*Int. J. of Applied Mechanics and Engineering, 2006, vol.11, No.1, pp.77-91*

# DYNAMIC ANALYSIS OF BALL BEARINGS WITH EFFECT OF PRELOAD AND NUMBER OF BALLS

R.K. PUROHIT* and K. PUROHIT
M.B.M. Engineering College, J.N.V. University „Shyamendra Bhavan”
D/1, Shastri Nagar, Jodhpur – 342003 (Raj.), INDIA
e-mail: ramkpurohit@yahoo.com

In this paper, the radial and axial vibrations of rigid shaft supported ball bearings are studied. In the analytical formulation the contacts between the balls and the races are considered as nonlinear springs, whose stiffness are obtained by using the Hertzian elastic contact deformation theory. The implicit type numerical integration technique Newmark-$\beta$ with Newton-Raphson method is used to solve the nonlinear differential equations iteratively. The effect on vibrations of varying preload and the number of balls in the bearings is investigated for prefect bearings. For perfect bearings, vibrations occur at the ball passage frequency. The amplitudes of these vibrations are shown to be considerably reduced if the preload and number of balls are correctly selected. All results are presented in the form of Fast Fourier Transformations (FFT).

## 1. Introduction

An analysis of ball bearing dynamie behavior is important to predict the system vibration responses. The behavior of nonlinear systems often demonstrates unexpected behavior patterns that are extremely sensitive to initial conditions. When rolling element bearings are operated at high speed, they generate vibrations and noise. The principle forces, which drive these vibrations, are time varying nonlinear contact forces, which exist between the various components of the bearings: rolling elements, races and shafts. In the shaft bearing assembly supported by perfect ball bearings, the vibration spectrum is dominated by the vibrations at the natural frequency and the ball passage frequency (BPF). The vibrations at this later frequency are called ball passage vibrations (BPV).

The first work on the ball passage vibrations was done by Perret (1950) and Meldau (1951) as a static running accuracy problem. They suggested that an increase in the number of balls in a bearing reduces its untoward effects. Gustafson *et al*. (1963) studied the effects of waviness and pointed out that lower order ring waviness affects the amplitude of the vibrations at the ball passage frequency. They observed that vibrations at higher harmonics of the ball passage frequency are also present in the vibration spectrum and their amplitudes depend on the radial load, radial clearance, rotational speed and the order of harmonics. The same conclusion was theoretically proved by Meyer *et al.* (1980) for perfect radial ball bearings with linear modeling of the spring characteristics of balls. Gad *et al*. (1984) showed that resonance occurs when BPF coincides with the frequency of the system and they also pointed out that for certain speeds, BPF can exhibit its sub and super harmonic vibrations for shaft ball bearing systems. Ji-Huan He (2000) dcveloped some analytical techniques for solving nonlinear equations. These techniques are used to increase numerical stability and decrease the computer time for system analysis.

El-Sayed (1980) derived a form of equation for the stiffness of bearings and determined total deflections of inner and outer races caused by an applied load, using the Hertz theory. Tamura and Tsuda (1980) performed a theoretical study of fluctuations of the radial spring characteristics of a ball bearing due to ball revolutions. Wardle and Poon (1983) pointed out the relations between the number of balls and waves for sever vibrations to occur. When the number of balls and waves are equal there would be severing vibrations. Wardle (1988a) showed that ball waviness produced vibrations in the axial and radial directions at different frequencies and also pointed out that only even orders of ball waviness produced vibrations.

* To whom correspondence should be addressed

Rahnejat and Gohar (1988) showed that even in the presence of elastohydrodynamic lubricating film between balls and the races, a peak at the BPF appears in the spectrum. Aktiirk (1999) presented the effect of surface waviness on vibrations associated with ball bearings and concluded that for outer race waviness most sever vibrations occur when the ball passage frequency (BPF) and its harmonics coincide with the natural frequency. Aktiirk ***et al.*** (1997) performed a theoretical investigation of the effect of varying the preload on the vibration characteristics of a shaft bearing system and also suggested that by taking the correct number of balls and amount of preload in a bearing the untoward effect of the BPV can be reduced.

In this paper, a theoretical model was made to observe the effect of varying the preload and number of balls on the vibration characteristics of a defect free system. A two-degree of freedom system is considered with the assumption that there is no friction between the balls and raceways and that both bearings are positioned symmetrically so that their moving parts are synchronized. To study the nonlinear dynamic responses of bearings FFT are obtained.

## 2. Modeling of the system

As a first step in investigating the vibrations characteristics of ball bearings, a model of a rotor-bearing assembly can be considered as a spring-mass system, where the rotor acts as a mass and the raceways and balls act as mass less nonlinear contact springs. In the model, the outer race of the bearing is fixed in a rigid support and the inner race is fixed rigidly with the rotor. A constant radial vertical force acts on the bearing. Therefore, the system undergoes nonlinear vibrations under dynamic conditions.

Elastic deformation between the race and ball gives a non-linear force deformation relation, which is obtained by using the Hertzian theory. Other sources of stiffness variation are the positive internal radial clearance, the finite number of balls whose position changes periodically and waviness at the inner and outer race. They cause periodic changes in stiffness of the bearing assembly. Taking into account these sources of stiffness variation the governing differential equations are obtained.

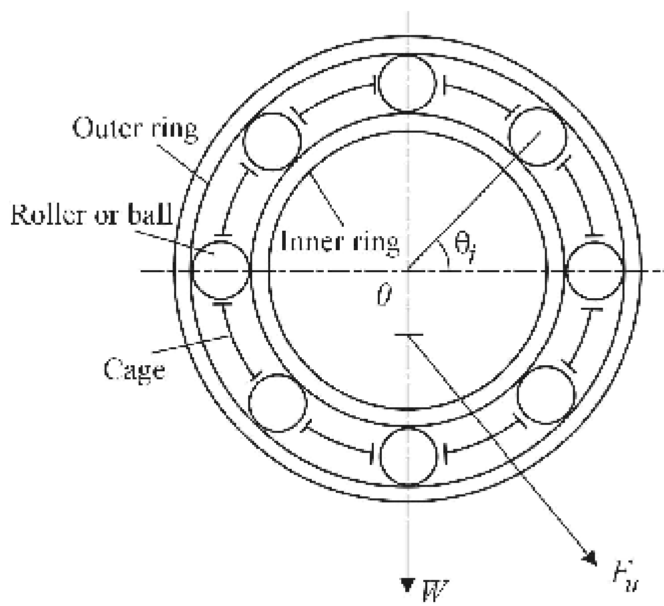



Fig.1. A schematic diagram of a rolling element bearing.

