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dedicated to Professor K. Walters on his seventieh Birthday

INFLUENCE OF RHEOLOGICAL PARAMETERS ON THE MECHANICAL PARAMETERS OF CURVILINEAR THRUST BEARING WITH ONE POROUS WALL LUBRICATED BY A COUPLE STRESS FLUID

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The flow of a couple stress fluid in the clearance of a curvilinear thrust bearing with a porous pad is considered. The porous pad is connected with an upper impermeable rotating surface which approaches the lower fixed bearing surface. The Reynolds and Poisson equations are uncoupled by using the Morgan-Cameron approximation and a closed-form solution is obtained. Expressions for the pressure and capacity load of the bearing are given. As an example the bearing modelled by two disks and two spherical surfaces is discussed.

Key words: curvilinear bearing, inertia effect, porous pad, squeeze film, couple stress fluid.

1. Introduction

In machine engines and installations of many industrial processes the phenomenon of flow of non-Newtonian fluids occurs. Channels by which the above mentioned fluids flow through can be created – in a general case – as rotating or fixed surfaces of revolution. The geometry of the channels depends on engineering requirements. The most often met channels are formed by two operating surfaces such as plane, spherical, quasi-spherical and cylindrical or quasi-cylindrical surfaces.

The example of machine engine units in which one uses such channels are sliding bearings in which the flowing medium is a lubricant. It is introduced between operating surfaces of a bearing to decrease the friction, remove the products of friction, etc.

In order to improve the conditions of lubrication of sliding bearings porous inserts are applied. They can be connected with the bearing bush or shaft, according to the constructional solution. The task of the porous insert is to store the lubricant and emit it to the bearing clearance while of bearing works.

A mathematical description of the lubricant flow is required both in the bearing clearance and it the porous insert to determine theoretically the machine properties of such a bearing.

Cameron and Morgan (Morgan and Cameron, 1957) were the first researchers who presented theoretical results on the porous bearings. They used a Darcy model to describe the flow in a porous medium.

Now, the rheological models are used in theoretical research to approach real lubricants. The model of a couple-stress fluid is an example. It is a mathematical model of a synovial fluid, recognized as the most effective lubricant in nature. The theory of the couple-stress fluid was presented in 1966 by Stokes (Bujurke and Jayaraman, 1982; Eringen, 1966; Stokes, 1966).

The purpose of the paper is to study pressure distributions and load-capacity of a thrust bearing with a porous insert lubricated by the couple-stress fluid. The bearing is created by two surfaces of revolution having a common axis of symmetry (Fig.1), the upper surface connected with the porous insert rotates with the angular velocity ω and the lower one is fixed. It is assumed that the porous region is defined by the Darcy model. In this work will, the influence of the rotational inertia effects of the flowing fluid, the fluid film squeezing and the permeability of the porous insert on the bearing mechanical parameters be estimated.

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2. Equation of motion in the bearing clearance

Consider a thrust bearing with one porous wall lubricated by a couple-stress fluid as shown in Fig.1. The fixed surfaces are described by the function $R(x)$, which denotes the radius of this surfaces. The bearing clearance is described by the function $h(x, \vartheta, t)$ which denotes the distance between the fixed surface and the lower surface of the porous layer, measured along normal to the fixed surface. The porous layer of thickness $H = \text{const}$ is connected with the rotating surface of the bearing. An intrinsic curvilinear orthogonal coordinate system x, ϑ, y is connected with the fixed surface, as shown in Fig.1.

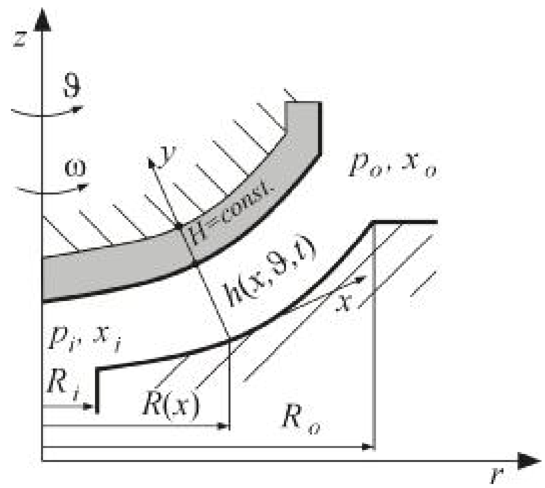

Fig.1. Coordinate system in the clearance of the bearing with one porous wall.

