\frac{d^2\tilde{\phi}}{dz^2} = R_{41}\tilde{u} + R_{43}\tilde{T} + R_{44}\tilde{\phi} + R_{46}\frac{d\tilde{w}}{dz} \tag{3.8}$$

where $R_{11}, R_{13}, R_{14}, R_{16}$ etc. are given in Appendix B.

The equations (3.5)-(3.8) can be written as

$$\frac{d}{dz}W(\xi,z) = A(\xi)W(\xi,z) \tag{3.9}$$

where

$$W = \begin{bmatrix} V \\ V' \end{bmatrix}, \qquad A = \begin{bmatrix} O & I \\ A_1 & A_2 \end{bmatrix}, \qquad V = \begin{bmatrix} \tilde{u} \\ \tilde{w} \\ \tilde{T} \\ \tilde{\phi} \end{bmatrix},$$

$$A_1 = \begin{bmatrix} R_{11} & 0 & R_{13} & R_{14} \\ 0 & R_{11} & 0 & 0 \\ R_{31} & 0 & R_{33} & R_{34} \\ R_{41} & 0 & R_{43} & R_{44} \end{bmatrix}, \qquad O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3.10}$$

$$A_2 = \begin{bmatrix} 0 & R_{16} & 0 & 0 \\ R_{16} & 0 & -\dfrac{R_{13}}{i\xi} & R_{28} \\ 0 & R_{36} & 0 & 0 \\ 0 & R_{46} & 0 & 0 \end{bmatrix}, \qquad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

To solve Eq.(3.9),we take

$$W(\xi,z) = X(\xi)e^{qz}, \tag{3.11}$$

so that

$$A(\xi)W(\xi,z) = qW(\xi,z), \tag{3.12}$$

which leads to an eigenvalue problem. The characterstic equation corresponding to the matrix A is given by

$$\det[A - qI] = 0, \tag{3.13}$$

which on expansion leads to

$$q^8 - \lambda_1 q^6 + \lambda_2 q^4 - \lambda_3 q^2 - \lambda_4 = 0 \tag{3.14}$$

where $\lambda_1,\ \lambda_2,\ \lambda_3, \lambda_4$ are presented in Appendix C.
The roots of Eq.(3.14) are $\pm q_{\mathbf{l}}(\mathbf{l} = 1,\ 2,\ 3, 4)$.

The eigenvalues of the matrix A are roots of Eq.(3.14). The eigenvector $X(\xi)$) corresponding to the eigenvalues $q_{\mathbf{l}}$ can be determined by solving the homogeneous equation

$$[A - qI]X(\xi) = 0\ . \tag{3.15}$$

The set of eigenvectors $X_{\mathbf{l}}(\xi)$, $(\mathbf{l} = 1, 2, 3, 4, 5, 6, 7, 8)$ are presented in Appendix D.
The solution of Eq.(3.9) is given by

$$W(\xi,\ z) = \sum_{\mathbf{l}=1}^{4} \left[B_{\mathbf{l}} X_{\mathbf{l}}(\xi)\exp(q_{\mathbf{l}} z) + B_{\mathbf{l}+4} X_{\mathbf{l}+4}(\xi)\exp(-q_{\mathbf{l}} z)\right] \tag{3.16}$$

where $B_l(\mathbf{l} = 1, 2, 3, 4, 5, 6, 7, 8)$ are arbitrary constants.

Thus Eq.(3.16) represents the solution of the general problem in the plane strain case of generalized homogeneous thermoelasticity by employing the eigenvalue approach and therefore can be applied to a broad class of problems in the Fourier transform.

For the half-space $z \geq 0$ the roots q_1, q_2, q_3, q_4 are related such that the real part of $(q_1, q_2, q_3, q_4) \geq 0$. With this consideration, the regularity conditions at infinity is satisfied and B_1, B_2, B_3, B_4 approaches zero for the domain as z approaches ∞

$$\tilde{u}(\xi,\ z) = \left(B_5 e^{-q_1 z} + B_6 e^{-q_2 z} + B_7 e^{-q_3 z} + B_8 e^{-q_4 z}\right), \tag{3.17}$$

$$\tilde{w}(\xi,\ z) = -\left(p_1 q_1 B_5\ e^{-q_1 z} + p_2 q_2 B_6\ e^{-q_2 z} + p_3 q_3 B_7\ e^{-q_3 z} + p_4 q_4 B_8\ e^{-q_4 z}\right), \tag{3.18}$$

$$\tilde{T}(\xi,\ z) = s_1 B_5\ e^{-q_1 z} + s_2 B_6\ e^{-q_2 z} + s_3 B_7\ e^{-q_3 z} + s_4 B_8\ e^{-q_4 z}\ , \tag{3.19}$$

$$\tilde{\phi}(\xi,\ z) = r_1 B_5\ e^{-q_1 z} + r_2 B_6\ e^{-q_2 z} + r_3 B_7\ e^{-q_3 z} + r_4 B_8\ e^{-q_4 z}\ . \tag{3.20}$$

4. Application

4.1. Normal point force on the surface of half-space

Boundary conditions in this case are given by

$$t_{zz}(x,\ z) = -P\delta\left(x^*\right),\quad t_{zx}(x,\ z) = 0\ ,\quad \frac{\partial\phi}{\partial z} = 0\ ,\quad \frac{\partial T}{\partial z} + hT = 0 \quad \text{at} \quad z = 0 \tag{4.1}$$

where P is the magnitude of the force, $\delta\left(x^*\right)$ is the Dirac delta function and h is the heat transfer coefficient.

Making use of Eqs (2.4)-(3.3), applying the transform defined by (3.4) in the boundary conditions (4.1) and with the help of Eqs (3.17)-(3.20), we obtain the expressions for displacement components, stresses, temperature distribution and change in the volume fraction field (see Appendix E).

Particular case (4.1a): If we neglect voids effect, i.e., $(\alpha = b = \xi_1 = \omega_0 = M = \chi = 0)$ in Eqs (E.1), the expressions for displacement components, stresses and temperature distribution are obtained in thermoelastic half-space (see Appendix F).

4.2. Thermal load on the surface of half-space

The boundary conditions in this case are

$$t_{zz} = 0, \quad t_{zx} = 0, \quad \frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = 0,$$

$$\frac{\partial T}{\partial z}(x, z = 0) = \delta(x^*) \tag{4.2}$$

for the temperature gradient boundary or

$$T(x, z = 0) = \delta(x^*)$$

for the temperature input boundary.

Making use of Eqs (2.4)-(3.3) applying the transform defined by (3.4) in the boundary conditions (4.2) and with the help of Eqs (3.17)-(3.20), we obtain the expressions for displacement components, stresses, temperature distribution and change in the volume fraction field which are presented in Appendix G.

On replacing Δ with $(T_0 \omega_1^* / c_1)\Delta_1^0$ and $T_0\, \Delta_2^0$ in Eqs (G.1), we obtain the expressions for temperature gradient boundary and temperature input boundary.

Particular case (4.2a): If we neglect voids effect, the expressions for displacement components, stresses and temperature distribution in generalized thermoelastic half-space are obtained by replacing $\overline{\Delta}_l$ with $\overline{\Delta}_l'' \,(\mathbf{l} = 1,2,3)$, $P = 1$ in Eqs (F.1) respectively, where

$$\overline{\Delta}_1'' = n_2 u_3 - n_3 u_2, \quad \overline{\Delta}_2'' = -(n_1 u_3 - n_3 u_1), \quad \overline{\Delta}_3'' = n_1 u_2 - n_2 u_1.$$

On replacing $\overline{\Delta}$ with $(T_0 \omega_1^* / c_1)\overline{\Delta}_1'$ and $T_0\, \Delta_2'$ in Eqs (F.1), we obtain the expressions for temperature gradient boundary and temperature input boundary.