A schematic diagram of a rolling element bearing is shown in Fig.1. In the mathematical model, the ball bearings are considered as a non-linear mass-spring system. Since the Hertzian forces arise only when there is contact deformation, the springs are required to act only in compression. In other words, the respective spring force comes into play when the instantaneous spring length is shorter than its unstressed length, otherwise the separation between the ball and race takes place and the resulting force is set to zero. An unbalance force (Fu) is due to rotating of the shaft with inner race. The assumptions made in the development of the mathematical model are as follows:

1. Balls are positioned equi-pitched around the inner race and there is no interaction between them.
2. The outer race is fixed rigidly to the support and the inner race is fixed rigidly to the shaft.
3. The ball, inner and outer races and the cage have motions in the plane of the bearing only. This eliminates any motion in the axial direction.
4. The bearings are assumed to operate under isothermal conditions.
5. There is no slipping of balls as they roll on the surface of races.
6. The races are flexurally rigid and undergo only local deformations due to the stresses in contacts,
7. Deformations occur according to the Hertzian theory of elasticity.

### 2.1. Calculation of restoring force

The local Hertzian contact force and deflection relationship for a bearing may be written as

$$F_{\theta_i} = k\left(r_{\theta i}\right)^p \quad \text{where} \quad p = 3/2\,. \tag{2.1}$$

Here $r_{\theta i}$ is the radial deflection due to misalignment of races.
$R_{\theta_i}$ is the displacement at the i^th ball, which is given as

$$R_{\theta i} = x\cos\theta i + y\sin\theta i\,. \tag{2.2}$$

Considering the internal radial clearance

$$R_{\theta i} = x\cos\theta i + y\sin\theta i = \left(r_{\theta i} + \gamma\right). \tag{2.3}$$

Substituting $r_{\theta i}$

$$F_{\theta i} = k\left[\left(x\cos\theta i + y\sin\theta i\right) - (\gamma)\right]_+^{3/2}. \tag{2.4}$$

If the expression inside the bracket is greater than zero, then the ball at the angular location $\theta i$ is loaded giving rise to a restoring force $F_{\theta i}$. If the expression in the bracket is negative or zero, then the ball is not in the load zone, and the restoring force $F_{\theta i}$ is set to zero. The total restoring force is the sum of the restoring force from each of the rolling elements. Thus the total restoring force components in the *X* and *Y* directions are

$$\begin{aligned} F_x &= \sum_{i=1}^{N_b} k\left[\left(x\cos\theta i + y\sin\theta i\right) - (\gamma)\right]_+^{3/2}\cos\theta i\,, \\ F_y &= \sum_{i=1}^{N_b} k\left[\left(x\cos\theta i + y\sin\theta i\right) - (\gamma)\right]_+^{3/2}\sin\theta i\,. \end{aligned} \tag{2.5}$$

### 2.2. Contact stiffness

Hertz considered the stress and deformation in the perfectly smooth, ellipsoidal, contacting elastic solids. The application of the classical theory of elasticity to the problem forms the basis of stress calculation for machine elements such as the ball and roller bearings. Therefore the point of contact between the race and ball develops into a contact area which has the shape of an ellipse with ***a*** and ***b*** as the semi major and semi minor axes respectively. The curvature sum and difference are needed in order to obtain the contact force of the ball. The curvature sum $\sum\rho$ obtained following Harris (1991) is expressed as

$$\sum\rho = \rho_{I1} + \rho_{I2} + \rho_{II1} + \rho_{II2} = \frac{1}{r_{I1}} + \frac{1}{r_{I2}} + \frac{1}{r_{II1}} + \frac{1}{r_{II2}}. \tag{2.6}$$

The curvature difference $F(\rho)$ is expressed as

$$F(\rho) = \frac{(\rho_{I1} - \rho_{I2}) + (\rho_{II1} - \rho_{II2})}{\sum\rho}. \tag{2.7}$$

The parameters $r_{I1}, r_{I2}, r_{II1}, r_{II2}, \rho_{I1}, \rho_{I2}, \rho_{II1}, \rho_{II2}$ are given dependent upon calculations referring to the inner and outer races as shown in Fig.2. If the inner race is considered

$$\begin{gathered} r_{I1} = D/2, \quad r_{I2} = D/2, \quad r_{II1} = d_i/2, \quad r_{II2} = r_i \quad \text{and} \\ \rho_{I1} = 2/D, \quad \rho_{I2} = 2/D, \quad \rho_{II1} = 2/d_i, \quad \rho_{II2} = -1/r_i \end{gathered} \tag{2.8}$$

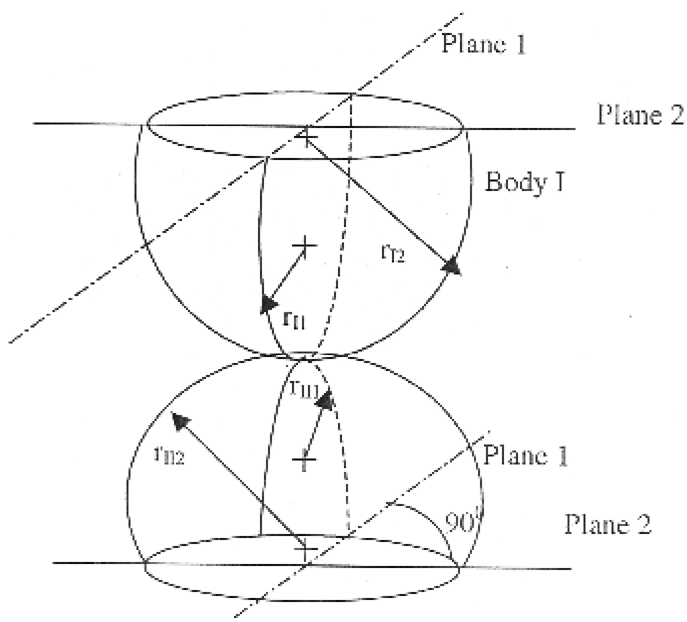



Fig.2. Geometry of contacting bodies.