The flow the couple-stress fluid in the clearance can be described by the following equations (Jurczak, 2004; Walicka, 2002a, 2002b; Walicki, 2005)

$$\frac{1}{R}\frac{\partial(R\upsilon_x)}{\partial x}+\frac{1}{R}\frac{\partial\upsilon_\vartheta}{\partial\vartheta}+\frac{\partial\upsilon_y}{\partial y}=0, \tag{2.1}$$

$$\mu\frac{\partial^2\upsilon_x}{\partial y^2}-\eta\frac{\partial^4\upsilon_x}{\partial y^4}=\frac{\partial p}{\partial x}-\rho\upsilon_\vartheta^2\frac{R'}{R}, \tag{2.2}$$

$$\mu\frac{\partial^2\upsilon_\vartheta}{\partial y^2}-\eta\frac{\partial^4\upsilon_\vartheta}{\partial y^4}=\frac{1}{R}\frac{\partial p}{\partial\vartheta}, \tag{2.3}$$

$$\frac{\partial p}{\partial y}=0, \tag{2.4}$$

the „prim" denotes derivation with respect to x.

The problem statement is complete upon specification of boundary conditions which are

$$\upsilon_x(x,\vartheta,0,t)=\left.\frac{\partial^2\upsilon_x}{\partial y^2}\right|_{y=0}=0, \qquad \upsilon_x(x,\vartheta,h,t)=\left.\frac{\partial^2\upsilon_x}{\partial y^2}\right|_{y=h}=0, \tag{2.5}$$

$$\upsilon_\vartheta(x,\vartheta,0,t)=\left.\frac{\partial^2\upsilon_\vartheta}{\partial y^2}\right|_{y=0}=0, \qquad \upsilon_\vartheta(x,\vartheta,h,t)=R\omega, \qquad \left.\frac{\partial^2\upsilon_\vartheta}{\partial y^2}\right|_{y=h}=0, \tag{2.6}$$

$$\upsilon_y(x,\vartheta,0,t)=0, \qquad \upsilon_y(x,\vartheta,h,t)=\omega\frac{\partial h}{\partial\vartheta}+V_h, \tag{2.7}$$

$$p(x_i,\vartheta)=p_i, \qquad p(x_o,\vartheta)=p_o. \tag{2.8}$$

Here V_h denotes the value of velocity on the boundary between the fluid film and porous layer.

Integrating Eq.(2.3) and taking into account the boundary conditions (2.6) we get

$$\upsilon_\vartheta=R\omega\frac{y}{h}+\frac{1}{2\mu R}\frac{\partial p}{\partial\vartheta}\left\{y^2-hy+2l^2\left[1-\mathrm{ch}\left(\frac{y}{l}\right)-\frac{1-\mathrm{ch}\left(\frac{h}{l}\right)}{\mathrm{sh}\left(\frac{h}{l}\right)}\mathrm{sh}\left(\frac{y}{l}\right)\right]\right\} \tag{2.9}$$

where

$$l^2=\frac{\eta}{\mu}. \tag{2.10}$$

We can observe that the main value of υ_ϑ is equal

$$\upsilon_\vartheta\approx R\omega\frac{y}{h}. \tag{2.11}$$

Putting this expression in Eq.(2.2) and integrating its result one gets

$$\begin{aligned}\upsilon_x=&\left(\frac{1}{2\mu}\frac{\partial p}{\partial x}-\frac{\rho RR'\omega^2 l^2}{\mu h^2}\right)\left\{y^2-hy+2l^2\left[1-\mathrm{ch}\left(\frac{y}{l}\right)-\frac{1-\mathrm{ch}\left(\frac{h}{l}\right)}{\mathrm{sh}\left(\frac{h}{l}\right)}\mathrm{sh}\left(\frac{y}{l}\right)\right]\right\}+\\&+\frac{\rho RR'\omega^2}{\mu}\left\{l^2\left[\frac{\mathrm{sh}\left(\frac{y}{l}\right)}{\mathrm{sh}\left(\frac{h}{l}\right)}-\frac{y}{h}\right]+\frac{1}{12h^2}\left(y^4-h^3y\right)\right\}.\end{aligned} \tag{2.12}$$