***Sub-case 1*:** If $h \to 0$, Eqs (E.1) yield the expression of displacements, stresses, temperature distribution and change in the volume fraction field for the insulated boundary.

***Sub-case 2*:** If $h \to \infty$, Eqs (E.1) yield the expression of displacements, stresses, temperature distribution and change in the volume fraction field for the isothermal boundary.

***Special case 1*:** By putting $k = 1$ and $\tau_1 = 0$ in Eqs (E.1), (F.1) and (G.1) we obtain the corresponding expressions of the thermoelastic half-space with and without voids, respectively, for L–S theory.

***Special case 2*:** For G–L theory, we recover the corresponding expressions of the thermoelastic half-space with and without voids, respectively, by substituting $k = 2$ in Eqs (E.1), (F.1) and (G.1).

5. Inversion of the transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms in Eqs (E.1), (F.1) and (G.1) for the two theories, i.e., L-S and G-L. These expressions are functions of z and the parameter of the Fourier transform ξ and hence are of the form $\tilde{f}(\xi, z)$. To get the function $f(x, z)$ in the physical domain, we invert the Fourier transform using

$$f(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}\, \tilde{f}(\xi, z)\, d\xi = \frac{1}{\pi}\int_{0}^{\infty}\left(\cos(\xi x)\, f_e - i\sin(\xi x) f_0\right) d\xi$$

where f_e and f_0 are, respectively, even and odd parts of the function $\tilde{f}(\xi, z)$. The method for evaluating this integral is described by Press *et al.* (1986), which involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

6. Numerical result and discussion

Following Dhaliwal and Singh (1980) we take the case of a magnesium crystal-like material for numerical calculations. The physical constants used are

$$\lambda = 2.17\times10^{10}\, Nm^{-2}, \quad T_0 = 298^0 K, \quad \mu = 3.278\times10^{10}\, Nm^{-2},$$

$$\rho = 1.74\times10^{3}\, kgm^{-3}, \quad K = 1.7\times10^{2}\, Wm^{-1}\text{degree}^{-1}, \quad \omega_1^{*} = 3.58x10^{11}\, s^{-1},$$

$$c_e = 1.04\times10^{3}\, Jkg^{-1}\text{degree}^{-1}, \quad \beta = 2.68\times10^{6}\, Nm^{-2}\text{degree}^{-1}, \quad P = 1,$$

and void parameters are

$$\alpha = 3.688\times10^{-5}\, N, \quad \xi_1 = 1.475\times10^{10}\, Nm^{-2}, \quad \chi = 1.753\times10^{-15}\, m^2,$$

$$b = 1.13849\times10^{10}\, Nm^{-2}, \quad \omega_0 = 0.0787\times10^{-3}\, Nm^{-2}, \quad M = 2\times10^{6}\, Nm^{-2}\text{degree}^{-1}.$$

A comparison of dimensionless normal stress t_{zz}, boundary temperature field T and change in the volume fraction field ϕ with distance x for Concentrated force (CF), is shown graphically in Figs 1-14, for L-S and G-L theories for non-dimensional relaxation times $\tau_0 = 0.02$, $\tau_1 = 0.05$. The solid lines with and without center symbols, with voids are denoted by LSV and GLV. The dashed lines with and without center symbols, without voids are denoted by LSWV and GLWV. In Figs 7 and 14 the solid lines with and without center symbols correspond to the case when $U < c_1$, the small dashed lines with and without center symbols correspond to the case $U > c_1$ and long dashed lines with and without center symbols correspond to the case when $U = c_1$ for LSV and GLV theories.

6.1. Normal point force on the boundary of half-space (Insulated boundary)

Case 1. $U < c_1$

Figure 2 shows the variation of normal stress t_{zz} with distance *x*. At the point of application of source due to the presence of voids the values of t_{zz} for LSV and GLV are greater than LSWV and GLWV. Due to the presence of voids the values of t_{zz} for LSV and GLV theories increase with an increase in distance *x* whereas for LSWV and GLWV the values of normal stress initially decrease and then become oscillatory in the whole range.

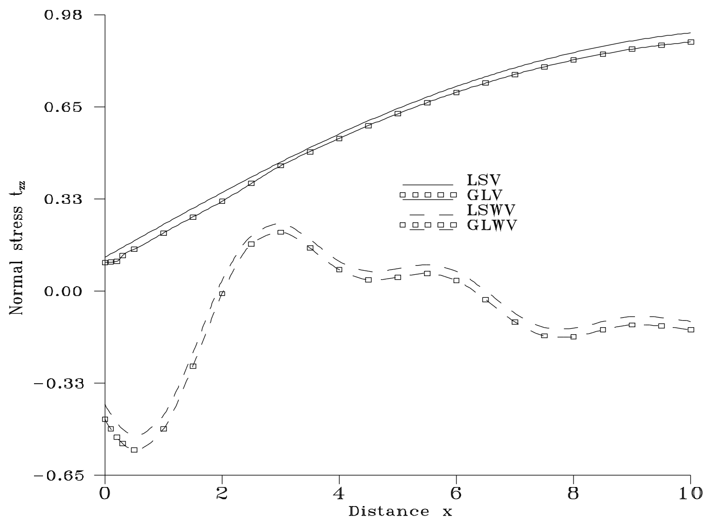

Fig.2. Variation of normal stress t_{zz} with distance *x*.

Figure 3 depicts the variation of temperature distribution T with distance *x*. Due to the presence of voids the values of T for LSV and GLV increase slowly in the range $0 \le x \le 6$ and decrease slowly in the range $6.1 \le x \le 10$. For LSWV and GLWV the values of T increase in the ranges $0 \le x \le 3$ and $7.5 \le x \le 10$ and decrease in other ranges.

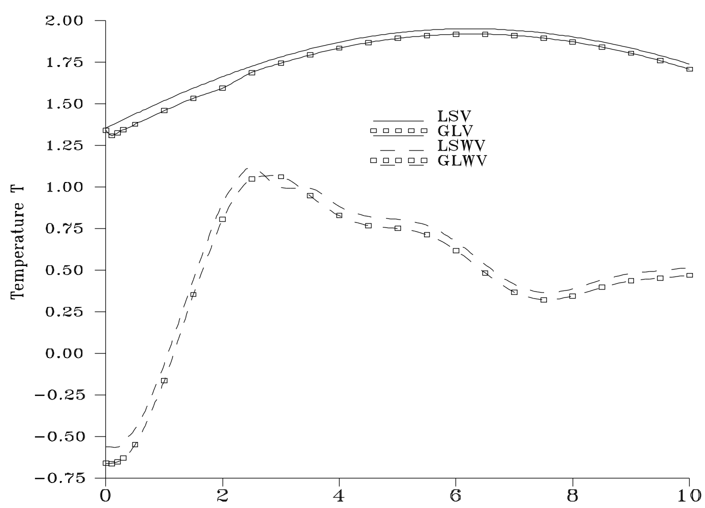

Fig.3. Variation of temperature T with distance *x*.

Case 2. $U > c_1$

The variation of normal stress t_{zz} with distance *x* is shown in Fig.4. The values of t_{zz} show opposite oscillatory behaviour in the whole range for LSWV and GLWV. The values of t_{zz} for LSV are greater than GLV but the trend of variation is same for both the theories in the whole range.

