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EFFECT OF TEMPERATURE-DEPENDENT VISCOSITY ON FERROCONVECTION IN A POROUS MEDIUM

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The effect of temperature-dependent viscosity on the threshold of ferroconvective instability in a porous medium is studied using the Brinkman model. It is found that the stationary mode of instability is preferred to the oscillatory mode. The critical values of the magnetic Rayleigh number marking the onset of ferroconvection are obtained using the Galerkin technique. It is found that the effect of magnetization is to destabilize the system and so is the effect of temperature-dependent viscosity. The porous medium is found to have a stabilizing influence on the onset of convection. The problem is important in energy conversion devices involving ferromagnetic fluids as working media.

Key words: ferroconvection, temperature-dependent viscosity, porous medium, Brinkman model, Galerkin technique.

1. Introduction

The last millennium has seen many fascinating materials that possess promising physical properties and which are technologically useful. The ferrofluid is one such material. The magnetic materials play an important role in the overall development of many scientific applications. The ferrofluid has to be synthesized and it has widespread applications in various fields ranging from physics, chemistry, electrical engineering, biomedicine and instrumentation to computer technology. Its commercial usage includes novel-zero leakage, rotary-shaft seals used in computer disc drives (Bailey, 1983), liquid cooled loudspeakers (Hathaway, 1979) and energy conversion devices (Berkovskii *et al.*, 1993).

The study of thermoconvective instability of ferrofluids has been the subject of investigation for the past four decades due to its remarkable applications. The magnetization of ferrofluids depends on the magnetic field, the temperature and density of the fluid. The variation of any one of these causes a change in the body force. This induces convection in ferromagnetic fluids in the presence of a magnetic field gradient. This mechanism, known as ferroconvection, is similar to the Rayleigh-Bénard convection in ordinary fluids (Chandrasekhar, 1961).

The convective instability of ferromagnetic fluids heated from below in the presence of a vertical uniform magnetic field was studied by Finlayson (1970). Lalas and Carmi (1971) made a nonlinear analysis of the convective instability problem in magnetic fluids using the energy method. Rosensweig *et al.* (1978) analyzed the penetration of ferrofluids in the Hele-Shaw cell.

Rayleigh-Bénard convection in liquids is known to be used for making measurements of properties of the fluid. It is advantageous, therefore, to have a steady dynamics in the fluid to facilitate the measurements. In this context, the problem of ferroconvection in a porous medium provides an ideal experimental situation. Lapwood (1948) analyzed the stability of convective flow using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through a porous medium was considered by Wooding (1960). The effect of variable viscosity on the setting up of convection currents in a

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porous medium was examined by Patil and Vaidyanathan (1981). Nield (1995) investigated the onset of convection in a fluid-saturated porous medium with time-periodic volumetric heating.

The influence of variable viscosity on laminar boundary layer flow and heat transfer to a continuously moving flat plate was examined by Pop *et al*. (1992). Siddheshwar (2004) analyzed the thermorheological effect on magnetoconvection in fluids with weak electrical conductivity. More recently, Siddheshwar and Chan (2005) studied the effects of thermorheology and thermomechanical anisotropy on the onset of Rayleigh-Bénard and Bénard-Marangoni convections in a porous medium.

The effect of a homogeneous magnetic field on the viscosity of a fluid with solid particles possessing intrinsic magnetic moments was investigated by Shliomis (1972). Siddheshwar (1995) studied convective instability of a ferromagnetic fluid with fluid-permeable and magnetic boundaries. The effect of a magnetic field on a fluid of variable viscosity in ferroconvection in a rotating medium was examined by Vaidyanathan *et al*. (2002). Siddheshwar and Abraham (2003) considered the thermal instability in a layer of a ferromagnetic fluid when the boundaries of the layer are subjected to synchronous/asynchronous imposed time-periodic boundary temperatures and time-periodic body force. The effect of anisotropy of porous medium on a fluid of variable viscosity in ferroconvection has been investigated by Ramanathan and Suresh (2004).