If the outer race is considered, they are given as

$$r_{I1} = D/2, \quad r_{I2} = D/2, \quad r_{II1} = d_o/2, \quad r_{II2} = r_o \quad \text{and}$$
$$\rho_{I1} = 2/D, \quad \rho_{I2} = 2/D, \quad \rho_{II1} = -2/d_o, \quad \rho_{II2} = -1/r_o \tag{2.9}$$

As per the sign convention followed, negative radius denotes a concave surface. Using Tab.2 we can calculate all the parameters including the curvature difference at the inner and outer race. For the contacting bodies made of steel, the relative approach between two contacting and deforming surfaces is given by

$$\delta = 2.787 \times 10^{-8} Q^{2/3} \left(\sum \rho\right)^{1/3} \delta^* \tag{2.10}$$

where $\delta^*$ is a function of $F(\rho)$.
Hence, the contact force $(Q)$ is

$$Q = \left\{3.587 \times 10^{7} \left(\sum \rho\right)^{1/2} \left(\delta^*\right)^{-3/2}\right\} \delta^{3/2} \quad (N). \tag{2.11}$$

The elastic modulus for the contact of a ball with the inner race is

$$K_i = 3.587 \times 10^{7} \left(\sum \rho\right)^{-1/2} \left(\delta_i^*\right)^{-3/2} \quad \left(\frac{N}{mm}\right). \tag{2.12}$$

And for the contact of a ball with the outer race is

$$K_o = 3.587 \times 10^{7} \left(\sum \rho_o\right)^{-1/2} \left(\delta_o^*\right)^{-3/2} \quad \left(\frac{N}{mm}\right). \tag{2.13}$$

Then the effective elastic modulus $K$ for the bearing system is written as

$$K = \frac{1}{\left( 1/K_i^{1/n} + 1/K_o^{1/n} \right)^n}. \tag{2.14}$$

In Eqs (2.12) and (2.13), the parameters $\delta_i^*$ and $\delta_o^*$ can be obtained from Tab.1, whereas the values of $F(\rho)_i$ and $F(\rho)_o$ are available from Tab.2. The effective elastic modulus $(K)$ for a bearing system using geometrical and physical parameters is written as

$$K = 7.055 \times 10^{5} \sqrt{\delta} \quad \left(\frac{N}{mm}\right). \tag{2.15}$$

Table 1. Dimensional contact parameters by Harris (1991).

| $F(\rho)$ | $\delta^*$ |
|---|---|
| 0 | 1 |
| 0.1075 | 0.997 |
| 0.3204 | 0.9761 |
| 0.4795 | 0.9429 |
| 0.5916 | 0.9077 |
| 0.6716 | 0.8733 |
| 0.7332 | 0.8394 |
| 0.7948 | 0.7961 |
| 0.83595 | 0.7602 |
| 0.87366 | 0.7169 |
| 0.90999 | 0.6636 |
| 0.93657 | 0.6112 |
| 0.95738 | 0.5551 |
| 0.97290 | 0.4960 |
| 0.983797 | 0.4352 |
| 0.990902 | 0.3745 |
| 0.995112 | 3176 |
| 0.997300 | 0.2705 |
| 0.9981847 | 0.2427 |
| 0.9989156 | 0.2106 |
| 0.9994785 | 0.17167 |
| 0.9998527 | 0.11995 |
| 1 | 0 |

Table 2. Geometrie and physical properties used for the ball bearings.

| | |
|---|---|
| Ball radius $(r_b)$ | $3.98\,mm$ |
| Inner race radius $(r_i)$ | $23\,mm$ |
| Outer race radius $(r_o)$ | $46\,mm$ |
| Internal radial clearance $(\gamma)$ | $0.1\,\mu m$ |
| Radial load $(W)$ | $6N$ |
| Mass of bearing $(m)$ | $0.6\,kg$ |
| Damping factor $(c)$ | $200\,Ns/m$ |
| Number of balls $(N_b)$ | $9$ |
| Inner race groove radius $(r_{gi})$ | $4.08\,mm$ |
| Outer race groove radius $(r_{go})$ | $4.61\,mm$ |
| Speed of the rotor $(N_r)$ | $5000\,rpm$ |
| Pitch radius of the ball set $(r_m)$ | $27\,mm$ |

### 2.3. Equation of motion

The system of governing equations accounting for inertia, the restoring and damping force and constant vertical force acting on the inner race are

$$m\ddot{x} + c\dot{x} + \sum_{i=1}^{N_b} k\left[(x\cos\theta i + y\sin\theta i) - (\gamma)\right]_+^{3/2}\cos\theta i = W + F_u\cos(\omega t), \tag{2.16a}$$

$$m\ddot{y} + c\dot{y} + \sum_{i=1}^{N_b} k\left[(x\cos\theta i + y\sin\theta i) - (\gamma)\right]_+^{3/2}\sin\theta i = F_u\sin(\omega t). \tag{2.16b}$$

Here $m$ is the mass of the rotor supported by the bearing and mass of the inner race. The system of Eqs.(16) is two coupled non-linear ordinary second order differential equations having parametric effect, the $3/2$ non-linearity and the summation term. The $'+'$ sign as subscript in these equations signifies that if the expression inside the bracket is greater than zero, then the rolling element at angular location $\theta i$ is loaded giving rise to the restoring force and if the expression inside the bracket is negative or zero, then the rolling element is not in the load zone, and the restoring force is set to zero. The damping in this system is represented by an equivalent viscous damping $c$. The damping force is proportional to velocity. The unbalance force $F_u$ is taken for the balanced rotor as zero. The damping of a ball bearing is very small. This damping is present because of friction and a small amount of lubrication.