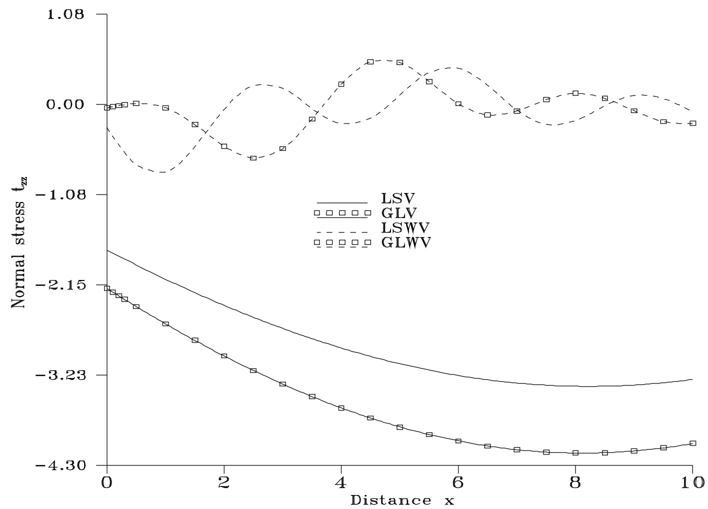

Fig.4. Variation of normal stress t_{zz} with distance *x.*

Figure 5 depicts the variation of temperature distribution *T* with distance *x.* Due to the presence of voids the values of *T* for LSV and GLV decrease sharply with an increase in distance *x*. For LSWV, initially the values of *T* increase sharply whereas for GLWV increase slowly and then become oscillatory for both theories in the whole range.

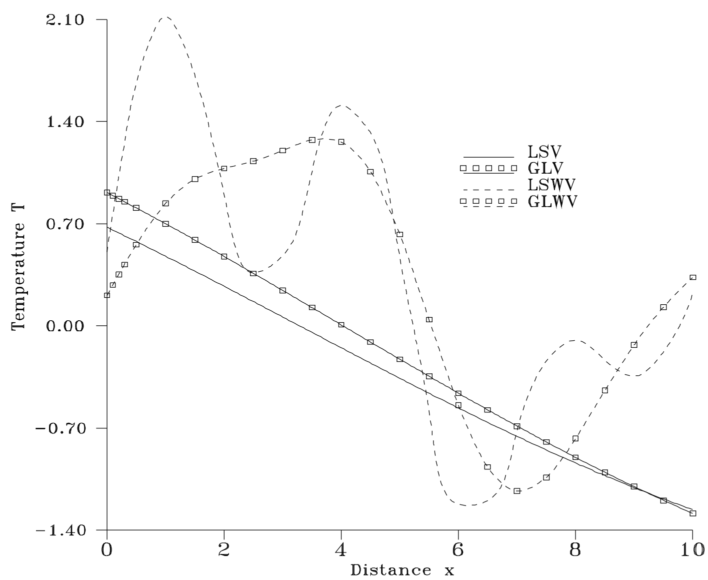

Fig.5. Variation of temperature *T* with distance *x.*

Case 3. $U = c_1$

The variation of normal stress t_{zz} with distance x are shown in Fig.6. The values of t_{zz} show an opposite oscillatory behaviour in the ranges $1.7 \le x \le 2.5$, $3.7 \le x \le 10$ and shows same behavior in the rest of the ranges for LSWV and GLWV whereas for LSV and GLV the values of t_{zz} show a small variation in the whole range. The values of t_{zz} for LSV are larger than GLV but the trend of variation is same for both the theories in the whole range.

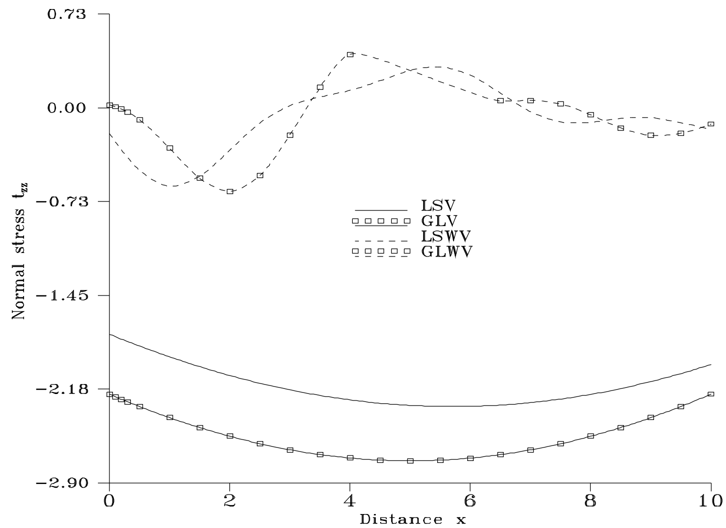

Fig.6. Variation of normal stress t_{zz} with distance *x*.

Figure 7 depicts the variation of temperature distribution *T* with distance *x*. Due to the presence of voids the values of *T* for LSV and GLV decrease sharply with an increase in distance *x*. For LSWV, initially the values of *T* increase sharply whereas for GLWV increase slowly and then become oscillatory for both theories in the whole range.

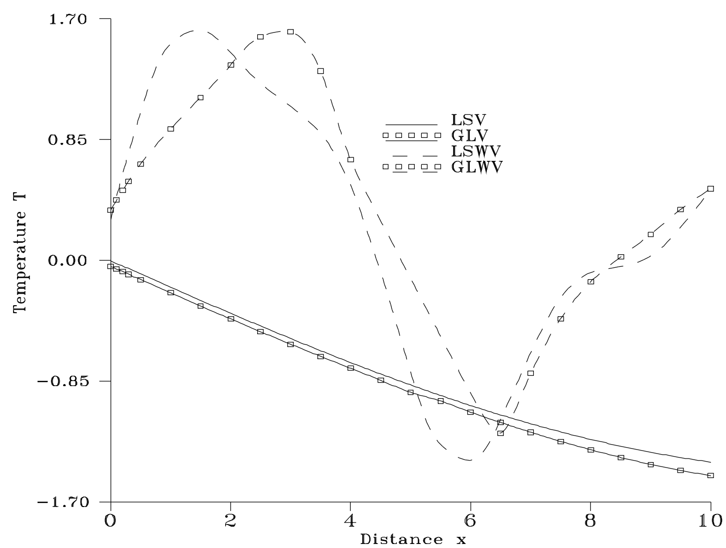

Fig.7. Variation of temperature *T* with distance *x*.

Figure 8 depicts the variation of change in the volume fraction field ϕ with distance *x*. The values of ϕ for the case $U < c_1$ are greater than $U > c_1$ and $U = c_1$ in the whole range. The values of ϕ for the case $U < c_1$ increase gradually and approach zero whereas for $U > c_1$ and $U = c_1$ the values of ϕ initially decrease and then increase in the whole range for both the theories.

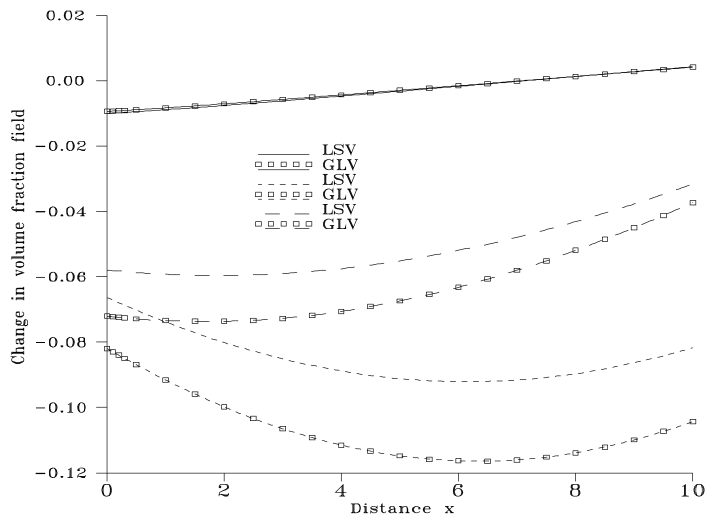

Fig.8. Variation of change in the volume fraction field with distance *x* due to mechanical source.

