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THE EFFECT OF VARIABLE VISCOSITY ON MHD NATURAL CONVECTION IN MICROPOLAR FLUIDS

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The influence of variable viscosity and a transverse magnetic field on natural convection in micropolar fluids is examined. The fluid viscosity is assumed to vary as an inverse linear function of temperature. Four different vertical flows have been analyzed, those adjacent to an isothermal surface and uniform heat flux surface, a plane plume and flow generated from a horizontal line energy source on a vertical adiabatic surface, or wall plume. By means of similarity solutions and deviation of the velocity, temperature and micro-rotation fields as well as the skin friction, heat transfer and wall couple stress results from their constant values are determined.

Key words: natural convection, micropolar fluids, variable viscosity.

1. Introduction

Most studies of the problems of heat transfer are based on the constant physical properties of the ambient fluid. However, it is known that these properties may change with temperature, especially for fluid viscosity. To accurately predict the flow and heat transfer rates, it is necessary to take account of this variation of viscosity. The influence of variable fluid properties on free convection laminar flow has been studied by Herwing *et al.* (1985), Jang and Lin (1988), Shang and Wang (1990), Pozzi and Lupo (1990), Spalding and Gruddace (1961) and Carey and Mollendorf (1978), (1980). Carey and Mollendorf (1978), (1980) have shown the mathematical forms of viscosity variation with temperature which result in similarity solutions for laminar natural convection from a vertical isothermal surface in liquids with temperature dependent viscosity. Considerably less work has been done concerning variable property effects on constant buoyancy natural convection flows: the plane plume above a horizontal line heat source and the flow above a horizontal line heat source on a vertical adiabatic surface. Takhar and Pop (1993) have studied the effects of temperature dependent viscosity on natural convection in axisymmetric flows around a heated vertical surface. Pop *et al.* (1992) have studied the effect of variable viscosity on the flow and heat transfer on a continuous moving flat plate. Mohammadien *et al.* (1998) have studied the effects of variable viscosity on natural convection in a micropolar fluid at an axisymmetric stagnation point on a heated vertical surface. El-Hakiem (1998) has studied the effect of a transverse magnetic field with temperature dependent viscosity in a micropolar fluid.

In this work, the effect of variable viscosity is considered for the flow and heat transfer on natural convection in the presence of a transverse magnetic field. The fluid viscosity is assumed to vary as an inverse linear function of temperature. Thus the analysis provides a more accurate picture of the momentum and thermal transport in this problem than the usual analysis with constant properties.

2. Analysis

We consider a steady, two-dimensional, vertical natural convection flow of a micropolar fluid. The absolute viscosity μ is taken as a variable in the force-momentum balance while the fluid volumetric coefficient of thermal expansion β, the acceleration to gravity g^* the density ρ, the thermal diffusivity k,

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the spin gradient viscosity γ, the vortex viscosity χ and the micro-inertia density j are assumed to be constant. The buoyancy force resulting from the concentration differences may assist or oppose the buoyancy force induced by the temperature variations in the fluid. The applied magnetic field is primary in the y-direction and is a function only of x. The analysis will be confined to species diffusion processes in which the diffusion-thermo and thermo-diffusion effects can be neglected. Under the Oberbeck-Boussinesq and boundary layer assumptions, the governing equations are given by:

Mass:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 . \tag{1}$$

Momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left[(\mu+\chi)\frac{\partial u}{\partial y}\right]+\frac{\chi}{\rho}\frac{\partial N}{\partial y}+g^{*}\beta(T-T_{\infty})-\sigma\frac{B_0^2}{\rho}u . \tag{2.2}$$

Angular momentum:

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{\chi}{\rho j}\left(\frac{\partial u}{\partial y}+2N\right). \tag{2.3}$$

Energy:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2} . \tag{2.4}$$

Proceeding with the analysis, we introduce the following transformations

$$\eta(x,y)=yb(x), \tag{2.5}$$

$$\psi(x,y)=\frac{\mu_f}{\rho}c(x)f(\eta,x), \tag{2.6}$$

$$\phi(\eta,x)=\frac{(T-T_{\infty})}{(T_0-T_{\infty})_0}, \qquad (T_0-T_{\infty})_0=d(x)=sx^n, \tag{2.7}$$

$$N(x,y)=\frac{\mu_f}{\rho}c(x)[b(x)]^2 g(\eta,x), \tag{2.8}$$

$$c(x)=4xb(x)=4\left[g^{*}\beta\rho^2 x^3\frac{(T_0-T_{\infty})_0}{4\mu_f^2}\right]^{1/4}=4\left[\frac{\mathrm{Gr}_x}{4}\right]^{1/4}, \tag{2.9}$$

$$\gamma_f=\left(\frac{1}{\mu}\frac{d\mu}{dT}\right)_f(T_0-T_{\infty})_0, \tag{2.10}$$

$$\mathrm{Gr}_x = \rho^2 g^* \beta x^3 \left(T_0 - T_\infty\right)_0 \big/ \mu_f^2 \,, \tag{2.11}$$

$$Mn = \frac{2\sigma}{\mu_f} \frac{B_0^2 x^2}{\sqrt{\mathrm{Gr}_x}} . \tag{2.12}$$