All the above works did not consider the effect of temperature-dependent viscosity on ferroconvection in a sparsely distributed porous medium. In this paper an attempt is made to study the effect of temperature-dependent viscosity on ferroconvection in a porous medium using the Brinkman model. The Oberbeck-Boussinesq approximation is used in obtaining the governing equations. The Boussinesq approximation, adopted by Chandrasekhar (1961), was, in fact, first introduced by Oberbeck. It is only recently that a rational explanation has been provided for the Oberbeck-Boussinesq approximation (Rajagopal *et al*., 1996). The Galerkin technique is employed to obtain the critical values marking the onset of ferroconvection in porous media.

2. Mathematical formulation

An infinitely spread horizontal layer of Oberbeck-Boussinesq, ferromagnetic fluid of thickness d saturating a sparsely distributed porous medium heated from below is considered. Let ΔT be the temperature difference between the upper and lower boundaries of the fluid. A Cartesian coordinate system is taken with the z-axis vertically upwards (Fig.1).

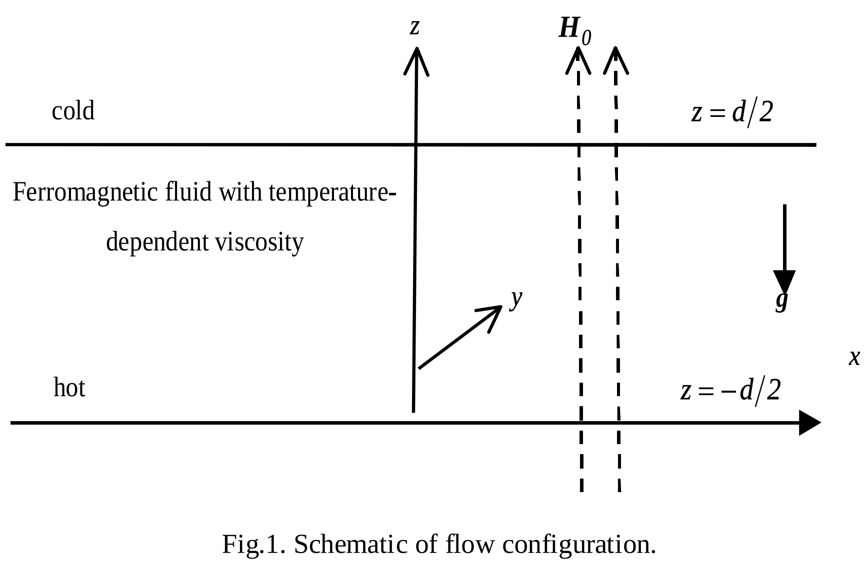

Fig.1. Schematic of flow configuration.

The fluid viscosity is assumed to be temperature-dependent in the following form (Straughan, 2004; Siddheshwar, 2004; Siddheshwar and Chan, 2005)

$$\mu(T)=\mu_1\left[1-\delta(T-T_a)^2\right] \tag{2.1}$$

where δ is a small positive quantity.

The governing equations used are (Finlayson, 1970; Siddheshwar, 2004)

$$\nabla\cdot\boldsymbol{q}=0\,, \tag{2.2}$$

$$\rho_0\left[\frac{\partial\boldsymbol{q}}{\partial t}+(\boldsymbol{q}\cdot\nabla)\boldsymbol{q}\right]=-\nabla p+\rho\boldsymbol{g}+\nabla\cdot(\boldsymbol{H}\cdot\boldsymbol{B})+\nabla\cdot\left[\mu(T)\left(\nabla\boldsymbol{q}+\nabla\boldsymbol{q}^{Tr}\right)\right]-\frac{\mu(T)}{k_0}\boldsymbol{q}\,, \tag{2.3}$$

$$\left[\rho_0 C_{v,H}-\mu_0\boldsymbol{H}\cdot\left(\frac{\partial\boldsymbol{M}}{\partial T}\right)_{V,H}\right]\frac{dT}{dt}+\mu_0 T\left(\frac{\partial\boldsymbol{M}}{\partial T}\right)_{V,H}\cdot\frac{\partial\boldsymbol{H}}{\partial t}=k_1\nabla^2 T\,, \tag{2.4}$$

$$\rho=\rho_0\left[1-\alpha(T-T_a)\right], \tag{2.5}$$

$$\nabla\cdot\boldsymbol{B}=0\,,\qquad \nabla\times\boldsymbol{H}=\boldsymbol{0}\,,\qquad \boldsymbol{B}=\mu_0(\boldsymbol{M}+\boldsymbol{H}) \tag{2.6}$$

where the superscript T_r in Eq.(2.3) denotes the transpose.