6.2. Thermal point source on the surface of half-space (Temperatue gradient boundary)

Case 1 $U < c_1$

Figure 9 shows the variations of normal stress t_{zz} with distance *x*. Due to voids effect, the values of t_{zz} for LSV and GLV are larger than LSWV and GLWV in the whole range. Due to voids effect the values of t_{zz} for LSV and GLV decrease with an increase in distance *x* whereas for LSWV and GLWV, the values of t_{zz} increase or decrease along an oscillatory path with an increase in distance *x*.

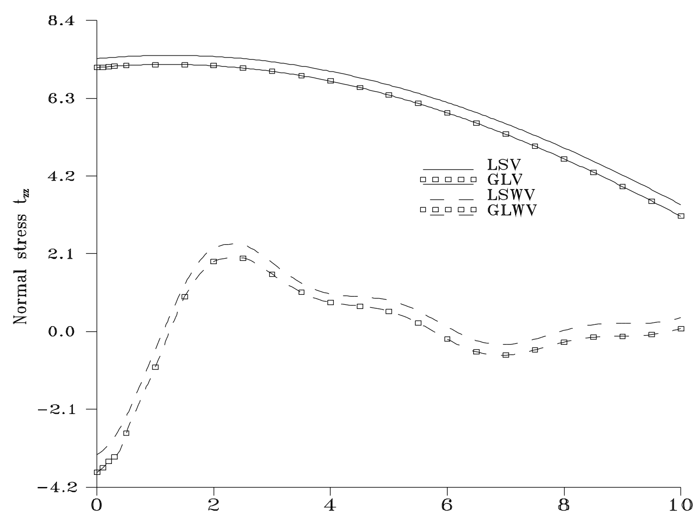

Fig.9. Variation of normal stress t_{zz} with distance *x*. (Temperature gradient boundary)

The variation of temperature distribution T are shown in Fig.10. Due to the presence of voids the values of T for LSV and GLV decrease slowly whereas for LSWV and GLWV, near the point of application of source, the values of T increase sharply and then become oscillatory in the whole range.

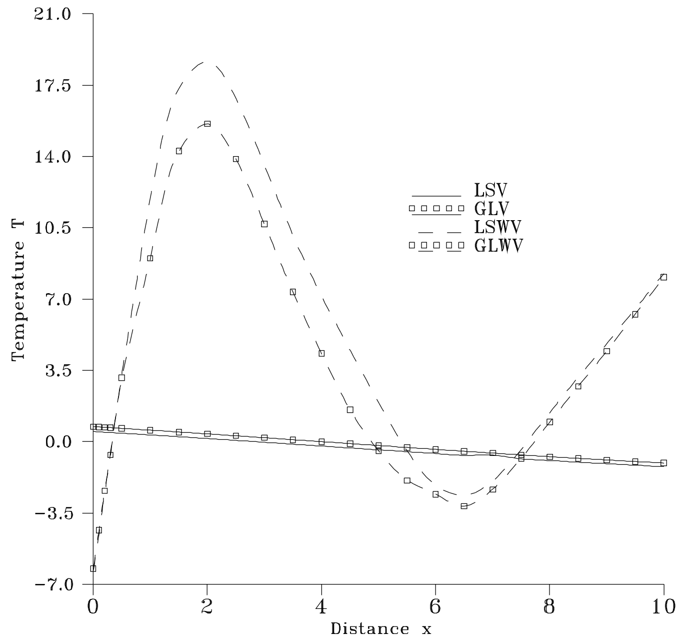

Fig.10. Variation of temperature T with distance x. (Temperature gradient boundary)

Case 2. $U > c_1$

Figure 11 depicts the variation of t_{zz} with distance x. Due to the presence of voids the values of t_{zz} for LSV and GLV increase slowly with an increase in distance x. Initially, the values of t_{zz} for GLWV are greater than LSWV and then become oscillatory in the whole range for both the theories.

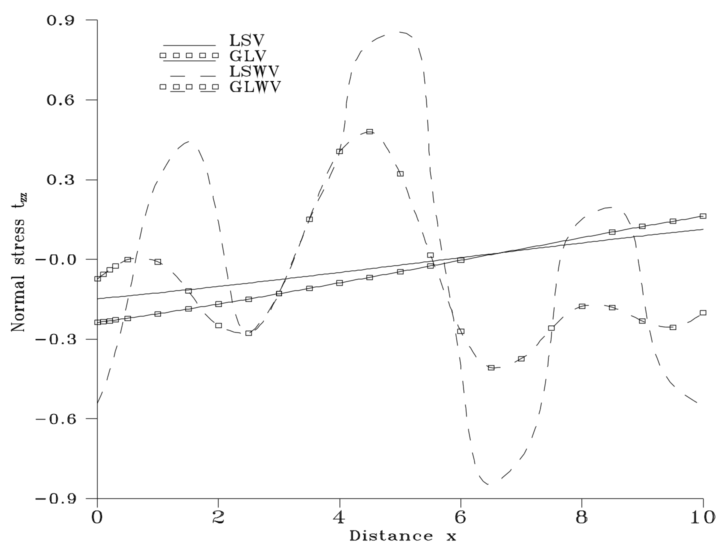

Fig.11. Variation of normal stress t_{zz} with distance x. (Temperature gradient boundary).

Figure 12 shows the variation of temperature distribution T with distance x. Due to the presence of voids the values of T for LSV and GLV increase slowly with an increase in distance x. The values of T show an oscillatory behaviour in the whole range for LSWV and GLWV.

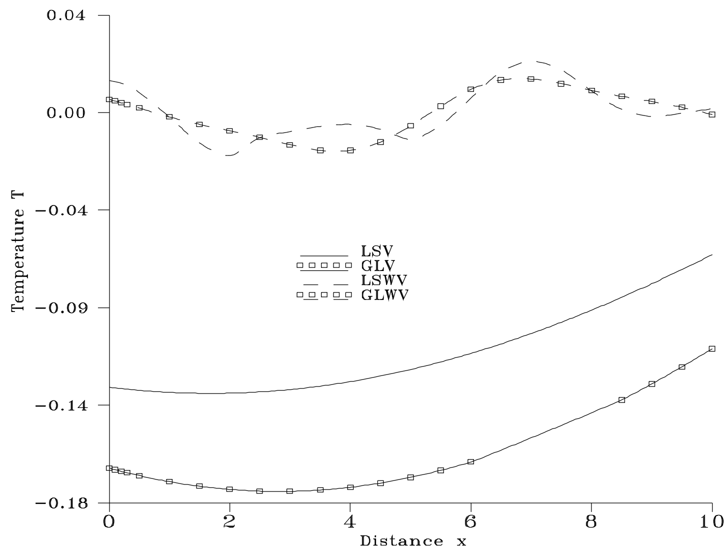

Fig.12. Variation of temperature T with distance x. (Temperature gradient boundary).

Case 3. $U = c_1$

The variations of normal stress t_{zz} are shown in Fig.13. Near the source application the values of GLWV are greater than LSWV and experience same oscillatory behavior in the whole range. For LSV and GLV the values of t_{zz} increase sharply with an increase in distance x.

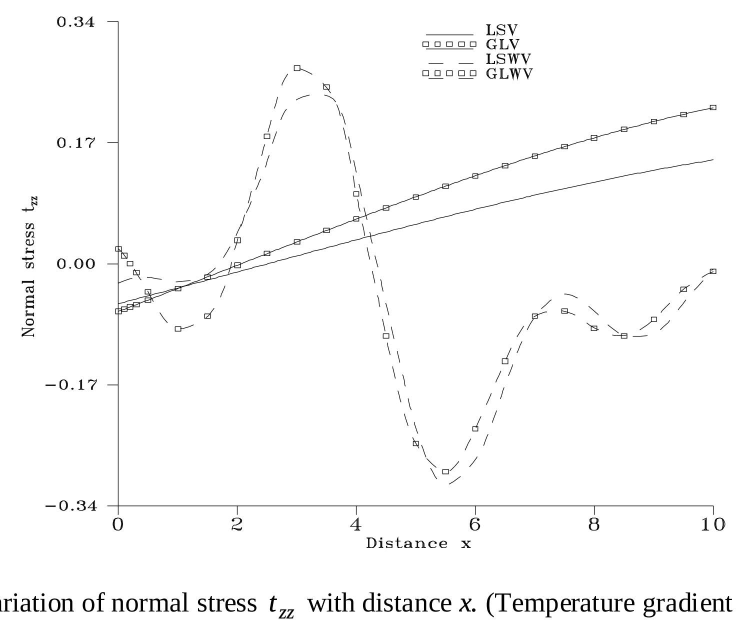

Fig.13. Variation of normal stress t_{zz} with distance x. (Temperature gradient boundary).