$\left(T_0 - T_\infty\right)_0$ is the downstream temperature difference (along the *x*-axis) and Gr_x is the Grashof number. The absolute viscosity μ is assumed to vary with temperature according to a general functional form

$$\frac{1}{\mu} = \frac{1}{\mu_f}\left[1 + \left(\frac{1}{\mu}\frac{d\mu}{dT}\right)_f \left(T - T_f\right)\right]. \tag{2.13}$$

The viscous shear term in (2) can be expanded

$$\frac{1}{\rho}\frac{\partial}{\partial y}\left[(\mu+\chi)\frac{\partial u}{\partial y}\right] = \frac{(\mu+\chi)}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho}\frac{d\mu}{dT}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y} \tag{2.14}$$

and after substitution, the momentum equation becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{(\mu+\chi)}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho}\frac{d\mu}{dT}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y} + \frac{\chi}{\rho}\frac{\partial N}{\partial y} + g^*\beta\left(T - T_\infty\right) - \sigma\frac{B_0^2}{\rho}u . \tag{2.15}$$

Expansions for the stream, temperature and microrotation functions $f(\eta, x)$, $g(\eta, x)$ and $\phi(\eta, x)$ are postulated as

$$f\left(\eta, \gamma_f\right) = f(\eta, x) = f_0(\eta) + \gamma_f(x) f_1(\eta) + \left[\gamma_f(x)\right]^2 f_2(\eta) + \ldots , \tag{2.16}$$

$$g\left(\eta, \gamma_f\right) = g(\eta, x) = g_0(\eta) + \gamma_f(x) g_1(\eta) + \left[\gamma_f(x)\right]^2 g_2(\eta) + \ldots , \tag{2.17}$$

$$\phi\left(\eta, \gamma_f\right) = \phi(\eta, x) = \phi_0(\eta) + \gamma_f(x)\phi_1(\eta) + \left[\gamma_f(x)\right]^2 \phi_2(\eta) + \ldots . \tag{2.18}$$

Here we consider only first order terms and therefore the expansions for μ, *f*, *g* and ϕ are truncated after terms of order γ_f.

Substituting Eqs (2.16)-(2.18) into Eqs (2.2)-(2.4) with the generalizations in Eq.(2.15), the equations for f_0, g_0, ϕ_0, f_1, g_1 and ϕ_1 are then determined for any value of *n*.

$$(1+\Delta) f_0''' + \Delta g_0' - 2(n+1) f_0'^2 + (n+3) f_0'' f_0 + \phi_0 - Mn f_0' = 0 , \tag{2.19}$$

$$\lambda g_0'' - \Delta B_1\left(f_0'' + 2g_0\right) - (3n+1) f_0' g_0 + (n+3) g_0' f_0 = 0 , \tag{2.20}$$

$$\phi_0''^2 + \sigma_f\left[(n+3)\phi_0' f_0 - 4n f_0' \phi_0\right] = 0 , \tag{2.21}$$

$$(1+\Delta)f_1''' + (1+2\Delta)\phi_0 f_0''' + \Delta g_1' - (8n+4)f_0' f_1' + (n+3)f_0 f_1'' + (5n+3)f_0'' f_1 + \\ -Mnf_1' + \phi_1 + 2\phi_0\left[\Delta g_0' - 2(n+1)f_0'^2 + (n+3)f_0'' f_0 + \phi_0 - Mnf'\right] - \phi_0' f_0'' = 0, \tag{2.22}$$

$$\lambda g_1'' - \Delta B_1 (f_1'' + 2g_1) - (7n+1)f_0' g_1 - (3n+1)f_1' g_0 + (5n+3)g_0' f_1 + (n+3)g_1' f_0 = 0, \tag{2.23}$$

$$\phi_1'' + \sigma_f\left[(5n+3)\phi_0' f_1 + (n+3)\phi_1' f_0 - 4n\phi_0 f_1' - 8nf_0'\phi_1\right] = 0, \tag{2.24}$$

In the above equations, a prime indicates differentiation with respect to η only and $\Delta = \chi/\mu_f$,

$\lambda = \gamma/(\mu_f j)$, $B_1 = 1/(jb^2)$, $\sigma_f = \mu_f C_p/k$.