The linearized magnetic equation of state for a single component fluid is

$$M=M_0+\chi(H-H_0)-K[T-T_a]. \tag{2.7}$$

The magnetic boundary conditions are that the normal component of the magnetic induction and tangential components of the magnetic field are continuous across the boundary. The temperature is assumed constant on each boundary, i.e., $T=T_0$ at $z=d/2$ and $T=T_1$ at $z=-d/2$. The basic state is assumed to be quiescent. Taking the components of magnetization and magnetic field in the basic state to be $[0,0,M_b(z)]$ and $[0,0,H_b(z)]$, we obtain the following basic state quantities

$$\left.\begin{array}{l}\mathbf{q}_b=0,\quad T_b(z)=T_a-\beta z,\quad p=p_b(z),\quad \rho_b(z)=\rho_0[1+\alpha\beta z],\\[2mm] \mu_b(z)=\mu_1\left[1-\delta\beta^2z^2\right],\quad \mathbf{H}_b=\left[H_0-\dfrac{K\beta z}{1+\chi}\right]\mathbf{k},\quad \mathbf{M}_b=\left[M_0-\dfrac{K\beta z}{1+\chi}\right]\mathbf{k}\end{array}\right\} \tag{2.8}$$

where $\boldsymbol{k}$ is the unit vector along the z-axis. In the succeeding section we study the stability of the basic state within the framework of the linear theory.

3. Linear stability analysis

The basic state is disturbed by a small thermal perturbation. Let the components of the perturbed magnetization and the magnetic field be $(M_1', M_2', M_b(z)+M_3')$ and $(H_1', H_2', H_b(z)+H_3')$ respectively.

The perturbed temperature and viscosity are taken to be $T_b(z)+T'$ and $\mu_b(z)+\mu'$ respectively. On linearization, and assuming $K\beta d\,(1+\chi)H_0$, and using the expressions for $\boldsymbol{H}_b$ and $\boldsymbol{M}_b$ in Eq.(2.8), Eqs (2.6) and (2.7) become

$$\left.\begin{aligned} &H_i' + M_i' = \left(1+\frac{M_0}{H_0}\right)H_i', \qquad (i=1,2), \\ &H_3' + M_3' = (1+\chi)H_3' - KT'. \end{aligned}\right\} \tag{3.1}$$

The second of Eq.(2.6) implies that $\boldsymbol{H}' = \nabla\phi'$, where ϕ' is the perturbed magnetic scalar potential. In a further analysis techniques as in Vaidyanathan *et al*. (1997) and Ramanathan and Suresh (2004), are used and the vertical component of the momentum equation becomes

$$\begin{aligned} &\rho_0 \frac{\partial}{\partial t}\left(\nabla^2 w'\right)\mu_b \nabla^4 w' - \frac{\partial^2 \mu_b}{\partial z^2}\left(\nabla_1^2 w' - \frac{\partial^2 w'}{\partial z^2}\right) + 2\frac{\partial \mu_b}{\partial z}\nabla^2\left(\frac{\partial w'}{\partial z}\right) + \\ &+ \alpha\rho_0 g \nabla_1^2 T' + \frac{\mu_0 K^2 \beta}{1+\chi}\nabla_1^2 T' - \mu_0 K\beta \frac{\partial}{\partial z}\left(\nabla_1^2 \phi'\right) - \frac{\mu_b}{k_0}\nabla^2 w' - \frac{1}{k_0}\frac{\partial \mu_b}{\partial z}\left(\frac{\partial w'}{\partial z}\right) \end{aligned} \tag{3.2}$$

where $\nabla_1^2 = \left(\partial^2/\partial x^2\right) + \left(\partial^2/\partial y^2\right)$ and $\nabla^2 = \nabla_1^2 + \left(\partial^2/\partial z^2\right)$. The linear form of Eq.(2.4) in the perturbed state becomes