Figure 14 depicts the variation of temperature distribution T with distance x. The values of T for LSWV and GLWV depict same oscillatory behaviour in the whole range whereas for LSV and GLV the values of T increase with an increase in distance x in the range $0 \le x \le 10$.

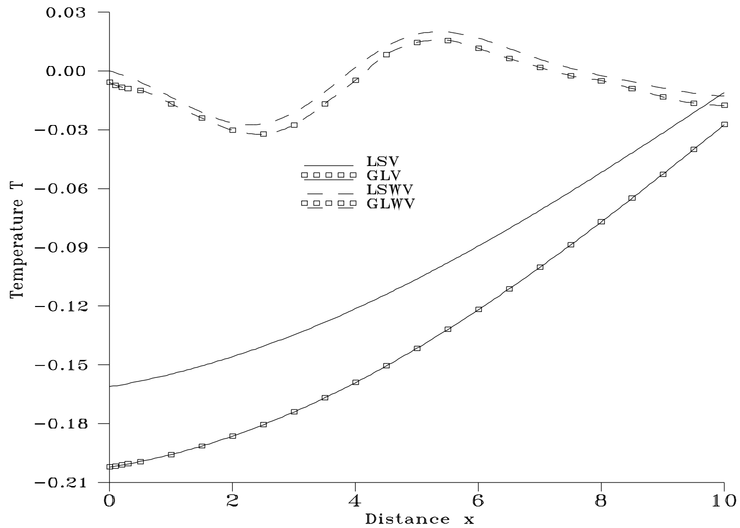

Fig.14. Variation of temperature T with distance x. (Temperature gradient boundary).

Figure 15 shows the variation of ϕ with distance x. For the case $U < c_1$, the values of ϕ for LSV are greater than GLV in the whole range, for $U > c_1$, the values of ϕ increase slowly whereas for $U = c_1$ the values of ϕ increase sharply with an increase in distance x for both the theories in the whole range.

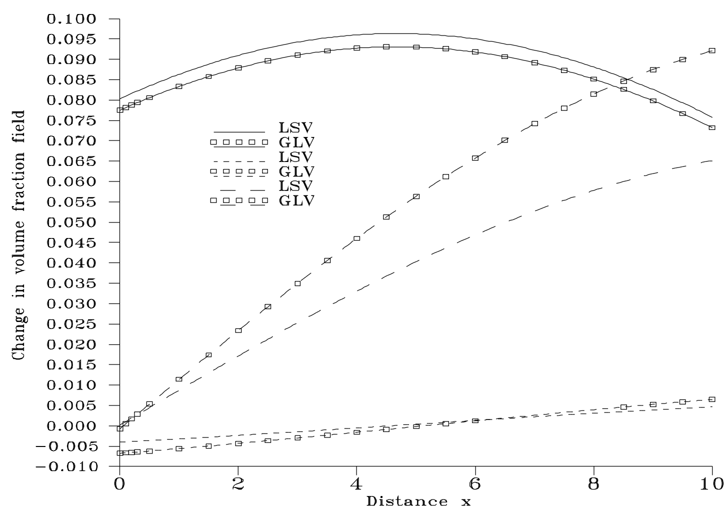

Fig.15. Variation of change in the volume fraction field with distance x due to mechanical source. (Temperature gradient boundary).

Conclusion

1. The Fourier transform technique is used to derive stresses, temperature distribution and change in the volume fraction field due to mechanical and thermal sources.
2. The voids effect in different theories of thermoelasticity, i.e, L-S and G-L for insulated and temperature gradient boundary for three different types of velocities are investigated.
3. It is observed that the magnitude of normal stress, temperature distribution and change in the volume fraction field attains a maximum near the point of application of the source.

Nomenclature

K – thermal conductivity
T – temperature change
T_0 – uniform temperature
t_{ij} – stress tensor
$\boldsymbol{u}$ – displacement vector
$\alpha, b, \xi_1, \omega_0, \chi, M$ – material constant due to the presence of voids
$\beta = (3\lambda + 2\mu)\alpha_t$
α_t – coefficient of linear thermal expansion
δ_{ij} – Kronecker delta
λ, μ – Lame’s constants
ρ, c_e – density and specific heat at constant strain
τ_0, τ_1 – thermal relaxation times
ϕ – change in volume fraction field

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Appendix A

For Lord-Shulman theory, $\tau_1 = 0$, $\delta_{1k} = 1$ and for Green-Lindsay theory $\tau_1 > 0$, $\delta_{1k} = 0$ (i.e., $k = 1$ for Lord-Shulman theory and $k = 2$ for Green-Lindsay theory). The thermal relaxations τ_0 and τ_1 satisfy the inequality $\tau_1 \geq \tau_0 > 0$ for the G-L theory only and a superposed dot represents differentiation with respect to time variable *t*.

Appendix B

$$R_{11} = -\xi^2\left(\frac{U^2}{c_1^2} - 1\right)\left(\frac{\lambda + 2\mu}{\mu}\right), \qquad R_{13} = -\frac{i\xi\beta T_0}{\mu}\left(1 - \frac{\tau_1 U\delta_{2k} i\xi}{c_1}\right),$$

$$R_{14} = \frac{i\xi b}{\chi\rho\omega_1^2}\left(\frac{\lambda + 2\mu}{\mu}\right), \qquad R_{16} = i\xi\left(\frac{\lambda + 2\mu}{\mu}\right),$$

$$R_{28}=-\frac{b(\lambda+2\mu)}{\mu\chi\rho\omega_1^{*2}},\quad R_{31}=\in_1\left(\frac{Ui\xi}{c_1}+\frac{\tau_0U^2\delta_{1k}\xi^2}{c_1^2}\right),$$

$$R_{33}=-\xi^2+\frac{c_e\rho\left(Uc_1i\xi+\tau_0U^2\,\xi^2\right)}{K\omega_1^*},\quad R_{34}=-\frac{MUc_2^3i\xi}{K\chi\omega_1^{*3}},$$

$$R_{36}=-\frac{R_{31}}{i\xi},\quad R_{41}=-\frac{i\xi b\chi}{\alpha},$$

$$R_{44}=\frac{\xi_1c_1^2}{\omega_1^{*2}\alpha}-\xi^2\left(\frac{\rho\chi U^2}{\alpha}-1\right)-\frac{i\xi\omega_0Uc_1}{\omega_1^*\alpha},\quad R_{43}=-\frac{MT_0\chi}{\alpha},\quad R_{46}=\frac{\omega_1^*b\chi}{\alpha c_1}.$$