The relevant boundary conditions for the four flows to be analyzed here are as follows:

An isothermal surface with a horizontal leading edge, $n = 0$ (a)

$$f_0(0) = f_1(0) = f_0'(0) = f_1'(0) = 0, \qquad g_0(0) = g_1(0) = 0,$$

$$1 - \phi_0(0) = \phi_1(0) = 0, \qquad f_0'(\infty) = f_1'(\infty) = 0,$$

$$g_0(\infty) = g_1(\infty) = 0, \qquad \phi_0(\infty) = \phi_1(\infty) = 0.$$

A uniform-flux surface with a horizontal leading edge, $n = 0.2$ (b)

$$f_0(0) = f_1(0) = f_0'(0) = f_1'(0) = 0, \qquad g_0(0) = g_1(0) = 0,$$

$$1 - \phi_0(0) = \phi_1'(0) = 0, \qquad f_0'(\infty) = f_1'(\infty) = 0,$$

$$g_0(\infty) = g_1(\infty) = 0, \qquad \phi_0(\infty) = \phi_1(\infty) = 0.$$

An adiabatic surface with a concentrated heat source along horizontal leading edge, $n = -0.6$ (c)

$$f_0(0) = f_1(0) = f_0'(0) = f_1'(0) = 0, \qquad g_0(0) = g_1(0) = 0,$$

$$1 - \phi_0(0) = \phi_0'(0) = \phi_1'(0) = 0, \qquad f_0'(\infty) = f_1'(\infty) = 0,$$

$$g_0(\infty) = g_1(\infty) = 0, \qquad \phi_0(\infty) = \phi_1(\infty) = 0.$$

A plane plume rising from a horizontal thermal source, $n = -0.6$ (d)

$$f_0(0) = f_1(0) = f_0''(0) = f_1''(0), \qquad g_0(0) = g_1(0) = 0,$$

$$1 - \phi_0(0) = \phi_0'(0) = \phi_1'(0) = 0, \qquad f_0'(\infty) = f_1'(\infty) = 0,$$

$$g_0(\infty) = g_1(\infty) = 0, \qquad \phi_0(\infty) = \phi_1(\infty) = 0.$$

For the isothermal condition, $n = 0$, and since $\phi_1(0) = 0$, the temperature at $y = 0$ is not altered by varying γ_f. Consequently, the film temperature, $T_f = (T_0 + T_\infty)/2$ and $T_0 - T_\infty$ are not altered by varying γ_f. Therefore, for the isothermal condition, γ_f is equal to $\gamma_f = (1/\mu)_f (d\mu/dT)_f (T_0 - T_\infty)$ as defined by

Carey and Mollendorf (1980). The values of n shown above for the other three flow conditions are determined by calculating the value of $Q(x)$ – the total heat convected in the flow at any downstream location *x*.

$$Q(x)=\int_0^\infty \rho c_p (T-T_\infty) u dy = \mu_f c_p c d \int_0^\infty f' \phi d\eta \propto x^{(3+5n)/4} . \tag{2.25}$$

This must increase linearly with x for the uniform heat flux surface condition, (b), and be independent of x for the adiabatic flows, (c) and (d) Therefore

$$n_a = 0 , \quad n_b = 0.2 , \quad n_c = n_d = -0.6 .$$

Including the first order terms in f and ϕ for $\gamma_f \neq 0$, $Q(x)$ is

$$Q(x)=\mu_f c_p c d \left[\int_0^\infty \phi_0 f_0' d\eta + \gamma_f \int_0^\infty (\phi_0 f_1' + \phi_1 f_0') d\eta \right] . \tag{2.26}$$

For the uniform flux condition, (b), and the adiabatic flows, (c) and (d), integration of the first order energy Eq.(24) shows that the second integral in Eq.(2.26) is zero. This is required to ensure that additional x dependence is not added to $Q(x)$ though γ_f may therefore be written as

$$Q(x)=\mu_f c_p c d\, I_Q \quad \text{where} \quad I_Q = \int_0^\infty \phi_0 f_0' d\eta . \tag{2.27}$$