### 2.4. Ball passage frequency

When the shaft is rotating, applied loads are supported by a few balls restricted to a narrow load region and the radial position of the inner race with respect to the outer race depends on the elastic deflections at the ball to raceways contacts. Balls are deformed as they enter the loaded zone where the mutual convergence of the bearing races takes place and the balls rebound as they move to the unloaded region. The time taken by the shaft to regain its initial position is

$$t = \frac{time\ \ for\ a\ complete\ rotation\ of\ \ cage}{N_b}. \tag{2.17}$$

As the time needed for a complete rotation of the cage is $2\pi/\omega_c$ the shaft will be excited at the frequency of $(N_b \times \omega_c)$ known as the ball passage frequency. Here $\omega_c$ is the speed of the cage.

$$\omega_c = \frac{\omega_{inner}}{2}\left(1 - \frac{d_b}{d_m}\right) + \frac{\omega_{outer}}{2}\left(1 + \frac{d_b}{d_m}\right). \tag{2.18}$$

Hence, ball passage freauency $(\omega_{bp})$ is

$$\omega_{bp} = \frac{1}{2}N_b\omega_{inner}\left[1 - \frac{d_b}{d_m}\right] + \frac{1}{2}N_b\omega_{outer}\left[1 + \frac{d_b}{d_m}\right]. \tag{2.19}$$

Since outer is assumed to be constant, the ball passage frequency is

$$\omega_{bp} = \frac{1}{2} N_b \omega_{inner} \left[ 1 - \frac{d_b}{d_m} \right]. \tag{2.20}$$

Vibrations associated with the ball passage frequency are known as the ball passage vibration (BPV) or the elastic compliance vibrations. The effect of the ball passage frequency can be the worst when it coincides with a natural frequency of the shaft bearing system. The axial preload is assumed to apply through the spring contact. Balls are preloaded and the preloaded contact angle $(\alpha_p)$ is calculated as

$$P_l = mK \left[ Bd_b \left( \frac{1}{\cos(\alpha_p)} - 1 \right) \right]^{3/2} \sin(\alpha_p). \tag{2.21}$$

Where $B$ is the total curvature equals to $(A/d_b)$ and

$$A = r_{go} + r_{gi} - d_b . \tag{2.22}$$

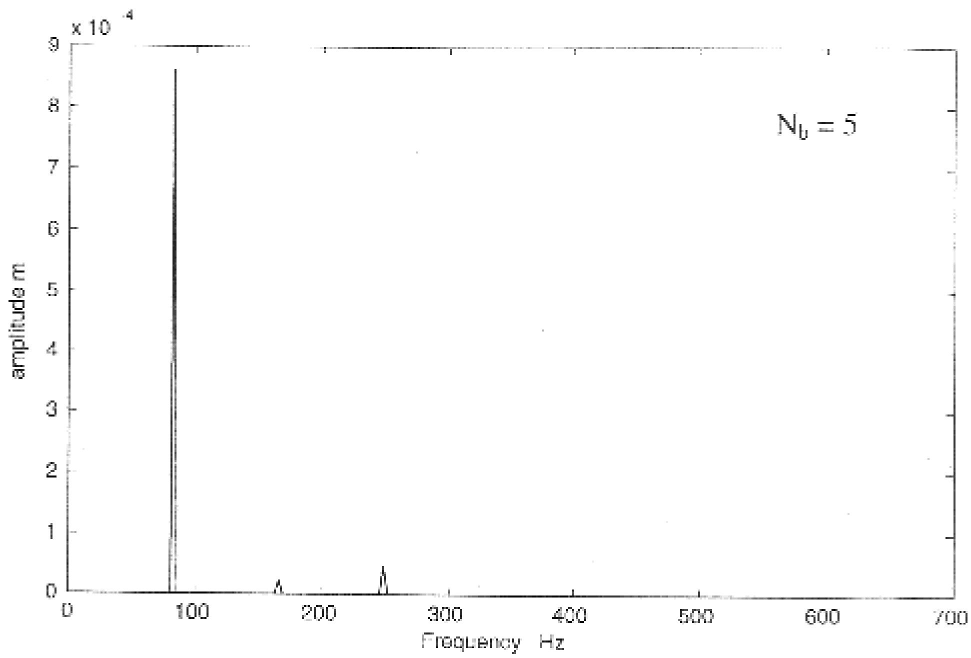
## 3. Results

The nonlinear governing equations of motion $(2.16)$ are solved by the Newmark-$\beta$ with Newton-Raphson method to obtain the axial and radial displacements of the rolling elements and the shaft. In order to study the effect of the number of balls and preload in a more detailed form, the shaft is assumed to be perfectly rigid and supported by two radial contact ball bearings. The numerical values of the parameters chosen for the numerical simulation are shown in Tab.2. The numerical stability in the result is obtained by assuming *0.00001* – radial angular rotation at each step. In order to eliminate the effect of the natural frequency an artificial damping was introduced into the system. With this damping, transient vibrations are eliminated. Thus, the peak steady state amplitude of vibration can be measured. An increase in preload or number of balls will result in stiffer ball bearings with steady state vibrations reached in a relatively longer time. The longer the time to reach the steady state vibrations, the longer CPU time needed and hence the more expensive the computation. A value of $c = 200\, Ns/m$ was chosen.

### 3.1. Effect of varying the number of balls

Increasing the number of balls means increasing the number of balls supporting the shaft therefore increasing the system stiffness and reducing the vibration amplitude. For a small number of balls, peak amplitudes of vibrations at the ball passage frequency are more significant. Figure 3a shows the response with *5* balls, the natural frequency coincides with the ball passage frequency. The natural frequency of the bearing system is *90 Hz*. Two superharmonics at the ball passage frequencies (at $2\omega_{bp} = 180\, Hz$, $3\omega_{bp} = 270\, Hz$) also appear in the spectrum. When the number of balls is *7*, the peak amplitude of vibration appears at the ball passage frequency $(\omega_{bp} = 120\, Hz)$ with two superharmonics which appear at the ($2\omega_{bp} = 240\, Hz$, $3\omega_{bp} = 360\, Hz$) as shown in Fig.3b. Figure 3c shows the response with *8* balls, the peak amplitude of vibration decreases and the natural frequency is pushed to a higher value of *135 Hz*. The peak amplitude of vibration appears at twice of ball passage frequency (at $2\omega_{bp} = 270\, Hz$) with a subharmonic which appears at the ball passage frequency *130 Hz*. When the number of balls further increased, the lower peak amplitude of vibration appears in the vibration spectrum. When the number of balls is 10, the peak amplitude of vibration appears at the ball passage frequency $(\omega_{bp} = 170\, Hz)$ with one superharmonic which appears at $(2\omega_{bp} = 340\, Hz)$ as shown in Fig.3d. For 12 balls, the peak amplitude of vibration appears at the ball passage frequency $(\omega_{bp} = 200\,Hz)$ with a lower amplitude of the superharmonic which appears at $(2\omega_{bp} = 400\,Hz)$ as shown in Fig.3e. When the number of balls is *15*, the peak amplitude of vibration appears at the ball passage frequency $(\omega_{bp} = 255\,Hz)$. The effects of superharmonic seem to be disappearing as shown in Fig.3f. Hence the nonlinear dynamic responses are found to be associated with the ball passage frequency.