$$\rho_0 C \frac{\partial T'}{\partial t} - \mu_0 T_a K \frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial z}\right) = k_1 \nabla^2 T' + \left[\rho_0 C\beta - \frac{\mu_0 T_a K^2 \beta}{1+\chi}\right] w' \tag{3.3}$$

where $\rho_0 C = \rho_0 C_{V,H} + \mu_0 K H_0$. Using Eq.(3.1) in the first of Eq.(2.6), we obtain

$$\left(1+\frac{M_0}{H_0}\right)\nabla_1^2 \phi' + (1+\chi)\frac{\partial^2 \phi'}{\partial z^2} - K\frac{\partial T'}{\partial z}0. \tag{3.4}$$

Following the analysis of Finlayson (1970), the normal mode solution of all dynamical variables can be written as

$$\begin{bmatrix} w' \\ T' \\ \phi' \end{bmatrix} = \begin{bmatrix} w(z,t) \\ T(z,t) \\ \phi(z,t) \end{bmatrix} \exp\left[i\left(k_x x + k_y y\right)\right] \tag{3.5}$$

where k is the wave number given by $k^2 = k_x^2 + k_y^2$.

Using Eq.(3.5), Eqs (3.2)-(3.4) become

$$\rho_0 \frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial z^2}-k^2\right)w=\mu_b\left(\frac{\partial^2}{\partial z^2}-k^2\right)^2 w+\frac{\partial^2\mu_b}{\partial z^2}\left(\frac{\partial^2}{\partial z^2}-k^2\right)w+$$
$$+2\frac{\partial\mu_b}{\partial z}\left(\frac{\partial^2}{\partial z^2}-k^2\right)\left(\frac{\partial w}{\partial z}\right)-\alpha\rho_0 g k^2 T+ \tag{3.6}$$
$$+\frac{\mu_0 K^2\beta}{1+\chi}k^2\left[(1+\chi)\frac{\partial\phi}{\partial z}-KT\right]-\frac{\mu_b}{k_0}\left(\frac{\partial^2}{\partial z^2}-k^2\right)w-\frac{1}{k_0}\frac{\partial\mu_b}{\partial z}\left(\frac{\partial w}{\partial z}\right),$$

$$\rho_0 C\frac{\partial T}{\partial t}-\mu_0 T_a K\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right)=k_1\left(\frac{\partial^2}{\partial z^2}-k^2\right)T+\left[\rho_0 C\beta-\frac{\mu_0 T_a K^2\beta}{1+\chi}\right]w, \tag{3.7}$$

$$(1+\chi)\frac{\partial^2\phi}{\partial z^2}-\left(1-\frac{M_0}{H_0}\right)k^2\phi-K\frac{\partial T}{\partial z}=0. \tag{3.8}$$

The following non-dimensional terms are introduced for further analysis

$$w^*=-\frac{d}{\nu}w,\quad t^*=\frac{\nu}{d^2}t,\quad T^*=\frac{k_1 a\sqrt{R}}{\rho_0 C\beta\nu d}T,$$
$$\phi^*=\frac{(1+\chi)k_1 a\sqrt{R}}{\rho_0 CK\beta\nu d^2}\phi,\quad a^*=kd,\quad z^*=\frac{z}{d} \tag{3.9}$$

where the quantities with asterisks are dimensionless. Using the above non-dimensional terms, Eqs (3.6)-(3.8) take the form

$$\frac{\partial}{\partial t}\left(D^2-a^2\right)w=\left(1-Vz^2\right)\left(D^2-a^2\right)w-2V\left(D^2+a^2\right)w-4Vz\left(D^2-a^2\right)Dw+$$
$$+a\sqrt{R}\left[M_1 D\phi-(1+M_1)T\right]-\mathrm{Da}\left(1-Vz^2\right)\left(D^2-a^2\right)w+2\mathrm{Da}VzDw, \tag{3.10}$$

$$\Pr\frac{\partial T}{\partial t}-\Pr M_2\frac{\partial}{\partial t}(D\phi)=\left(D^2-a^2\right)T+(1-M_2)a\sqrt{R}w, \tag{3.11}$$