The mass flow per unit width of surface, $\dot{m}$, becomes

$$\dot{m} = \int_0^\infty \rho u\, dy = \mu_f c \int_0^\infty f' d\eta = \mu_f c \left[f_0(\infty) + \gamma_f f_1(\infty) \right] , \tag{2.28}$$

and the momentum flux in the x direction is given by

$$M(x)=\int_0^\infty \rho u^2 dy = \frac{\mu_f^2 c^2 b}{\rho} \int_0^\infty f'^2 d\eta = \frac{\mu_f^2 c^2 b}{\rho} \left[I_{M0} + \gamma_f I_{M1} \right] \tag{2.29}$$

where

$$I_{M0} = \int_0^\infty f_0'^2 d\eta , \qquad I_{M1} = \int_0^\infty 2 f_0' f_1'^2 d\eta . \tag{2.30}$$

The stream function, as defined here, is based on the film viscosity. For the flows adjacent to a vertical surface, the shear stress at the surface, τ_w is, therefore a function of γ_f directly, as well as $f''(0)$

$$\tau_w = \left[(\mu + K)\frac{\partial u}{\partial y} + KN \right]_{y=0} . \tag{2.31}$$

The surface heat flux, q'', and the local Nusselt number, Nu_x, are determined as

$$q'' = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} , \tag{2.32}$$

$$\mathrm{Nu}_x = \frac{q''}{(T_0 - T_\infty)}\frac{x}{k} = \left[-\frac{\phi'(0,\gamma_f)}{[\phi(0,\gamma_f)]^{5/4}} \right]\frac{(\mathrm{Gr}'_x)}{\sqrt{2}} \tag{2.33}$$

where $\phi(0) = \phi_0(0) + \gamma_f \phi_1(0)$ and $\mathrm{Gr}'_x = \mathrm{Gr}_x \phi(0)$, $\mathrm{Gr}'_x = \rho^2 g^* \beta x^3 (T_0 - T_\infty)/\mu_f^2$ is the actual physical Grashof number.

The last relation is rewritten for convenience as

$$N' = \frac{\mathrm{Nu}_x \sqrt{2}}{(\mathrm{Gr}'_x)^{1/4}} , \tag{2.34}$$

we have

$$N' = \left[-\frac{\phi'(0,\gamma_f)}{[\phi(0,\gamma_f)]^{5/4}} \right].$$

Results and discussion .3

The system of Eqs (2.19)-(2.24) with the boundary conditions (a-d) was solved numerically by the fourth order Runge-Kutta integration scheme. Calculations were carried out for the value of the Prandtl number *10*, the magnetic parameter ranged from *0* to *2* and the micropolar parameter $\Delta = 0.5, 1.5, 5.0$ are summarized with $B_1 = 0.1$ and $\lambda = 0.5$.

We compared these results with the results in EL-Hakiem (1998) to show the difference when the fluid viscosity is assumed to vary as an inverse linear function of temperature. Table 1 displays the results for the isothermal wall boundary condition. They show the surface values of velocity, temperature and the microrotation gradient components. These are proportional to the friction factor, the Nusselt number and the wall couple stress respectively. The results indicate that as Δ increases the friction factor, the Nusselt number and the wall couple stress decrease. A similar behavior is noticed when the magnetic field *Mn* increases. Tables 2-4 display results for the constant surface heat flux, an unbounded plume and the wall plume respectively. The results indicate that the temperature dependent viscosity has a significant effect on the temperature, velocity and angular velocity fields as well as the heat transfer rate and drag. The strong effect of temperature dependent viscosity on the flow field would suggest the possibility of significant effects on the stability and transition of such a flow.

Table 1. Case of an isothermal surface with a horizontal leading edge, $n = 0$.

$-g(0)x10^2$	$-\phi'(0)$	$f''(0)$	Mn	Δ	γ_f
0.00000	*1.00876*	*0.16900*	*0.0*	*0.0*	*-0.8*
0.00000	*0.96933*	*0.16342*	*0.5*		