Appendix C

$$\lambda_1=-\left(R_{11}-R_{22}+R_{33}-R_{44}-R_{16}^2-R_{28}R_{46}+\frac{R_{13}R_{36}}{i\xi}\right),$$

$$\begin{aligned}\lambda_2=&R_{11}(R_{22}+R_{33}+R_{44})-R_{14}R_{41}-R_{28}(R_{11}R_{46}+R_{16}R_{41})+\\&-\frac{R_{13}R_{36}}{i\xi}\left(\xi^2+i\xi R_{16}+R_{11}\right)-\frac{R_{13}}{i\xi}\{R_{44}R_{36}+R_{46}R_{34}+R_{16}R_{36}i\xi+\\&-R_{33}\left(R_{28}R_{46}+R_{16}^2-R_{44}\right)+R_{16}^2R_{44}-R_{43}(R_{34}-R_{28}R_{36})+R_{22}(1-R_{44})\},\end{aligned}$$

$$\begin{aligned}\lambda_3=&-[R_{14}R_{16}(R_{33}R_{46}-R_{43}R_{36})-R_{13}R_{16}(R_{46}R_{34}-R_{36}R_{44})+\\&-R_{16}^2(R_{44}R_{33}-R_{43}R_{34})-R_{22}(R_{33}R_{44}-R_{34}R_{43})+\\&-\frac{R_{13}}{i\xi}\{-R_{11}(R_{46}R_{34}+R_{44}R_{36})+R_{14}R_{36}(R_{41}+i\xi R_{46})+\\&-R_{16}(i\xi R_{44}R_{36}+R_{41}R_{34})\}-R_{26}\{R_{11}(R_{46}R_{33}-R_{43}R_{36})+\\&+R_{13}R_{36}(R_{41}+i\xi R_{46})-R_{16}(i\xi R_{43}R_{36}+R_{41}R_{33})\}-\{R_{11}(R_{44}R_{33}-R_{43}R_{34})+\\&-R_{13}(R_{34}R_{41}+i\xi R_{44}R_{36})-R_{36}(i\xi R_{43}R_{36}+R_{41}R_{33})\}+\\&-R_{22}(R_{11}R_{44}-R_{14}R_{41}+R_{11}R_{33}-i\xi R_{13}R_{36})],\end{aligned}$$

$$\begin{aligned}\lambda_4=&-R_{22}\{R_{11}(R_{44}R_{33}-R_{43}R_{34})-R_{13}(R_{34}R_{41}+i\xi R_{44}R_{36})+\\&-R_{36}(i\xi R_{43}R_{36}+R_{41}R_{33})\},\end{aligned}$$

Appendix D

$$\mathrm{X}_{\mathbf{1}}(\xi)=\ X_l(\xi)=\begin{bmatrix}X_{\mathbf{1}1}(\xi)\\X_{\mathbf{1}2}(\xi)\end{bmatrix}$$

where

$$X_{\mathbf{l}1}(\xi)=\begin{bmatrix}1\\ q_{\mathbf{l}}p_{\mathbf{l}}\\ s_{\mathbf{l}}\\ r_{\mathbf{l}}\end{bmatrix},\quad X_{\mathbf{l}2}(\xi)=\begin{bmatrix}q_{\mathbf{l}}\\ q_{\mathbf{l}}^{2}\,p_{\mathbf{l}}\\ q_{\mathbf{l}}s_{\mathbf{l}}\\ q_{\mathbf{l}}r_{\mathbf{l}}\end{bmatrix},\quad q=q_{\mathbf{l}},\quad \mathbf{l}=1,2,3,4,$$

$$X_{l_a1}(\xi)=\begin{bmatrix}1\\ -q_{\mathbf{l}}p_{\mathbf{l}}\\ s_{\mathbf{l}}\\ r_{\mathbf{l}}\end{bmatrix},\quad X_{\mathbf{l}_a2}(\xi)=\begin{bmatrix}-q_{\mathbf{l}}\\ q_{\mathbf{l}}^{2}p_{\mathbf{l}}\\ -q_{\mathbf{l}}s_{\mathbf{l}}\\ -q_{\mathbf{l}}r_{\mathbf{l}}\end{bmatrix},\quad \mathbf{l}_a=\mathbf{l}+4,\quad q=-q_{\mathbf{l}},\quad \mathbf{l}=1,2,3,4,$$

$$p_{\mathbf{l}}=-\frac{m_{\mathbf{l}1}m_{\mathbf{l}6}-m_{\mathbf{l}3}m_{\mathbf{l}4}}{m_{\mathbf{l}2}m_{\mathbf{l}6}-q_{\mathbf{l}}m_{\mathbf{l}3}m_{\mathbf{l}5}},\qquad s_{\mathbf{l}}=-\frac{m_{\mathbf{l}1}m_{\mathbf{l}5}q_{\mathbf{l}}^{2}-m_{\mathbf{l}2}m_{\mathbf{l}4}}{m_{\mathbf{l}3}m_{\mathbf{l}5}q_{\mathbf{l}}^{2}-m_{\mathbf{l}2}m_{\mathbf{l}6}},$$

$$r_{\mathbf{l}}=\frac{m_{\mathbf{l}7}m_{\mathbf{l}10}-q_{\mathbf{l}}m_{\mathbf{l}8}m_{\mathbf{l}9}}{m_{\mathbf{l}6}m_{\mathbf{l}10}+q_{\mathbf{l}}m_{\mathbf{l}11}m_{\mathbf{l}8}},$$

$$m_{\mathbf{l}1}=R_{28}\left(R_{11}-q_{\mathbf{l}}^{2}\right)+R_{14}R_{16},\qquad m_{\mathbf{l}2}=R_{16}R_{28}q_{\mathbf{l}}^{2}-R_{14}\left(R_{22}-q_{\mathbf{l}}^{2}\right),$$

$$m_{\mathbf{l}3}=\frac{R_{28}}{\mu}\left(\mu R_{13}+R_{14}\right),\qquad m_{\mathbf{l}4}=R_{31}\left(R_{44}-q_{\mathbf{l}}^{2}\right)-R_{41}R_{34},$$

$$m_{\mathbf{l}5}=-\frac{R_{31}}{i\xi}\left(R_{44}-q_{\mathbf{l}}^{2}\right)-R_{46}R_{34},\qquad m_{\mathbf{l}6}=\left(R_{44}-q_{\mathbf{l}}^{2}\right)\left(R_{33}-q_{\mathbf{l}}^{2}\right)+R_{43}R_{14},$$

$$m_{\mathbf{l}7}=R_{43}R_{31}-\left(R_{33}-q_{\mathbf{l}}^{2}\right)R_{41},\qquad m_{\mathbf{l}8}=-\frac{R_{43}R_{31}}{i\xi}-\left(R_{33}-q_{\mathbf{l}}^{2}\right)R_{46},$$

$$m_{\mathbf{l}9}=\left(R_{11}-q_{\mathbf{l}}^{2}\right)\frac{\beta a_4}{\mu}+q_{\mathbf{l}}R_{16},\qquad m_{\mathbf{l}10}=\left(q_{\mathbf{l}}^{2}-R_{11}\right)R_{13}-\frac{q_{\mathbf{l}}R_{16}R_{13}}{i\xi},$$

$$m_{\mathbf{l}11}=q_{\mathbf{l}}R_{28}R_{13}-\frac{R_{14}R_{13}}{i\xi},\qquad (\mathbf{l}=1,2,3,4).$$