$$D^2\phi-M_3 a^2\phi-DT=0 \tag{3.12}$$

where the asterisks have been dropped for simplicity and $D=\partial/\partial z$. We recover the system of equations obtained by Finlayson (1970) from Eqs (3.10)-(3.12) when $V=0$ and $\mathrm{Da}=0$. The typical value of M_2 is 10^{-6} (Finlayson, 1970) and hence it is assumed negligible. It can be shown, following the analysis of Ramanathan and Suresh (2004), that oscillatory instability does not occur for the problem under consideration. Thus we limit our consideration to stationary instability. The fact that viscosity increases due to the presence of suspended particles supports the contention that oscillatory instability can be discounted in ferromagnetic fluids.

The boundary conditions are (Finlayson, 1970)

$$w = D^2 w = T = D\phi = 0 \quad \text{at} \quad z = \pm\frac{1}{2}. \tag{3.13}$$

The boundary conditions on w and T signify stress-free and isothermal boundaries respectively. The boundary condition on ϕ indicates that the magnetic susceptibility χ in respect of the perturbed field is very large at the boundaries. It should be mentioned that it is no longer possible to obtain a closed form solution to the problem at hand owing to the presence of space varying coefficients in Eq.(3.10). We therefore use the Galerkin technique to solve the eigenvalue problem pertaining to stationary instability.

Application of the Galerkin technique (Finlayson, 1970) yields the eigenvalue R for the stationary convection in the case of stress-free, isothermal, magnetic boundaries in the form

$$R = \frac{(X_1 + \mathrm{Da}X_2)X_5 X_7}{a^2 X_3[M_1 X_4 X_6 + (1+M_1)X_3 X_7]} \tag{3.14}$$

where

$$X_1 = \left\langle w_1(1-Vz^2)D^4 w_1\right\rangle - 2a^2\left\langle w_1(1-Vz^2)D^2 w_1\right\rangle + a^4\left\langle w_1(1-Vz^2)w_1\right\rangle +$$
$$+ 2V\left\langle (Dw_1)^2\right\rangle - 2a^2\left\langle w_1^2\right\rangle - 4V\left\langle w_1 z D^3 w_1\right\rangle + 4Va^2\left\langle w_1 z D w_1\right\rangle,$$

$$X_2 = a^2\left\langle w_1(1-Vz^2)w_1\right\rangle - \left\langle w_1(1-Vz^2)D^2 w_1\right\rangle + 2V\left\langle w_1 z D w_1\right\rangle,$$

$$X_3 = \left\langle w_1 T_1\right\rangle, \quad X_4 = \left\langle w_1 D\phi_1\right\rangle, \quad X_5 = \left\langle (DT_1)^2\right\rangle + a^2\left\langle T_1^2\right\rangle,$$

$$X_6 = \left\langle \phi_1 DT_1\right\rangle, \quad X_7 = \left\langle (D\phi_1)^2\right\rangle + M_3 a^2\left\langle \phi_1^2\right\rangle$$

where $\langle u\, v\rangle = \int_{-1/2}^{1/2} u\, v\, dz$ and w_1, T_1 and ϕ_1 are trial functions that satisfy the boundary conditions. The velocity, temperature and magnetic potential trial functions that satisfy the boundary conditions in Eq.(3.13) are

$$w_1 = \cos\pi z, \quad T_1 = \cos\pi z, \quad \phi_1 = \sin\pi z.$$

The above choice of trigonometric functions tacitly implies the use of a higher order Galerkin technique.

For M_1 very large, we obtain the magnetic Rayleigh number N in the form

$$N = RM_1 = \frac{(X_1 + \mathrm{Da}X_2)X_5 X_7}{a^2 X_3[X_4 X_6 + X_3 X_7]}. \tag{3.15}$$

4. Results and discussion

The effect of temperature-dependent viscosity on ferroconvection in a porous medium has been studied using the Brinkman model. The reason for pursuing a linear stability analysis is due to the fact thatsub-critical instabilities are discounted in ferromagnetic fluids with no external constraints (Lalas and Carmi, 1971; Straughan, 2004). Due to the elegance of the method, the Galerkin technique is employed to obtain the critical eigenvalues. However, one can also use the shooting technique or such other methods but at the expense of more computational time.