Next, integrating the continuity Eq.(2.1) across the film thickness and taking into account Eqs (2.9), (2.2) and the boundary conditions (2.7) we obtain

$$\frac{1}{R}\frac{\partial}{\partial x}\left(Rh^3 f(l,h)\left\{\frac{\partial p}{\partial x}-\frac{3\rho\omega^2 RR'}{10}\left[\frac{g(l,h)}{f(l,h)}-40\left(\frac{l}{h}\right)^2\right]\right\}\right)+$$
$$+\frac{1}{R^2}\frac{\partial}{\partial\vartheta}\left[h^3 f(l,h)\frac{\partial p}{\partial\vartheta}\right]=6\mu\omega\frac{\partial h}{\partial\vartheta}+12\mu V_h \tag{2.13}$$

where

$$f(l,h)=1-12\left(\frac{l}{h}\right)^2+24\left(\frac{l}{h}\right)^3\mathrm{th}\left(\frac{h}{2l}\right),\qquad g(l,h)=1+40\left(\frac{l}{h}\right)^3\left[\mathrm{cth}\left(\frac{h}{l}\right)-\frac{h}{2l}\right]. \tag{2.14}$$

Let us consider now the flow of the couple-stress fluid in the porous insert. Assuming that the fluid in the layer rotates at the same angular velocity as the upper surface one can write according to the Darcy law equations (Jurczak, 2004; Walicka, 2002a, 2002b; Walicki, 2005)

$$\overline{\upsilon}_x=-\frac{\Phi}{\mu}\left(\frac{\partial\overline{p}}{\partial x}-\rho\omega^2 RR'\right), \tag{2.15}$$

$$\overline{\upsilon}_\vartheta=-\frac{\Phi}{\mu}\frac{1}{R}\frac{\partial\overline{p}}{\partial\vartheta}, \tag{2.16}$$

$$\overline{\upsilon}_y=\frac{\partial h}{\partial t}-\frac{\Phi}{\mu}\frac{\partial\overline{p}}{\partial y}, \tag{2.17}$$

where Φ represents the permeability of the insert. The continuity equation for the porous region has the same form as Eq.(2.1)

$$\frac{1}{R}\frac{\partial\left(R\overline{\upsilon}_x\right)}{\partial x}+\frac{1}{R}\frac{\partial\overline{\upsilon}_\vartheta}{\partial\vartheta}+\frac{\partial\overline{\upsilon}_y}{\partial y}=0. \tag{2.18}$$

Since the cross velocity component must be continuous at the porous wall-film interface

$$V_h=\overline{\upsilon}_y\Big|_{y=h},$$

one obtains from Eqs (2.15) and (2.18) the modified Reynolds equation

$$\frac{1}{R}\frac{\partial}{\partial x}\left(Rh^3 f(l,h)\left\{\frac{\partial p}{\partial x}-\frac{3\rho\omega^2 RR'}{10}\left[\frac{g(l,h)}{f(l,h)}-40\left(\frac{l}{h}\right)^2\right]\right\}\right)+$$
$$+\frac{1}{R^2}\frac{\partial}{\partial\vartheta}\left[h^3 f(l,h)\frac{\partial p}{\partial\vartheta}\right]=6\mu\omega\frac{\partial h}{\partial\vartheta}+12\mu\left[\frac{\partial h}{\partial t}-\frac{\Phi}{\mu}\left(\frac{\partial\overline{p}}{\partial y}\right)\Bigg|_{y=h}\right]. \tag{2.19}$$

By substituting formulae (2.15)-(2.17) into Eq.(2.18) we obtain Poisson's equation for the pressure distribution in the porous insert

$$\frac{1}{R}\frac{\partial}{\partial x}R\left(\frac{\partial \overline{p}}{\partial x}-\rho\omega^2 RR'\right)+\frac{1}{R^2}\frac{\partial^2 \overline{p}}{\partial \vartheta^2}+\frac{\partial^2 \overline{p}}{\partial y^2}=0\,. \tag{2.20}$$

The boundary conditions are given by the formulae

$$\begin{gathered}\overline{p}(x_i,\vartheta,y)=p_i, \qquad \overline{p}(x_o,\vartheta,y)=p_o,\\ \left.\frac{\partial \overline{p}}{\partial y}\right|_{y=H}=0, \qquad p(x,\vartheta)=\overline{p}(x,\vartheta,h).\end{gathered} \tag{2.21}$$