a)

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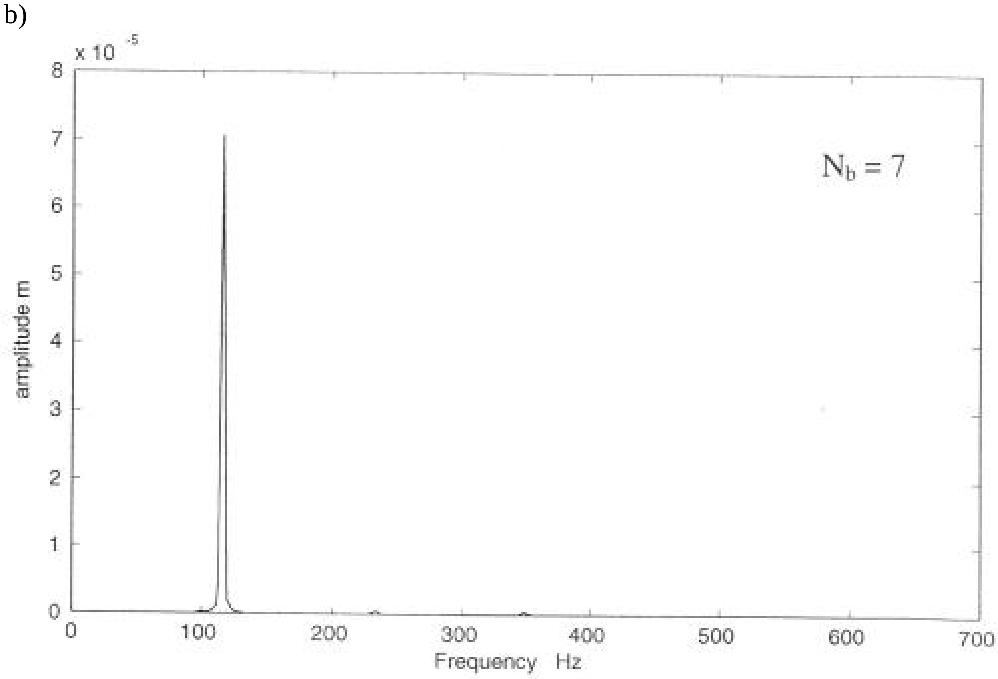