Before discussing the important results of the problem, we turn our attention to the possible range of values of various parameters arising in the study. The parameter M_1 is a ratio of the magnetic to gravitational forces. M_1 is taken to be *1000* (Finlayson, 1970). The parameter M_3 measures the departure of linearity in the magnetic equation of state. The chosen values of M_3 are *1*, *5*, *10*, and *25* (Finlayson, 1970). The range of values of the Darcy number Da and the temperature-dependent viscosity parameter *V* are *0* to *100* (Walker and Homsy, 1977) and *0* to *0.5* (Straughan, 2004) respectively.

In what follows we analyze the effect of the magnetization parameter M_3, the temperature-dependent viscosity parameter *V* and the Darcy number Da on the critical eigenvalue N_C and the critical wave number a_C. The other two magnetization parameters M_1 and M_2 do not come into the picture in the problem, as discussed earlier, due to the fact that they are quite large and quite small respectively. The results of the numerical calculations are depicted through Figs 2-5.

Figure 2 is a plot of the critical magnetic Rayleigh number N_C versus *V* for different values of M_3. It is quite explicit that the effect of the departure from linearity in the magnetic equation of state, reflected by increasing values of M_3, is to destabilize the system. Further, it is amply clear that the effect of temperature-dependent viscosity is to reinforce the destabilizing effect of M_3. Figure 3 spells out the stabilizing nature of the porous matrix in addition to reiterating the destabilizing effect of *V* for a fixed value of M_3.

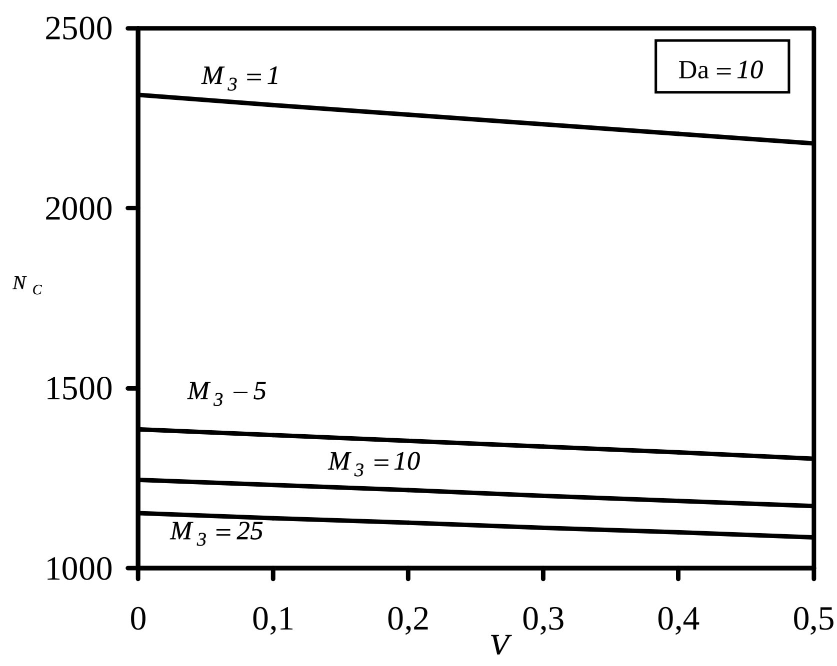

Fig.2. Plot of critical magnetic number N_C versus temperature-dependent viscosity parameter *V* for different values of the magnetization parameter M_3.