0.00000	0.94029	0.15923	1.0		
0.00000	0.89521	0.15248	2.0		
0.50118	0.97073	0.18797	0.0	0.5	
0.40220	0.93273	0.17931	0.5		
0.35166	0.90546	0.17314	1.0		
0.29524	0.86359	0.16377	2.0		
1.46425	0.90291	0.16468	0.0	1.5	
1.23217	0.86873	0.15669	0.5		
1.10884	0.84488	0.15118	1.0		
0.96038	0.80870	0.14295	2.0		
3.36696	0.77215	0.10200	0.0	5.0	
3.01110	0.74574	0.09756	0.5		
2.80680	0.72802	0.09459	1.0		
2.54040	0.70156	0.09018	2.0		0.0
0.00000	1.16933	0.41920	0.0	0.0	
0.00000	1.11548	0.39250	0.5		
0.00000	1.07567	0.37346	1.0		
0.00000	1.01456	0.34534	2.0		
0.57512	1.07185	0.31451	0.0	0.5	
0.45815	1.02495	0.29551	0.5		
0.39679	0.99127	0.28228	1.0		
0.32700	0.94002	0.26284	2.0		
1.61200	0.95905	0.21798	0.0	1.5	
1.34848	0.92011	0.20589	0.5		
1.20734	0.89298	0.19765	1.0		
1.03820	0.85205	0.18555	2.0		
3.56339	0.79253	0.11372	0.0	5.0	
3.17609	0.76451	0.10850	0.5		0.8
2.95420	0.74575	0.10502	1.0		
2.66627	0.71782	0.09990	2.0		
0.00000	1.32990	0.66940	0.0	0.0	
0.00000	1.26163	0.62158	0.5		
0.00000	1.21105	0.58769	1.0		
0.00000	1.13391	0.53820	2.0		
0.64906	1.17297	0.44105	0.0	0.5	
0.51410	1.11717	0.41171	0.5		
0.44192	1.07708	0.39142	1.0		
0.35876	1.01645	0.36191	2.0		
1.75975	1.01519	0.27128	0.0	1.5	
1.46479	0.97149	0.25509	0.5		
1.30584	0.94108	0.24412	1.0		
1.11602	0.89540	0.22815	2.0		
3.75982	0.81291	0.12544	0.0	5.0	
3.34108	0.78328	0.11944	0.5		
3.10160	0.76348	0.11545	1.0		
2.79214	0.73408	0.10962	2.0		

Table 2. Case of a uniform-flux surface with a horizontal leading edge, $n = 0.2$.

$-g(0)x10^2$	$\phi'(0)$	$f''(0)$	Mn	Δ	γ_f
0.00000	1.10846	0.19029	0.0	0.0	-0.8

0.00000	1.10265	0.18275	0.5			
0.00000	1.09818	0.17703	1.0			
0.00000	1.09126	0.16802	2.0			
0.48329	1.07438	0.19302	0.0	0.5		
0.38985	1.07043	0.18401	0.5			
0.34105	1.06752	0.17754	1.0			
0.28495	1.06311	0.16774	2.0	1.5		
1.37383	1.04607	0.16189	0.0			
1.15996	1.04370	0.15426	0.5			
1.04516	1.04202	0.14896	1.0			
0.90629	1.03954	0.14104	2.0	5.0		
3.11261	1.02018	0.09760	0.0			
2.79121	1.01919	0.09354	0.5			
2.60566	1.01855	0.09082	1.0			
2.36336	1.01763	0.08677	2.0	0.0	0.0	
0.00000	1.00000	0.39503	0.0			
0.00000	1.00000	0.37134	0.5			
0.00000	1.00000	0.35435	1.0			
0.00000	1.00000	0.32911	2.0	0.5		
0.53366	1.00000	0.29622	0.0			
0.42693	1.00000	0.27935	0.5			
0.37059	1.00000	0.26755	1.0	1.5		
0.30601	1.00000	0.25012	2.0			
1.47409	1.00000	0.20518	0.0			
1.23776	1.00000	0.19446	0.5			
1.11044	1.00000	0.18711	1.0	5.0		
0.95726	1.00000	0.17628	2.0			
3.24908	1.00000	0.10704	0.0			
2.90554	1.00000	0.10241	0.5		0.8	
2.70778	1.00000	0.09932	1.0	0.0		
2.45044	1.00000	0.09474	2.0			
0.00000	0.89154	0.59977	0.0			
0.00000	0.89735	0.55993	0.5			
0.00000	0.90182	0.53167	1.0	0.5		
0.00000	0.90874	0.49020	2.0			
0.58403	0.92562	0.39942	0.0			
0.46401	0.92957	0.37469	0.5	1.5		
0.40013	0.93248	0.35756	1.0			
0.32707	0.93689	0.33250	2.0			
1.57435	0.95393	0.24847	0.0			
1.31556	0.95630	0.23466	0.5	5.0		
1.17572	0.95798	0.22526	1.0			
1.00823	0.96046	0.21152	2.0			
3.38555	0.97982	0.11648	0.0			
3.01987	0.98081	0.11128	0.5			
2.80990	0.98145	0.10782	1.0			
2.53752	0.98237	0.10271	2.0			

Case of an adiabatic surface with a concentrated heat source along a horizontal leading edge, $n = -0.6$. Table 3.