The problem consists in solving Eqs (2.19) and (2.20) with the boundary conditions (2.8) and (2.21). It is impossible to find a general solution to these equations, but if an approximation is incorporated, the solution to this system of equations is possible. Assuming that

$$H << R(x)$$

and integrating Eq.(2.20) with respect to y over the porous insert and using the Morgan-Cameron (Morgan and Cameron, 1957) one obtains

$$\left.\left(\frac{\partial \overline{p}}{\partial y}\right)\right|_{y=h}=\frac{1}{R}\frac{\partial}{\partial x}HR\left(\frac{\partial p}{\partial x}-\rho\omega^2 RR'\right)+\frac{H}{R^2}\frac{\partial^2 p}{\partial \vartheta^2}\,. \tag{2.22}$$

By substituting Eq.(2.22) into Eq.(2.19) the modified Reynolds equation governing the film pressure may be obtained

$$\begin{gathered}\frac{1}{R}\frac{\partial}{\partial x}\left\{R\left[h^3 f(l,h)+12\Phi H\right]\frac{\partial p}{\partial x}\right\}+\frac{1}{R^2}\frac{\partial}{\partial \vartheta}\left\{\left[h^3 f(l,h)+12\Phi H\right]\frac{\partial p}{\partial \vartheta}\right\}=\\ =\frac{3\rho\omega^2}{10}\frac{1}{R}\frac{\partial}{\partial x}\left(\left\{h^3\left[g(l,h)-40\left(\frac{l}{h}\right)^2 f(l,h)\right]+40\Phi H\right\}R^2R'\right)+6\mu\omega\frac{\partial h}{\partial \vartheta}+12\mu\frac{\partial h}{\partial t}.\end{gathered} \tag{2.23}$$

Generally, it is impossible to find the solution to Eq.(2.23) for a general case of the bearing configuration. The solution can be obtained only in two cases, namely: for the axial symmetry and for the steady flow.

In the case of axial symmetry of the flow the solution of Eq.(2.23) takes the form

$$p(x,t)=B(x,t)+\frac{\left[A(x,t)-A_o\right]\left(p_i-B_i\right)-\left[A(x,t)-A_i\right]\left(p_o-B_o\right)}{A_i-A_o} \tag{2.24}$$

where

$$A(x,t)=\int\frac{dx}{R\left[h^3 f(l,h)+12\Phi H\right]}\,, \qquad A_i=A(x_i,t), \qquad A_o=A(x_o,t),$$

$$B(x,t)=C^{(0)}(x,t)+D(x,t), \qquad B_i=B(x_i,t), \qquad B_o=B(x_o,t),$$

$$C^{(0)}(x,t)=\frac{3\rho\omega^2}{10}\int\frac{\left\{h^3\left[g(l,h)-40\left(\frac{l}{h}\right)^2 f(l,h)\right]+40\Phi H\right\}}{\left[h^3 f(l,h)+12\Phi H\right]}RR'dx, \tag{2.25}$$

$$D(x,t)=12\mu\int\frac{A_t(x,t)}{R\left[h^3 f(l,h)+12\Phi H\right]}dx, \qquad A_t(x,t)=\int R\frac{\partial h}{\partial t}dx.$$

Knowing the pressure distribution one may find the load capacity from the following formula

$$N=\pi R_i^2 p_i+2\pi\int_{x_i}^{x_o} pR\cos\varphi dx\,. \tag{2.26}$$

The sense of the angle φ arises from Fig.2.

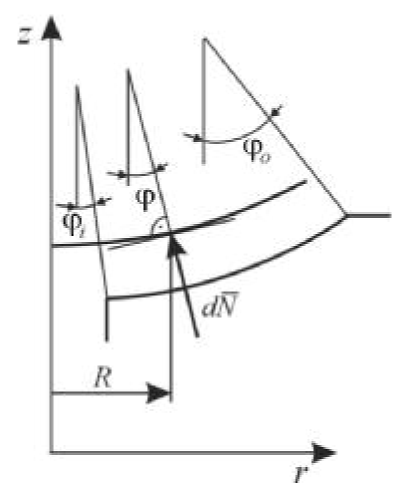

Fig.2. Elementary load acting on the bearing surface.

3. Example of application

3.1. The flow in a plane bearing

Let us consider a thrust bearing modelled by two disks as shown in Fig.3.