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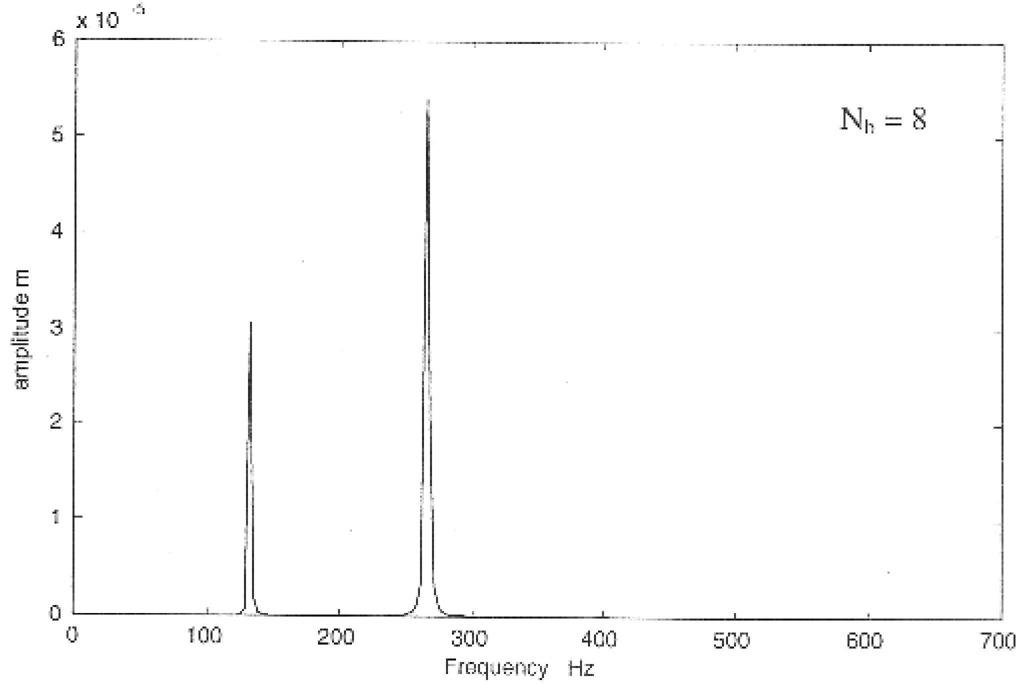
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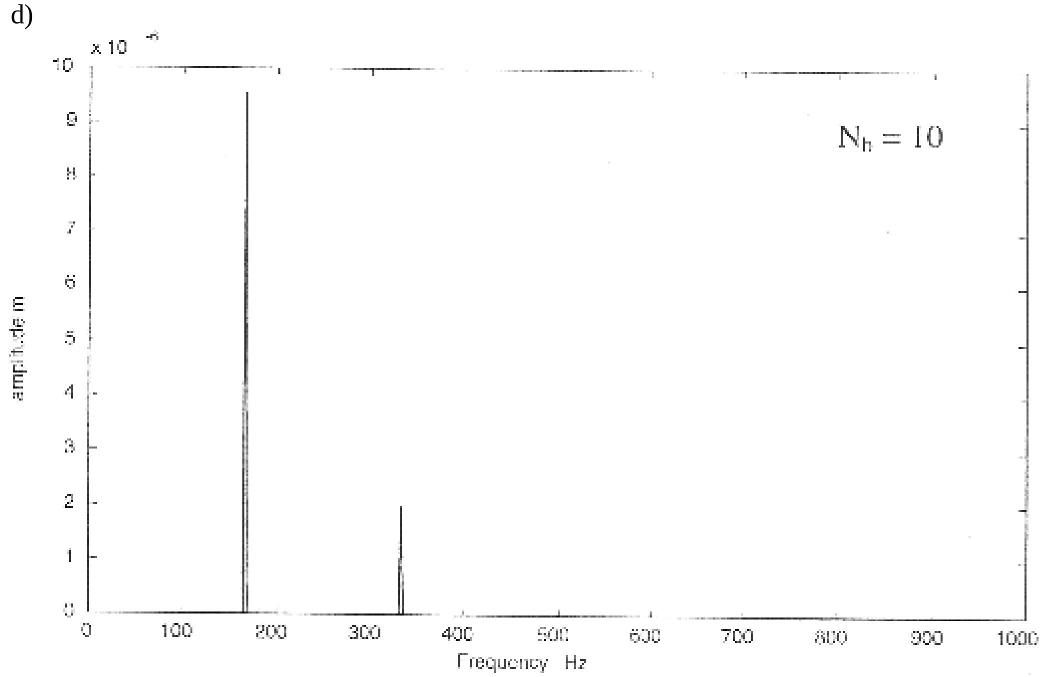
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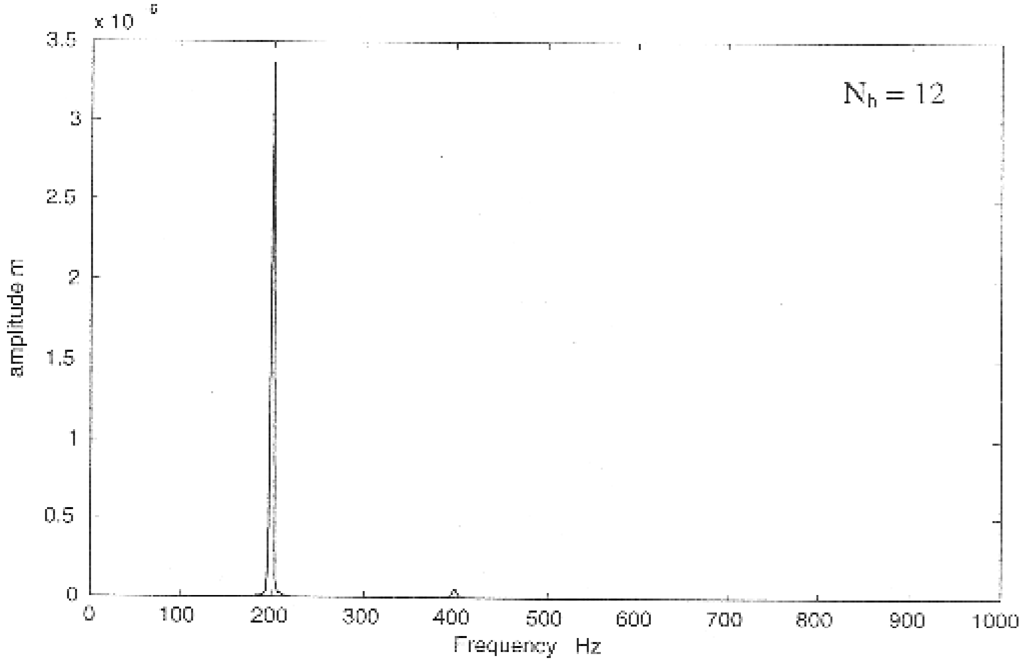


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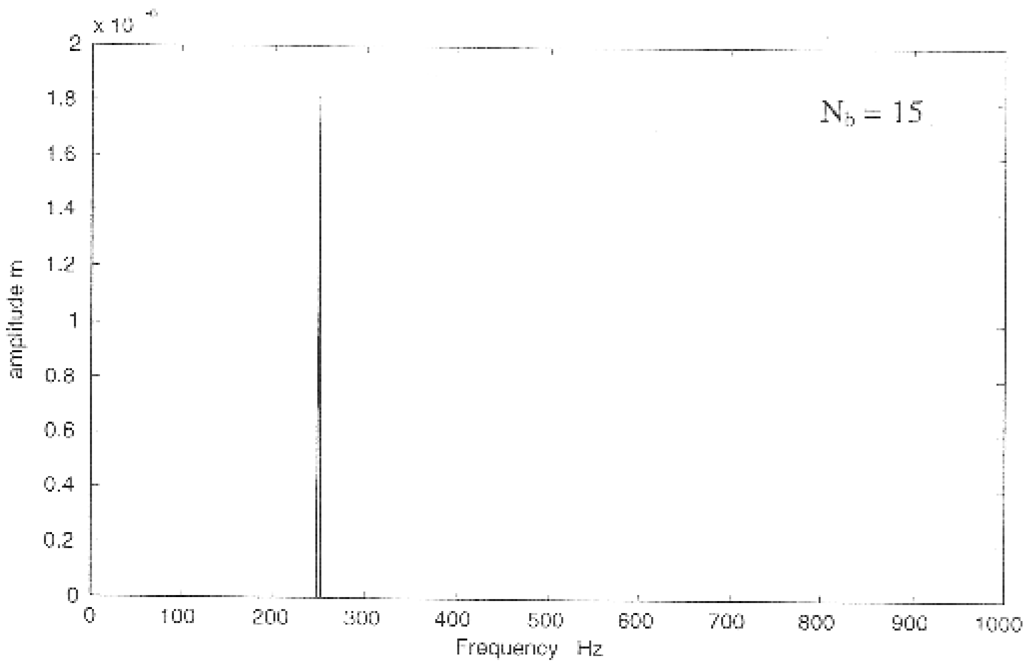
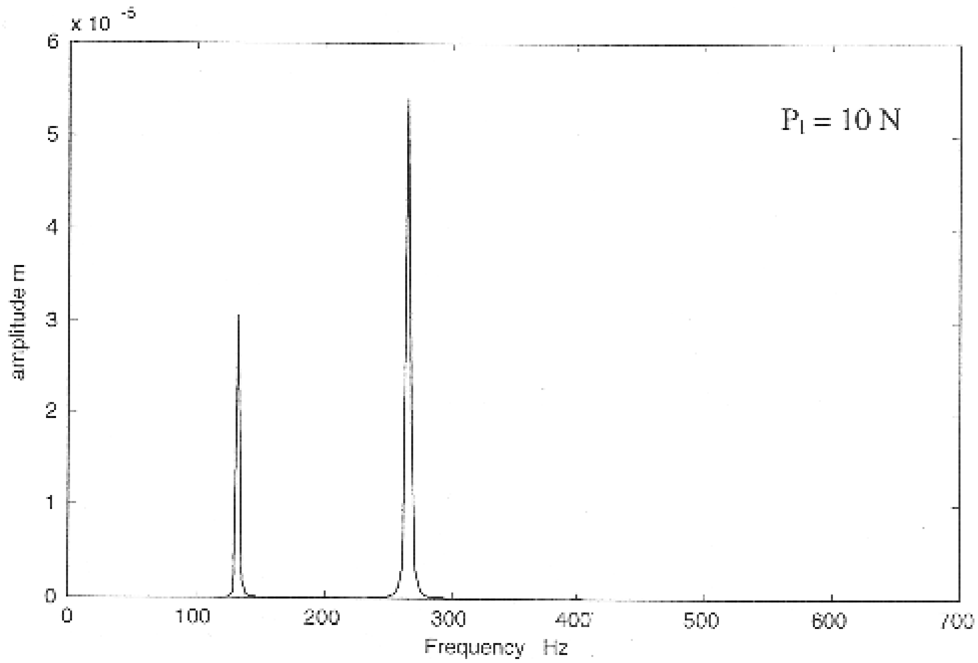