γ_f	Δ	Mn	$f'(0)$	$\phi(0)$	$-g'(0)x10^2$	I_Q	I_{M0}	I_{M1}

-0.8	0.0	0.0	0.34490	1.21874	0.00000	0.10625	0.04899	0.02208
		0.5	0.31841	1.16812	0.00000	0.09847	0.03248	0.01066
		1.0	0.29877	1.14056	0.00000	0.09278	0.02464	0.00673
		2.0	0.26987	1.10786	0.00000	0.08435	0.01688	0.00123
	0.5	0.0	0.32806	1.15842	0.83241	0.09879	0.04505	0.01490
		0.5	0.30290	1.12259	0.64904	0.09181	0.02861	0.00686
		1.0	0.28497	1.10350	0.55491	0.08686	0.02152	0.00430
		2.0	0.25887	1.08083	0.45066	0.07958	0.01465	0.00189
	1.5	0.0	0.26679	1.10365	2.37549	0.08980	0.03963	0.00858
		0.5	0.24729	1.08131	2.00585	0.08381	0.02403	0.00382
		1.0	0.23388	1.06956	1.79499	0.07969	0.01796	0.00240
		2.0	0.21443	1.05551	1.53645	0.07368	0.01230	0.00131
	5.0	0.0	0.15842	1.05206	5.73548	0.07649	0.03195	0.00359
		0.5	0.14827	1.04114	5.07744	0.07185	0.01807	0.00148
		1.0	0.14156	1.03565	4.68674	0.06881	0.01333	0.00092
		2.0	0.13186	1.02916	4.17509	0.06442	0.00916	0.00051
0.0	0.0	0.0	0.61060	1.00000	0.00000	0.10625	0.04899	0.02208
		0.5	0.55871	1.00000	0.00000	0.09847	0.03248	0.01066
		1.0	0.52173	1.00000	0.00000	0.09278	0.02464	0.00673
		2.0	0.46829	1.00000	0.00000	0.08435	0.01688	0.00123
	0.5	0.0	0.46138	1.00000	0.81784	0.09879	0.04505	0.01490
		0.5	0.42400	1.00000	0.65296	0.09181	0.02861	0.00686
		1.0	0.39800	1.00000	0.56286	0.08686	0.02152	0.00430
		2.0	0.36055	1.00000	0.45973	0.07958	0.01465	0.00189
	1.5	0.0	0.32245	1.00000	2.57890	0.08980	0.03963	0.00858
		0.5	0.29823	1.00000	2.14467	0.08381	0.02403	0.00382
		1.0	0.28177	1.00000	1.90538	0.07969	0.01796	0.00240
		2.0	0.25805	1.00000	1.61784	0.07368	0.01230	0.00131
	5.0	0.0	0.16988	1.00000	6.09375	0.07649	0.03195	0.00359
		0.5	0.15904	1.00000	5.34624	0.07185	0.01807	0.00148
		1.0	0.15186	1.00000	4.91452	0.06881	0.01333	0.00092
		2.0	0.14148	1.00000	4.35817	0.06442	0.00916	0.00051
0.8	0.0	0.0	0.87630	0.78126	0.00000	0.10625	0.04899	0.02208
		0.5	0.79901	0.83188	0.00000	0.09847	0.03248	0.01066
		1.0	0.74469	0.85944	0.00000	0.09278	0.02464	0.00673
		2.0	0.66671	0.89214	0.00000	0.08435	0.01688	0.00123
	0.5	0.0	0.59470	0.84158	0.80327	0.09879	0.04505	0.01490
		0.5	0.54510	0.87741	0.65688	0.09181	0.02861	0.00686
		1.0	0.51103	0.89650	0.57081	0.08686	0.02152	0.00430
		2.0	0.46223	0.91917	0.46880	0.07958	0.01465	0.00189
	1.5	0.0	0.37811	0.89635	2.78231	0.08980	0.03963	0.00858
		0.5	0.34917	0.91869	2.28349	0.08381	0.02403	0.00382
		1.0	0.32966	0.93044	2.01577	0.07969	0.01796	0.00240
		2.0	0.30167	0.94449	1.69923	0.07368	0.01230	0.00131
	5.0	0.0	0.18134	0.94794	6.45202	0.07649	0.03195	0.00359
		0.5	0.16981	0.95886	5.61504	0.07185	0.01807	0.00148
		1.0	0.16216	0.96435	5.14230	0.06881	0.01333	0.00092
		2.0	0.15110	0.97084	4.54125	0.06442	0.00916	0.00051

Table 4. Case of a plane plume rising from a the horizontal thermal source, $n = -0.6$.