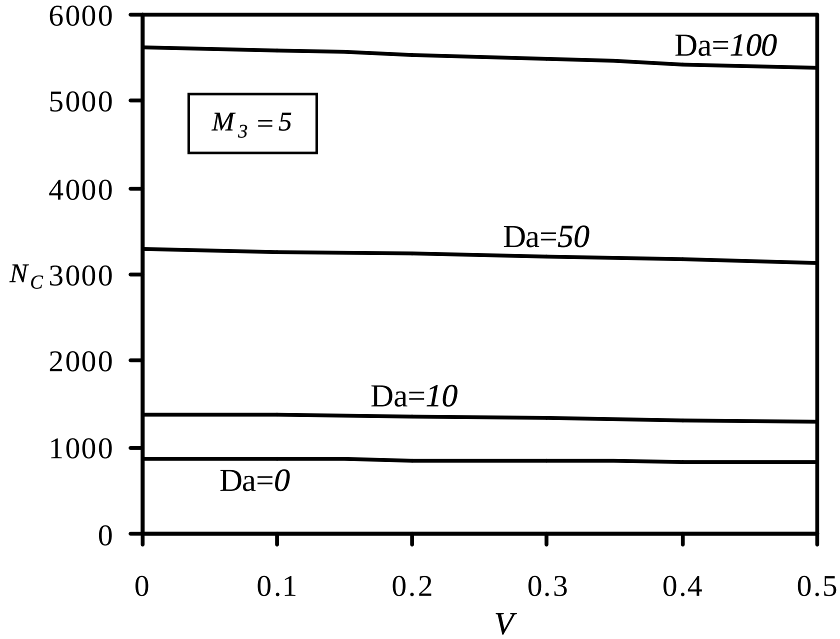

Fig.3. Plot of N_C versus V for different values of the Darcy number Da.

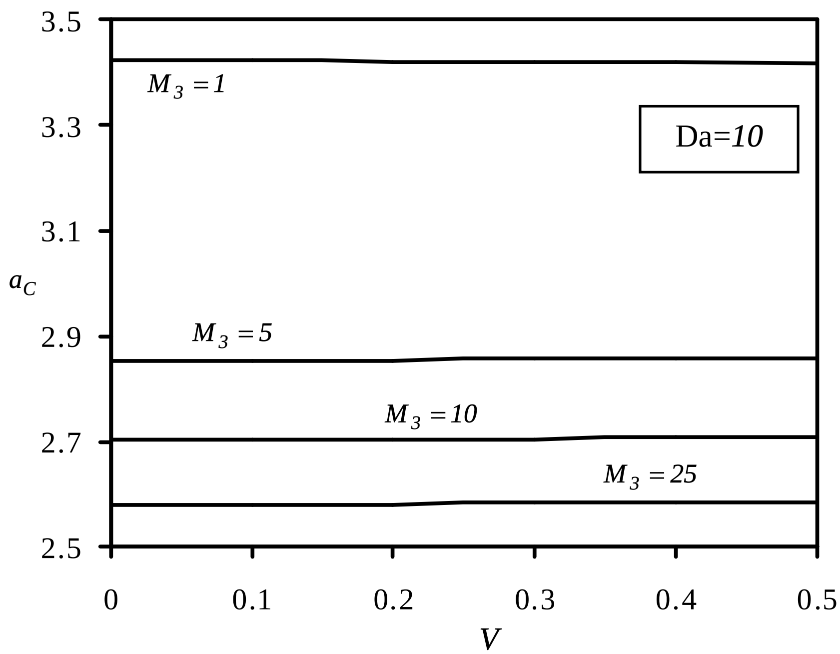

Fig.4. Plot of critical wve number a_C versus V for different values of M_3.

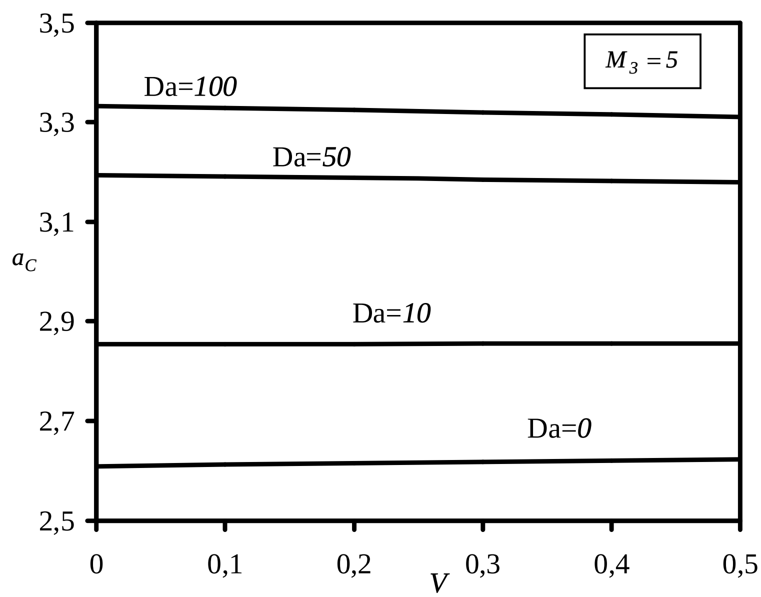

Fig.5. Plot of a_C versus *V* for different values of Da.