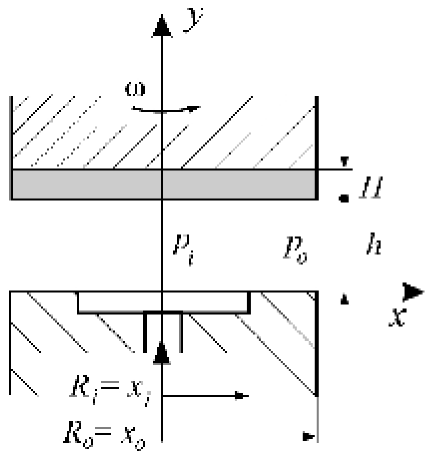

Fig.3. Thrust plane bearing modelled by two disks.

The dimensionless pressure distribution and the load capacity for the plane bearing are as follows

$$\tilde{p} = \frac{p}{p_o} = 1 + \Pi_p\left(\tilde{x}^2 - 1\right) + \frac{\delta - 1 + \Pi_p\left(1 - \lambda^2\right)}{\ln\lambda}\ln\tilde{x}\ , \tag{3.1}$$

$$\tilde{N} = \frac{N - \pi x_o^2 p_o}{\pi x_o^2 p_o} = -\frac{1-\lambda^2}{2\ln\lambda}\left\{\delta - 1 + \Pi_p\left[1 - \lambda^2 + \left(1+\lambda^2\right)\ln\lambda\right]\right\} \tag{3.2}$$

where

$$\Pi_p = \left(GP_c + F\mathrm{St}\right), \qquad F = \frac{1}{f\left(l^*\right) + K^3}, \qquad G = \left[g\left(l^*\right) - 40 l^{*2} f\left(l^*\right) + \frac{10}{3}K^3\right]F,$$

$$P_c = \frac{3\rho\omega^2 x_o^2}{20}, \quad \mathrm{St} = \frac{3\mu x_o^2}{h^3 p_o}\frac{\partial h}{\partial t}, \quad K = \left(\frac{\Phi H}{h_o^3}\right)^{\frac{1}{3}}, \tag{3.3}$$

$$\tilde{x} = \frac{x}{x_o}, \quad \lambda = \frac{R_i}{R_o}, \quad \delta = \frac{p_i}{p_o}, \quad l^* = \frac{l}{h_o};$$

here the introduced magnitudes represent the parameters:

Π_p - pressure, P_c - inertia, St - squeeze, K - permeability.

Figure 4 presents the plots of Π_p versus K. The parameter Π_p specifies the relation between inertia, squeeze and permeability. From the plots we can read the values of the parameter Π_p for steady values of P_c, St, K. Figs.5-7 present the dimensionless pressure distributions $\tilde{p}$ for different values of Π_p and different longitude of the clearance bearing and Fig.8 presents nondimensional load capacity $\tilde{N}$.

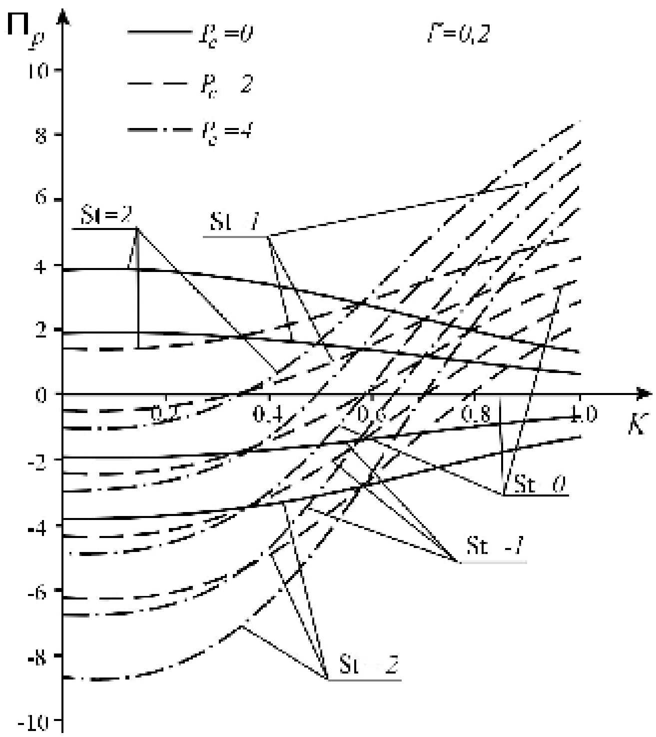

Fig.4. The graph of Π_p versus K for $l^* = 0{,}2$.