γ_f	Δ	Mn	$f''(0)$	$\phi(0)$	$g'(0)x10^2$	I_Q	I_{M0}	$-I_{M1}$

-0.8	*0.0*	*0.0*	*0.39158*	*0.98960*	*0.00000*	*0.15387*	*0.12735*	*0.00954*
		0.5	*0.31620*	*0.99728*	*0.00000*	*0.13708*	*0.08135*	*0.00370*
		1.0	*0.26718*	*1.00018*	*0.00000*	*0.12459*	*0.05706*	*0.00159*
		2.0	*0.20720*	*1.00160*	*0.00000*	*0.10718*	*0.03374*	*0.00027*
	0.5	*0.0*	*0.37051*	*0.99513*	*2.05396*	*0.14982*	*0.13187*	*0.00610*
		0.5	*0.29762*	*0.99957*	*1.67236*	*0.13279*	*0.10228*	*0.00244*
		1.0	*0.25228*	*1.00130*	*1.52195*	*0.12084*	*0.05537*	*0.00116*
		2.0	*0.19676*	*1.00218*	*1.23603*	*0.10429*	*0.03224*	*0.00024*
	1.5	*0.0*	*0.34336*	*0.99810*	*4.07448*	*0.14482*	*0.13745*	*0.00341*
		0.5	*0.27440*	*1.00049*	*3.43169*	*0.12795*	*0.07989*	*0.00145*
		1.0	*0.23203*	*1.00146*	*3.00896*	*0.11626*	*0.05351*	*0.00074*
		2.0	*0.18191*	*1.00207*	*2.45369*	*0.10072*	*0.03061*	*0.00022*
	5.0	*0.0*	*0.30124*	*0.99969*	*5.09565*	*0.13675*	*0.14617*	*0.00131*
		0.5	*0.23635*	*1.00063*	*4.33978*	*0.11977*	*0.07823*	*0.00059*
		1.0	*0.19903*	*1.00109*	*3.87513*	*0.10872*	*0.05045*	*0.00033*
		2.0	*0.15672*	*1.00146*	*3.26928*	*0.09462*	*0.02807*	*0.00012*
0.0	*0.0*	*0.0*	*0.41395*	*1.00000*	*0.00000*	*0.15387*	*0.12735*	*0.00954*
		0.5	*0.34041*	*1.00000*	*0.00000*	*0.13708*	*0.08135*	*0.00370*
		1.0	*0.29076*	*1.00000*	*0.00000*	*0.12459*	*0.05706*	*0.00159*
		2.0	*0.22780*	*1.00000*	*0.00000*	*0.10718*	*0.03374*	*0.00027*
	0.5	*0.0*	*0.38229*	*1.00000*	*2.15014*	*0.14982*	*0.13187*	*0.00610*
		0.5	*0.31083*	*1.00000*	*1.79858*	*0.13279*	*0.10228*	*0.00244*
		1.0	*0.26544*	*1.00000*	*1.65136*	*0.12084*	*0.05537*	*0.00116*
		2.0	*0.20875*	*1.00000*	*1.35747*	*0.10429*	*0.03224*	*0.00024*
	1.5	*0.0*	*0.34872*	*1.00000*	*4.24996*	*0.14482*	*0.13745*	*0.00341*
		0.5	*0.28066*	*1.00000*	*3.63560*	*0.12795*	*0.07989*	*0.00145*
		1.0	*0.23846*	*1.00000*	*3.21643*	*0.11626*	*0.05351*	*0.00074*
		2.0	*0.18804*	*1.00000*	*2.64835*	*0.10072*	*0.03061*	*0.00022*
	5.0	*0.0*	*0.30278*	*1.00000*	*5.28165*	*0.13675*	*0.14617*	*0.00131*
		0.5	*0.23827*	*1.00000*	*4.53512*	*0.11977*	*0.07823*	*0.00059*
		1.0	*0.20108*	*1.00000*	*4.06956*	*0.10872*	*0.05045*	*0.00033*
		2.0	*0.15879*	*1.00000*	*3.45248*	*0.09462*	*0.02807*	*0.00012*
0.8	*0.0*	*0.0*	*0.43632*	*1.01040*	*0.00000*	*0.15387*	*0.12735*	*0.00954*
		0.5	*0.36462*	*1.00272*	*0.00000*	*0.13708*	*0.08135*	*0.00370*
		1.0	*0.31434*	*0.99982*	*0.00000*	*0.12459*	*0.05706*	*0.00159*
		2.0	*0.24840*	*0.99840*	*0.00000*	*0.10718*	*0.03374*	*0.00027*
	0.5	*0.0*	*0.39407*	*1.00487*	*2.24632*	*0.14982*	*0.13187*	*0.00610*
		0.5	*0.32404*	*1.00043*	*1.92480*	*0.13279*	*0.10228*	*0.00244*
		1.0	*0.27860*	*0.99870*	*1.78077*	*0.12084*	*0.05537*	*0.00116*
		2.0	*0.22074*	*0.99782*	*1.47891*	*0.10429*	*0.03224*	*0.00024*
	1.5	*0.0*	*0.35408*	*1.00190*	*4.42544*	*0.14482*	*0.13745*	*0.00341*
		0.5	*0.28692*	*0.99951*	*3.83951*	*0.12795*	*0.07989*	*0.00145*
		1.0	*0.24489*	*0.99854*	*3.42390*	*0.11626*	*0.05351*	*0.00074*
		2.0	*0.19417*	*0.99793*	*2.84301*	*0.10072*	*0.03061*	*0.00022*
	5.0	*0.0*	*0.30432*	*1.00031*	*5.46765*	*0.13675*	*0.14617*	*0.00131*
		0.5	*0.24019*	*0.99937*	*4.73046*	*0.11977*	*0.07823*	*0.00059*
		1.0	*0.20313*	*0.99891*	*4.26399*	*0.10872*	*0.05045*	*0.00033*
		2.0	*0.16086*	*0.99854*	*3.63568*	*0.09462*	*0.02807*	*0.00012*