Fig.3. The effect of number of balls on BPV $\left(P_1 = 10N, c = 200\ Ns/m\right)$.

### 3.2. Effect of varying the preload

When preload is applied axially, the deflection will change and by increasing preload the initial axial displacement increases. Therefore for larger preloads, the vibration amplitudes associated with the ball passage frequency will be reduced.

a)

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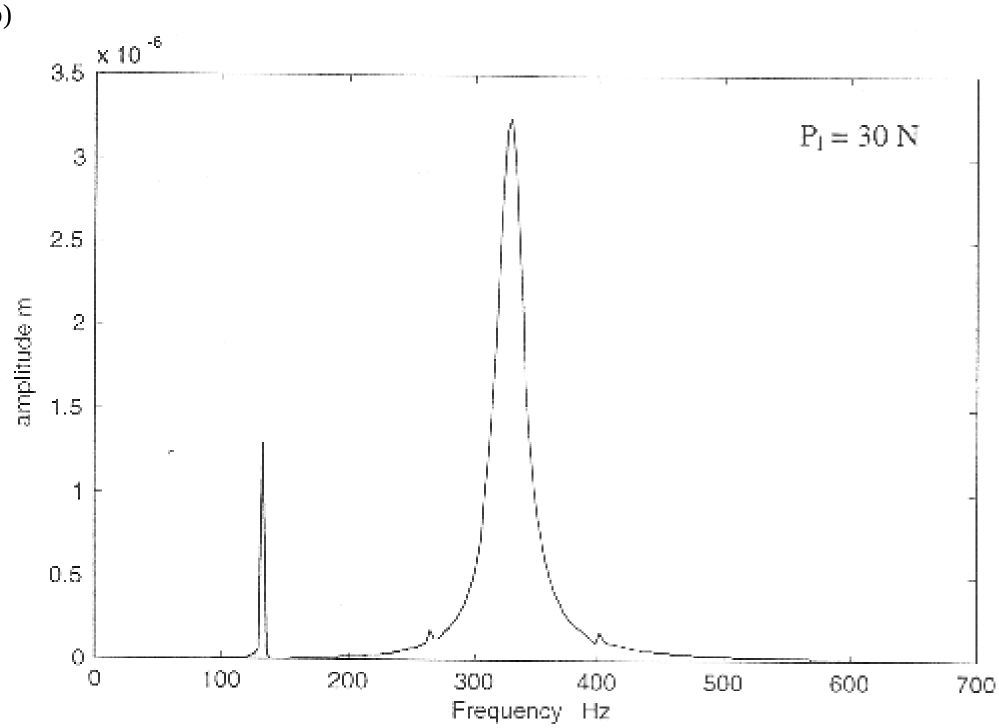

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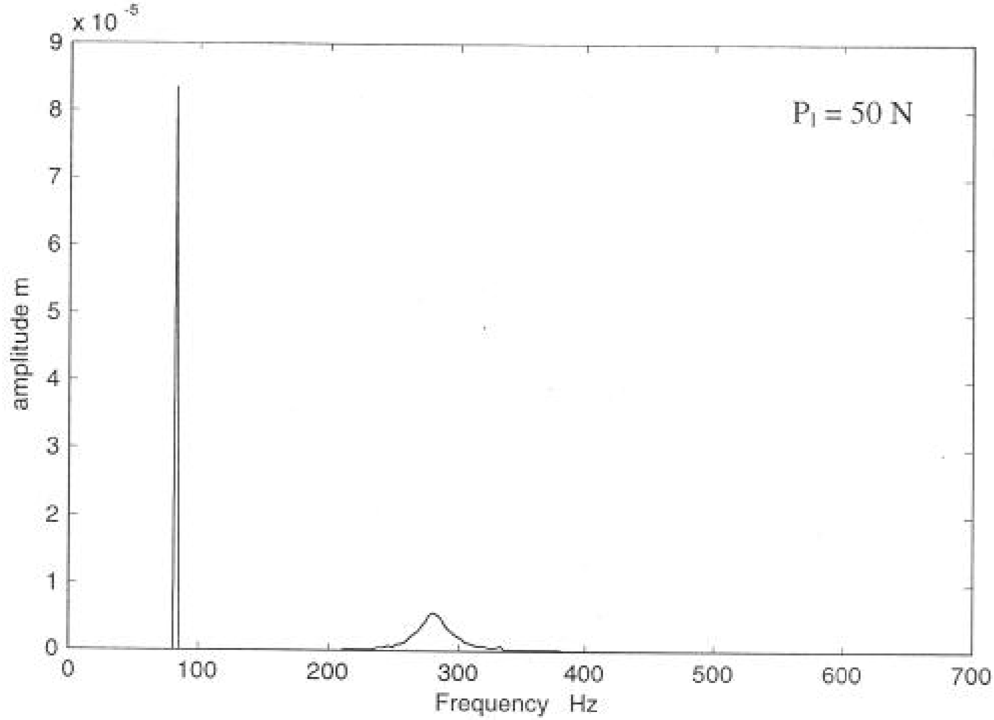