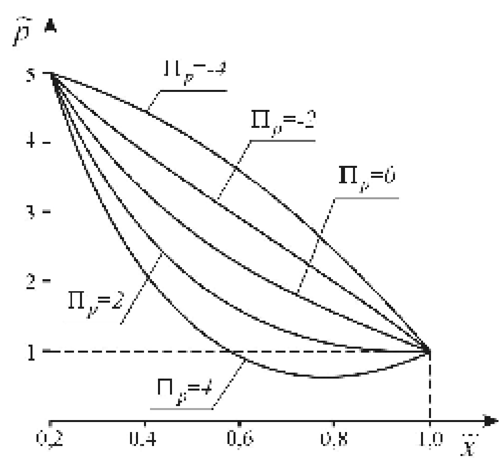

Fig.5. Dimensionless pressure distribution $\tilde{p}$ for the long clearance.

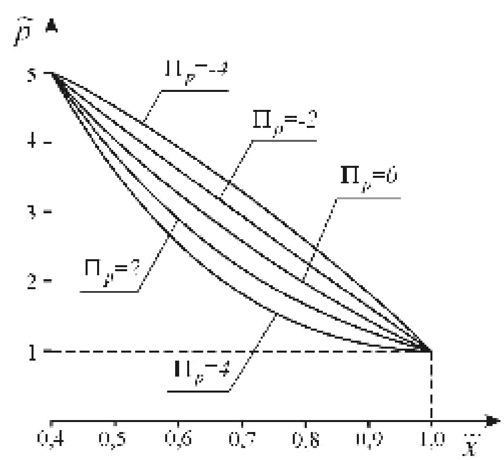

Fig.6. Dimensionless pressure distribution $\tilde{p}$ for average length of clearance.

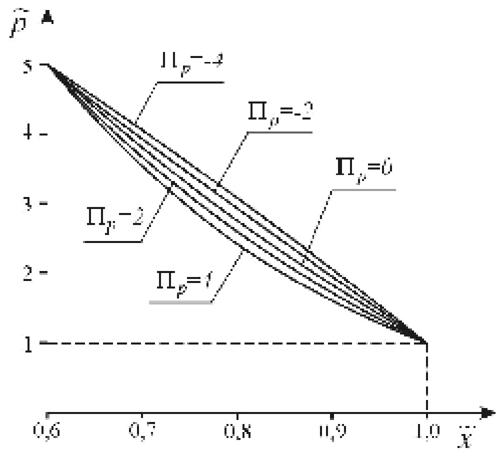

Fig.7. Dimensionless pressure distribution $\tilde{p}$ for the short clearance.

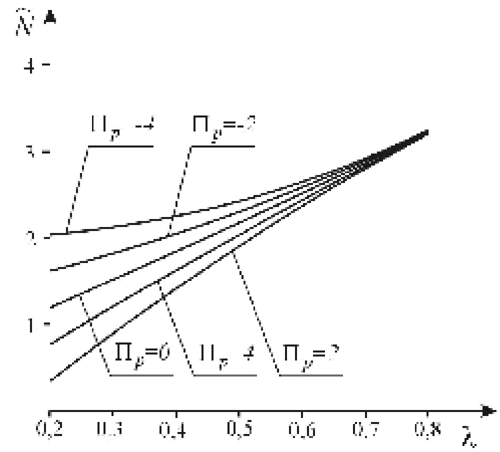

Fig.8. Dimensionless load-capacity $\tilde{N}$ of a plane bearing.

3.1. The flow in a spherical bearing

Let us consider a spherical bearing shown in Fig.9. Assuming that the flow is steady we have

$$A_t(x,t)=0, \qquad D(x,t)=0\,. \tag{3.4}$$

Resolving Eqs.(2.24) and (2.25) to the dimensionless form one obtains

$$\tilde{p}=1+P_cG\left(\sin^2\varphi-\sin^2\varphi_o\right)+\left(\mathrm{lntg}\frac{\varphi}{2}-\mathrm{lntg}\frac{\varphi_o}{2}\right)\frac{\delta-1+P_cG\left(\sin^2\varphi_o-\sin^2\varphi_i\right)}{\mathrm{lntg}\frac{\varphi_i}{2}-\mathrm{lntg}\frac{\varphi_o}{2}}, \tag{3.5}$$