Appendix E

$$\tilde{u}=-\frac{P}{\Delta}\left(\Delta_1'\bar{e}^{q_1z}-\Delta_2'\bar{e}^{q_2z}+\Delta_3'\bar{e}^{q_3z}-\Delta_4'\bar{e}^{q_4z}\right),$$

$$\tilde{w}=\frac{P}{\Delta}\left(q_1p_1\Delta_1'\bar{e}^{q_1z}-q_2p_2\Delta_2'\bar{e}^{q_2z}+q_3p_3\Delta_3'\bar{e}^{q_3z}-q_4p_4\Delta_4'\bar{e}^{q_4z}\right), \tag{E.1}$$

$$\tilde{T} = -\frac{P}{\Delta}\left(s_1\Delta_1'\bar{e}^{q_1 z} - s_2\Delta_2'\bar{e}^{q_2 z} + s_3\Delta_3'\bar{e}^{q_3 z} - s_4\Delta_4'\bar{e}^{q_4 z}\right),$$

$$\tilde{\phi} = -\frac{P}{\Delta}\left(r_1\Delta_1'\bar{e}^{q_1 z} - r_2\Delta_2'\bar{e}^{q_2 z} + r_3\Delta_3'\bar{e}^{q_3 z} - r_4\Delta_4'\bar{e}^{q_4 z}\right), \tag{E.1}$$

$$\tilde{t}_{zz} = -\frac{P}{\Delta}\left(e_1\Delta_1'\bar{e}^{q_1 z} - e_2\Delta_2'\bar{e}^{q_2 z} + e_3\Delta_3'\bar{e}^{q_3 z} - e_4\Delta_4'\bar{e}^{q_4 z}\right),$$

$$\tilde{t}_{zx} = -\frac{P}{\Delta}\left(d_1\Delta_1'\bar{e}^{q_1 z} - d_2\Delta_2'\bar{e}^{q_2 z} + d_3\Delta_3'\bar{e}^{q_3 z} - d_4\Delta_4'\bar{e}^{q_4 z}\right)$$

where

$$\Delta = \Delta_1^0 + h\Delta_2^0,$$

$$\begin{aligned}\Delta_1^0 = [&e_1\{d_2q_3q_4(r_3s_4 - r_4s_3) - d_3q_2q_4(r_2s_4 - r_4s_2) + d_4q_2q_3(r_2s_3 - r_3s_2)\} + \\ &- e_2\{d_1q_3q_4(r_3s_4 - r_4s_3) - d_3q_1q_4(r_1s_4 - r_4s_1) + d_4q_1q_3(r_1s_3 - r_3s_1)\} + \\ &+ e_3\{d_1q_2q_4(r_2s_4 - r_4s_2) - d_2q_1q_4(r_1s_4 - r_4s_1) + d_4q_1q_2(r_1s_2 - r_2s_1)\} + \\ &- e_4\{d_1q_2q_3(r_2s_3 - r_3s_2) - d_2q_1q_3(r_1s_3 - r_3s_1) + d_3q_1q_2(r_1s_2 - r_2s_1)\}],\end{aligned}$$

$$\begin{aligned}\Delta_2^0 = [&e_1\{d_2(r_4s_3q_4 - r_3s_4q_3) - d_3(r_3s_2q_4 - r_2s_4q_2) + d_4(r_3s_2q_3 - r_2s_3q_2)\} + \\ &- e_2\{d_1(r_4s_3q_4 - r_3s_4q_3) - d_3(r_4s_1q_4 - r_1s_4q_1) + d_4(r_3s_1q_3 - r_1s_3q_1)\} + \\ &+ e_3\{d_1(r_4s_2q_4 - r_2s_4q_2) - d_2(r_4s_1q_4 - r_1s_4q_1) + d_4(r_2s_1q_2 - r_1s_2q_1)\} + \\ &- e_4\{d_1(r_3s_2q_3 - r_2s_3q_2) - d_2(r_3s_1q_3 - r_1s_3q_1) + d_3(r_2s_1q_2 - r_1s_2q_1)\}],\end{aligned}$$

$$\begin{aligned}\Delta_1' = [&\{d_2q_3q_4(r_3s_4 - r_4s_3) - d_3q_2q_4(r_2s_4 - r_4s_2) + d_2q_3q_2(r_2s_3 - r_3s_2) + \\ &+ e_2q_3q_4(r_3s_4 - r_4s_3) - e_3q_2q_4(r_2s_4 - r_4s_2) + e_2q_3q_2(r_2s_3 - r_3s_2)\} + \\ &+ h\{d_2(r_4q_4s_3 - r_3q_3s_4) - d_3(r_4q_4s_2 - r_2q_2s_4) + d_4(r_3q_3s_2 - r_2q_2s_3) + \\ &+ e_2(r_4q_4s_3 - r_3q_3s_4) - e_3(r_4q_4s_2 - r_2q_2s_4) + e_4(r_3q_3s_2 - r_2q_2s_3)\}],\end{aligned}$$

$$\begin{aligned}\Delta_2' = [&\{d_1q_3q_4(r_3s_4 - r_4s_3) - d_3q_1q_4(r_2s_1 - r_1s_4) + d_4q_3q_1(r_1s_3 - r_3s_1) + \\ &+ e_1q_3q_4(r_3s_4 - r_4s_3) - e_3q_1q_4(r_2s_1 - r_1s_4) + e_4q_3q_1(r_1s_3 - r_3s_1)\} + \\ &+ h\{d_1(r_4q_4s_3 - r_3q_3s_4) - d_3(r_4q_4s_1 - r_1q_1s_4) + d_4(r_3q_3s_1 - r_1q_1s_3) + \\ &+ e_1(r_4q_4s_3 - r_3q_3s_4) - e_3(r_4q_4s_1 - r_1q_1s_4) + e_4(r_3q_3s_1 - r_1q_1s_3)\}],\end{aligned}$$

$$\begin{aligned}\Delta_3' = [&\{d_1q_2q_4(r_2s_4 - r_4s_2) - d_2q_1q_4(r_1s_4 - r_4s_1) + d_4q_2q_1(r_1s_2 - r_2s_1) + \\ &+ e_1q_2q_4(r_2s_4 - r_4s_2) - e_2q_1q_4(r_1s_4 - r_4s_1) + e_4q_2q_1(r_1s_2 - r_2s_1)\} + \\ &+ h\{d_1(r_4q_4s_2 - r_2q_2s_4) - d_2(r_4q_4s_1 - r_1q_1s_4) + d_4(r_2q_2s_1 - r_1q_1s_2) + \\ &+ e_1(r_4q_4s_2 - r_2q_2s_4) - e_2(r_4q_4s_1 - r_1q_1s_4) + e_4(r_2q_2s_1 - r_1q_1s_2)\}],\end{aligned}$$

$$\Delta_4' = [\{d_1q_2q_3(r_2s_3 - r_3s_2) - d_2q_1q_3(r_1s_3 - r_3s_1) + d_3q_2q_1(r_1s_2 - r_2s_1) + \\ + e_1q_2q_3(r_2s_3 - r_3s_2) - e_2q_1q_3(r_1s_3 - r_3s_1) + e_3q_2q_1(r_1s_2 - r_2s_1)\} + \\ + h\{d_1(r_3q_3s_2 - r_2q_2s_3) - d_2(r_3q_3s_1 - r_1q_1s_3) + d_3(r_2q_2s_1 - r_1q_1s_2) + \\ + e_1(r_3q_3s_2 - r_2q_2s_3) - e_2(r_3q_3s_1 - r_1q_1s_3) + e_3(r_2q_2s_1 - r_1q_1s_2)\}],$$

$$e_{\mathbf{l}} = -i\xi\lambda + (\lambda + 2\mu)q_{\mathbf{l}}^2 p_{\mathbf{l}} + \frac{bc_1^2 r_{\mathbf{l}}}{\omega_1^* \chi} - \beta T_0\left(1 - \frac{i\xi U \tau_1 \delta_{2k}}{c_1}\right)s_{\mathbf{l}},$$

$$d_{\mathbf{l}} = q_{\mathbf{l}}(1 - i\xi p_{\mathbf{l}}), \qquad \mathbf{l} = 1, 2, 3, 4\ .$$