Figures 4 and 5 clearly bring out the fact that the porous matrix greatly influences the size of the ferroconvective cell at the onset of convection whereas *V* has just a marginal influence. The parameter M_3 also significantly increases the cell size as demonstrated by a decreasing value of a_C with increasing M_3. Considering the fact that the porous matrix (through Da) and the temperature-dependent viscosity (through *V*) signify antagonistic influence on ferroconvection, we may conclude that by an appropriate choice of the porous matrix and temperature difference between the boundaries, it is possible to create a situation conducive to measurements. Coupled with the above observation is the regulatory nature of the magnetic field on ferroconvection that comes handy in the control of ferroconvection.

5. Conclusion

The two important conclusions that render the present study physically useful are:

1. External regulation of ferroconvection is possible in temperature-sensitive liquids.
2. The problem considered in a porous medium ensures that stationary convection is the preferred mode and hence measurements are easy to handle.

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Nomenclature

a – dimensionless wave number
$\boldsymbol{B}$ – magnetic induction
$C_{V,H}$ – effective heat capacity at constant volume and magnetic field

$\boldsymbol{d}$ – thickness of the fluid layer

Da – Darcy number, $\dfrac{d^2}{k_0}$

$\boldsymbol{g}$ – gravitational acceleration, $(0, 0, -g)$

$\boldsymbol{H}$ – magnetic field

k – dimensional wave number

k_1 – thermal conductivity

k_0 – permeability of the porous medium

k_x, k_y – wave number in the x and y directions

K – pyromagnetic coefficient, $-\left(\dfrac{\partial M}{\partial T}\right)_{H_0, T_a}$

$\boldsymbol{M}$ – magnetization

M_0 – mean value of the magnetization at $H = H_0$ and $T = T_a$

M_1 – ratio of the magnetic force due to the temperature fluctuation to the gravitational force, $\dfrac{\mu_0 K^2 \beta}{(1+\chi)\alpha \rho_0 g}$

M_2 – ratio of thermal flux due to magnetization to magnetic flux, $\dfrac{\mu_0 T_a K^2}{(1+\chi)\rho_0 C}$

M_3 – measure of non-linearity in the magnetization, $\left(1 + \dfrac{M_0}{H_0}\right) \Big/ (1+\chi)$

N – magnetic Rayleigh number

p – hydrodynamic pressure

Pr – Prandtl number, $\dfrac{\nu \rho_0 C}{k_1}$

$\boldsymbol{q}$ – velocity of the fluid, (u, v, w)

R – Rayleigh number, $\dfrac{\alpha g \beta d^4 \rho_0 C}{\nu k_1}$

t – time

T – temperature

T_a – average temperature of the lower and upper surfaces, $\dfrac{T_0 + T_1}{2}$

T_0 – constant temperature at the lower surface of the layer

T_1 – constant temperature at the upper surface of the layer

V – temperature-dependent viscosity parameter, $\delta \beta^2 d^2$

(x, y, z) – Cartesian coordinates

∇ – vector differential operator

α – coefficient of volume expansion

β – adverse basic temperature gradient

ρ – density of the fluid

ρ_0 – reference density at $T = T_a$

μ – dynamic viscosity

μ_0 – magnetic permeability of vacuum

μ_1 – reference viscosity at $T = T_a$

ν – kinematic viscosity, $\dfrac{\mu_1}{\rho_0}$

ϕ – magnetic scalar potential

χ – magnetic susceptibility, $\left(\dfrac{\partial M}{\partial H}\right)_{H_0, T_a}$

Subscripts

b – basic state quantity
c – critical value

Superscript

$'$ – infinitesimal perturbation

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