$$\tilde{N}=-\frac{P_cG}{2}\frac{\sin^4\varphi_o-\sin^4\varphi_i}{\sin^2\varphi_o}+\frac{\cos\varphi_o-\cos\varphi_i}{\sin^2\varphi_o}\frac{\delta-1+P_cG\left(\sin^2\varphi_o-\sin^2\varphi_i\right)}{\mathrm{lntg}\frac{\varphi_i}{2}-\mathrm{lntg}\frac{\varphi_o}{2}} \tag{3.6}$$

where

$$P_c=\frac{3}{20}\frac{\rho\omega^2x_o^2}{p_o}, \qquad K=\left(\frac{\Phi H}{h^3}\right)^{\frac{1}{3}} \qquad F=\frac{1}{f\left(l^*\right)+K^3},$$

$$G=\left[g\left(l^*\right)-40l^{*2}f\left(l^*\right)+\frac{10}{3}K^3\right]F, \qquad \tilde{p}=\frac{p}{p_o}, \qquad \tilde{N}=\frac{N-\pi R_o^2p_o}{\pi R_o^2p_o}, \qquad \delta=\frac{p_i}{p_o}\,. \tag{3.7}$$

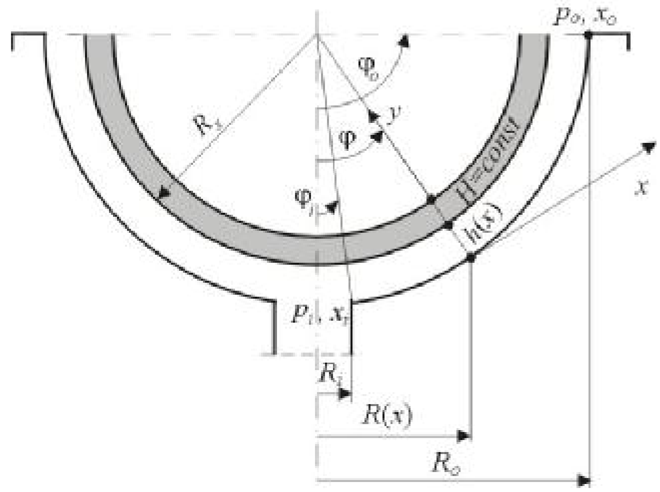

Fig.9. Thrust plane bearing modelled by two spherical surfaces.

Figures 10-12 presents the dimensionless pressure distributions $\tilde{p}$ for clearances of different length and for two different values of the parameter of permeability $K: K=0$ and $K=0.2$. Figure 13 presents nondimensional load capacity $\tilde{N}$.

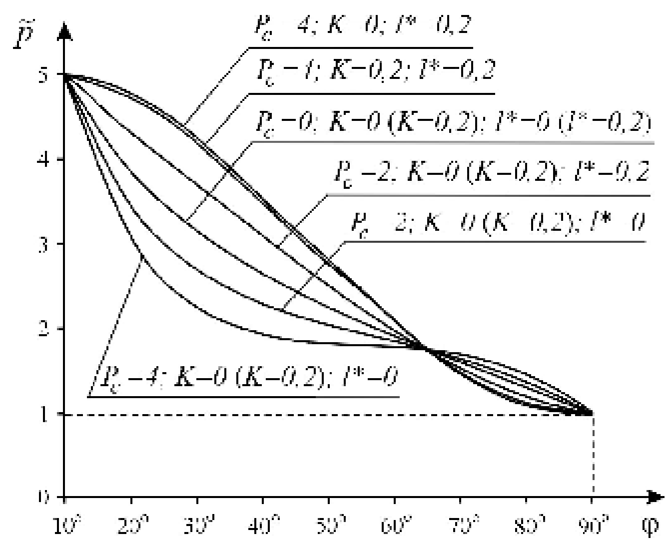

Fig.10. Dimensionless pressure distribution $\tilde{p}$ for the long clearance.