Appendix F

$$\tilde{u} = -\frac{P}{\overline{\Delta}}\left(\overline{\Delta}_1 \bar{e}^{q_1' z} - \overline{\Delta}_2 \bar{e}^{q_2' z} + \overline{\Delta}_3 \bar{e}^{q_3' z}\right),$$

$$\tilde{w} = \frac{P}{\overline{\Delta}}\left(b_1 q_1' \overline{\Delta}_1 \bar{e}^{q_1' z} - b_2 q_2' \overline{\Delta}_2 \bar{e}^{q_2' z} + b_3 q_3' \overline{\Delta}_3 \bar{e}^{q_3' z}\right),$$

$$\tilde{T} = -\frac{P}{\overline{\Delta}}\left(a_1 \overline{\Delta}_1 \bar{e}^{q_1' z} - a_2 \overline{\Delta}_2 \bar{e}^{q_2' z} + a_3 \overline{\Delta}_3 \bar{e}^{q_3' z}\right), \tag{F.1}$$

$$\tilde{t}_{zz} = -\frac{P}{\overline{\Delta}}\left(n_1 \overline{\Delta}_1 \bar{e}^{q_1' z} - n_2 \overline{\Delta}_2 \bar{e}^{q_2' z} + n_3 \overline{\Delta}_3 \bar{e}^{q_3' z}\right),$$

$$\tilde{t}_{zx} = -\frac{P}{\overline{\Delta}}\left(u_1 \overline{\Delta}_1 \bar{e}^{q_1' z} - u_2 \overline{\Delta}_2 \bar{e}^{q_2' z} + u_3 \overline{\Delta}_3 \bar{e}^{q_3' z}\right)$$

where

$$\overline{\Delta} = \overline{\Delta}_1' + h\overline{\Delta}_2',$$

$$\overline{\Delta}_1' = n_1(u_2q_3'R_{14} - u_3q_2'R_{16}) - n_2(u_1q_3'R_{14} + u_3q_1'R_{11}) + n_3(u_1q_2'R_{16} + u_2q_1'R_{11}),$$

$$\overline{\Delta}_2' = n_1(u_3R_{16} - u_2R_{14}) + n_2(u_3R_{11} + u_1R_{14}) - n_3(u_2R_{11} + u_1R_{16}),$$

$$\overline{\Delta}_1 = (u_2a_3q_3' - u_3a_2q_2' + n_2a_3q_3' - n_3a_2q_2') - h(u_2a_3 - u_3a_2 + n_2a_3 - n_3a_2),$$

$$\overline{\Delta}_2 = (u_1a_3q_3' - u_3a_2q_2' + n_1a_3q_3' - n_3a_2q_2') - h(u_1a_3 - u_3a_2 + n_1a_3 - n_3a_2),$$

$$\overline{\Delta}_3 = (u_1a_2q_2' - u_2a_1q_1' + n_1a_2q_2' - n_2a_1q_1') - h(u_1a_2 - u_2a_1 + n_1a_2 - n_2a_1),$$

$$u_{\mathbf{l}} = (1 - i\xi b_{\mathbf{l}})q_{\mathbf{l}}',$$

$$n_{\mathbf{l}} = -i\xi\lambda + \mu q_{\mathbf{l}}'^{2} b_{\mathbf{l}} - \beta T_0\left(1 - \frac{i\xi U\tau_1\delta_{2k}}{c_1}\right)a_{\mathbf{l}},$$

$$a_{\mathbf{l}} = \frac{\mu i\xi\left\{R_{16}^2 q_{\mathbf{l}}'^2 - \left(q_{\mathbf{l}}'^2 - R_{11}\right)\left(q_{\mathbf{l}}'^2 - R_{22}\right)\right\}}{\overline{\Delta}^{\otimes}},$$

$$b_{\mathbf{l}} = \frac{q_{\mathbf{l}}' \left\{\mu R_{13}\left(q_{\mathbf{l}}'^2 - R_{11}\right) - \xi^2\beta R_{16}\right\}}{\overline{\Delta}^{\otimes}},$$

$$\overline{\Delta}^{\otimes} = \mu R_{13} R_{16} q_{\mathbf{l}}'^2 - \left(q_{\mathbf{l}}'^2 - R_{22}\right)\xi^2\beta, \qquad \mathbf{l} = 1, 2, 3,$$

and $q_{\mathbf{l}}'$ are the roots of the equation

$$\left(\frac{d^6}{dz^6} + \lambda_1'\frac{d^4}{dz^4} + \lambda_2'\frac{d^2}{dz^2} + \lambda_3'\right)\left(\tilde{u}, \tilde{w}, \tilde{T}\right) = 0,$$

$$\lambda_1' = 1 - R_{33} - R_{22} - R_{16}^2 + \frac{R_{13}R_{36}}{i\xi},$$

$$\lambda_2' = \frac{R_{13}R_{36}}{i\xi} - \frac{\xi^2\beta R_{36}}{\mu R_{11}} - R_{33}\left(1 - R_{22} - R_{16}^2\right) - R_{22},$$

$$\lambda_3' = \left(\frac{\xi^2\beta R_{36}}{\mu R_{11}} + R_{33}\right)R_{22} - \frac{R_{16}R_{13}R_{36}}{R_{11}}.$$

Appendix G

$$\begin{aligned}
\tilde{u} &= \left(\Delta_1'' \bar{e}^{q_1 z} + \Delta_2'' \bar{e}^{q_2 z} + \Delta_3'' \bar{e}^{q_3 z} + \Delta_4'' \bar{e}^{q_4 z}\right)/\Delta, \\
\tilde{w} &= -\left(q_1 p_1 \Delta_1'' \bar{e}^{q_1 z} + q_2 p_2 \Delta_2'' \bar{e}^{q_2 z} + q_3 p_3 \Delta_3'' \bar{e}^{q_3 z} + q_4 p_4 \Delta_4'' \bar{e}^{q_4 z}\right)/\Delta, \\
\tilde{T} &= \left(s_1 \Delta_1'' \bar{e}^{q_1 z} + s_2 \Delta_2'' \bar{e}^{q_2 z} + s_3 \Delta_3'' \bar{e}^{q_3 z} + s_4 \Delta_4'' \bar{e}^{q_4 z}\right)/\Delta, \\
\tilde{\phi} &= \left(r_1 \Delta_1'' \bar{e}^{q_1 z} + r_2 \Delta_2'' \bar{e}^{q_2 z} + r_3 \Delta_3'' \bar{e}^{q_3 z} + r_4 \Delta_4'' \bar{e}^{q_4 z}\right)/\Delta, \\
\tilde{t}_{zz} &= \left(e_1 \Delta_1'' \bar{e}^{q_1 z} + e_2 \Delta_2'' \bar{e}^{q_2 z} + e_3 \Delta_3'' \bar{e}^{q_3 z} + e_4 \Delta_4'' \bar{e}^{q_4 z}\right)/\Delta, \\
\tilde{t}_{zx} &= \left(d_1 \Delta_1'' \bar{e}^{q_1 z} + d_2 \Delta_2'' \bar{e}^{q_2 z} + d_3 \Delta_3'' \bar{e}^{q_3 z} + d_4 \Delta_4'' \bar{e}^{q_4 z}\right)/\Delta
\end{aligned} \tag{G.1}$$

where

$$\Delta_1'' = e_3\left(d_2 r_4 q_4 - d_4 r_2 q_2\right) - e_2\left(d_3 r_4 q_4 - d_4 r_3 q_3\right) - e_4\left(d_2 r_3 q_3 - d_3 r_2 q_2\right),$$

$$\Delta_2'' = e_4\left(d_1 r_3 q_3 - d_3 r_1 q_1\right) - e_1\left(d_3 r_4 q_4 - d_4 r_3 q_3\right) - e_3\left(d_1 r_4 q_4 - d_4 r_1 q_1\right),$$

$$\Delta_3'' = e_2\left(d_1 r_4 q_4 - d_4 r_1 q_1\right) - e_1\left(d_2 r_4 q_4 - d_4 r_2 q_2\right) - e_4\left(d_1 r_2 q_2 - d_2 r_1 q_1\right),$$

$$\Delta_4'' = e_3\left(d_1 r_3 q_3 - d_3 r_3 q_1\right) - e_2\left(d_1 r_3 q_3 - d_3 r_1 q_1\right) + e_1\left(d_2 r_3 q_3 - d_3 r_2 q_2\right).$$

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