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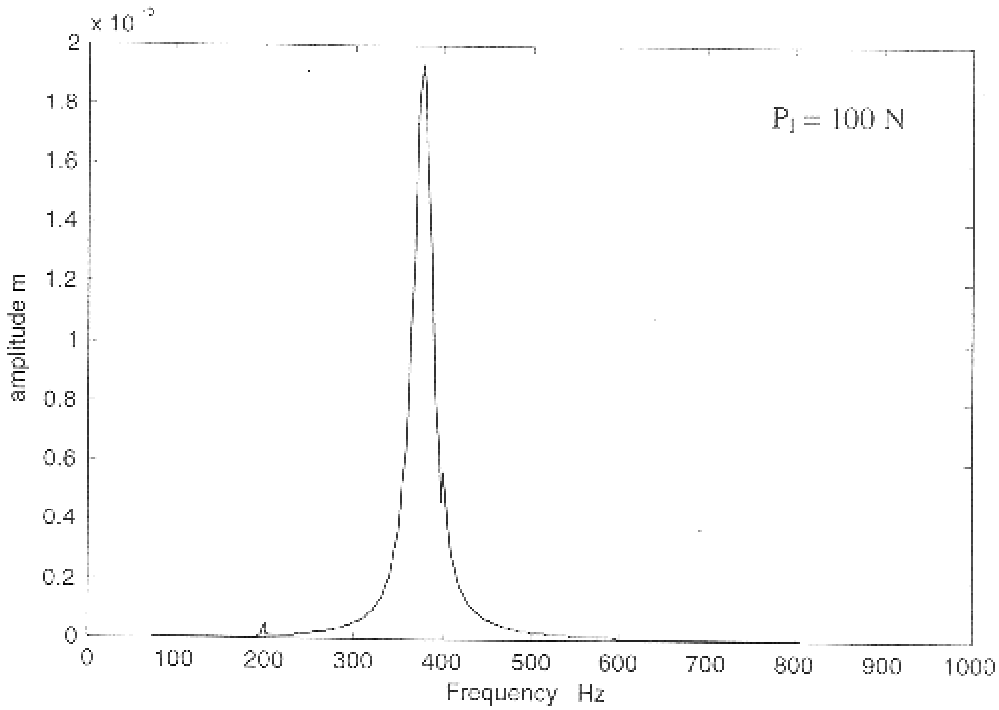


Fig.4. The effect of varying preload $\left(Nb = 10N,\, c = 200\, Ns/m\right)$.

Figure 4 shows the vibration responses at $(N_b = 8)$. When preload is increased there is a sharp decrease in the peak amplitude of vibration at the ball passage frequency and this is also proved experimentally by Wardle (1983). It is easily predicted because by increasing preload the balls get stiffer and they allow lower vibration amplitudes in radial and axial directions. Figure 4a shows the response at preload $(10N)$, the peak amplitude of vibration is high at the ball passage frequency and the first superharmonic of the ball passage frequency (at $2\omega_{bp}$) also appears in the spectrum. As preload is increased from *10 N* to *30 N*, the peak amplitude of vibration decreases and two subharmonics appear in the vibration spectrum as shown in Fig.4b. In Fig.4c, the preload value is *50 N* and the peak amplitude of vibration is slightly increased but the natural frequency in the spectrum is shifted to *100 Hz*. For a value of heavy preload (*100N*), the superharmonic of the ball passage frequency appears and the peak amplitude of vibration at the natural frequency increases as shown in Fig.4d. This is because the heavy preload results in a high stiffness.

## Conclusion

In the present investigation, an analytical model of a rotor bearing system has been developed to obtain the nonlinear vibration response due to varying the number of balls and preload. Nonlinear dynamie responses are found to be associated with the ball passage frequency. The ball passage frequency is the system characteristics and the prediction about system behavior can be made by BPF to avoid resonance. Preload is one of the important parameters in the dynamic analysis of rotor bearing systems and it is useful for controlling the vibrations of the system. The number of balls is also an important parameter for the vibration analysis of rotor bearing systems and should be considered at the design stage. It is also shown that the system exhibits dynamic behaviors that are extremely sensitive to small variations of the system parameters. such as the number of balls and preload.

## Nomenclature

$A$ – distance between centers of curvature of inner and outer race grooves, *mm*
$B$ – total curvature $(A/d_b)$
$c$ – eqivalent viscous damping factor, $Ns/m$
$F_{\theta i}$ – local Hertzian contact force, *N*
$F_u$ – force due to unbalanced rotor, *N*
$k$ – contact for Hertzian contat elastic deformation, $N/m^{3/2}$
$L$ – arc length of the wave of surface waviness, *mm*
$N_w$ – number of wave lobes
$N_b$ – number of balls
$N_r$ – speed of the balanced rotor, *rpm*
$p$ – constant for Hertzian contact elastic deformation
$r$ – inner race radius, *mm*
$R$ – outer race radius, *mm*
$r_{gi}$ – radius of inner groove, *mm*
$r_{go}$ – radius of outer groove, *mm*
$r_{\theta i}$ – radial displacement due to misalignment of races, *mm*
$R_{\theta i}$ – displacement at $i^{th}$ ball
$t$ – time, *sec*
$V_{cage}$ – translational velocity at the cage center, $m/s$
$V_{in}$ – translational velocity of the inner race, $m/s$
$V_{out}$ – translational velocity of the outer race, $m/s$
$W$ – radial load, *N*

$\alpha_p$ – preloaded contact angle, *rad*
$\gamma$ – internal radial clearance, $\mu m$
$\omega_{cage}$ – angular speed of the cage, $rad/sec$
$\omega_{in}$ – angular speed of the inner race, $rad/sec$
$\omega_{out}$ – angular speed of the outer race, $rad/sec$
$\theta_i$ – angular location of i$^{th}$ ball
BPF – ball passage frequency
BPV – ball passage vibration

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Received: March 22, 2004
Revised: April 22, 2005