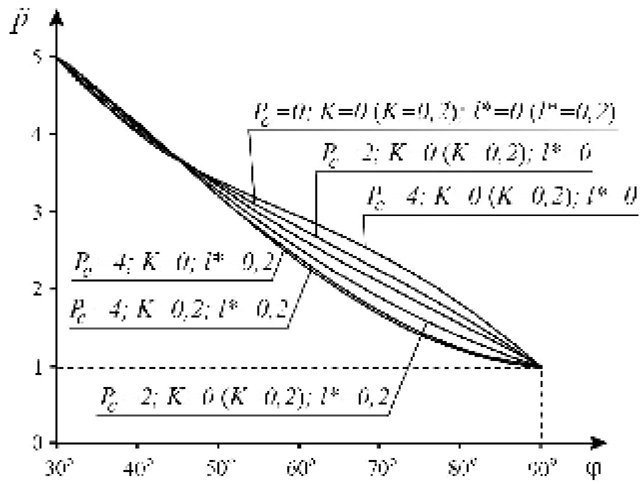

Fig.11. Dimensionless pressure distribution $\tilde{p}$ for the clearance of mean length.

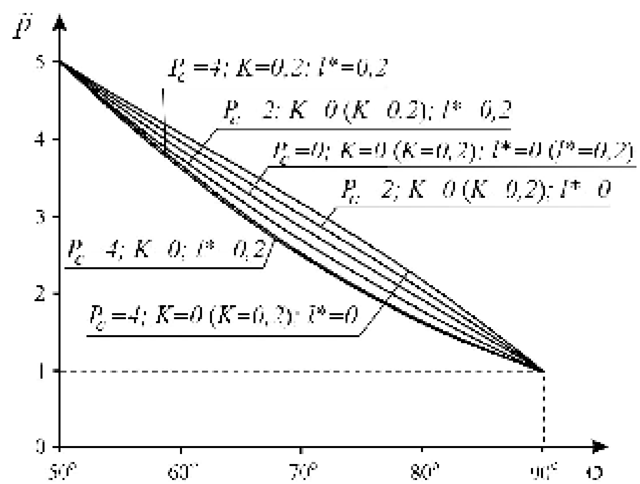

Fig.12. Dimensionless pressure distribution $\tilde{p}$ for the clearance of short length.

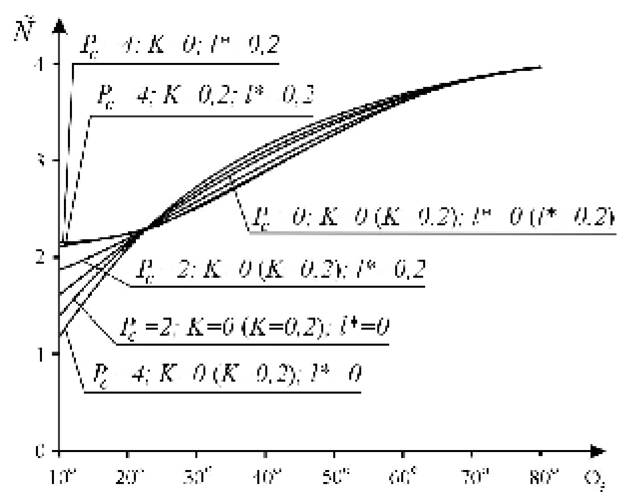

Fig.13. Dimensionless load-capacity $\tilde{N}$ a spherical bearing.

4. Conclusion

From general considerations and graphs presented for the plane bearing lubricated by a couple-stress fluid we can draw the following conclusions:

- the couple-stress fluid $\left(l^* \neq 0\right)$ is characterized by larger pressures than the Newtonian fluid $\left(l^* = 0\right)$, so is the case with the load-capacity of a plane bearing,
- for $l^* = 0$ near the long clearance of a bearing the inertia effects P_c reduce pressure, however, for $l^* \neq 0$ the pressure increases is observed,
- for the short bearings an opposite is observed for $l^* \neq 0$ the inertia effects cause a faster drop in pressure than for $l^* = 0$,
- the porosity $K \neq 0, K << 1$ produces drops of pressure, however, the smaller the length of the clearance, the smaller the influence of porosity on pressure distribution.

From the general considerations and graphs presented for a spherical bearing one can deduce that:

– for one rotating surface:
  - the flow of the couple-stress fluid ($l^* = 0$) is characterized by larger pressures than those of the Newtonian flow,
  - the porosity reduces the pressure values;

– for two fixed surfaces:
  - the couple-stress properties do not have any influence on the flow.

Nomenclature

H	– thickness of the porous layer
K	– porosity
N	– load-capacity
p	– pressure
R	– radius of the bearing surface
Φ	– permeability of porous layer
μ	– coefficient of plastic viscosity
η	– couple-stress viscosity
ρ	– density
ω	– the angular velocity
~	– dimensionless values

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