Figures 1-3 display results for the variation of f_0, f_1, ϕ_0, ϕ_1, g_0 and g_1 within the boundary layer for the isothermal surface conditions with $\sigma_f = 10.0$ for various Mn and Δ.

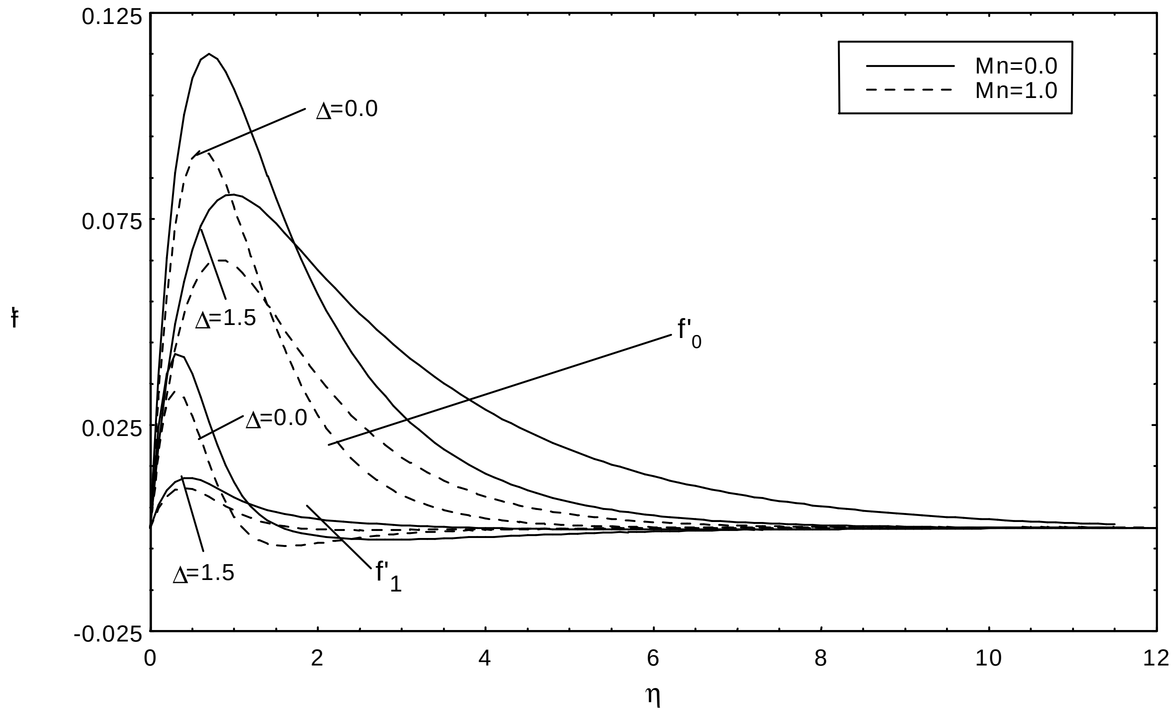


Fig.1. Velocity profiles for isothermal surface with horizontal leading edge.

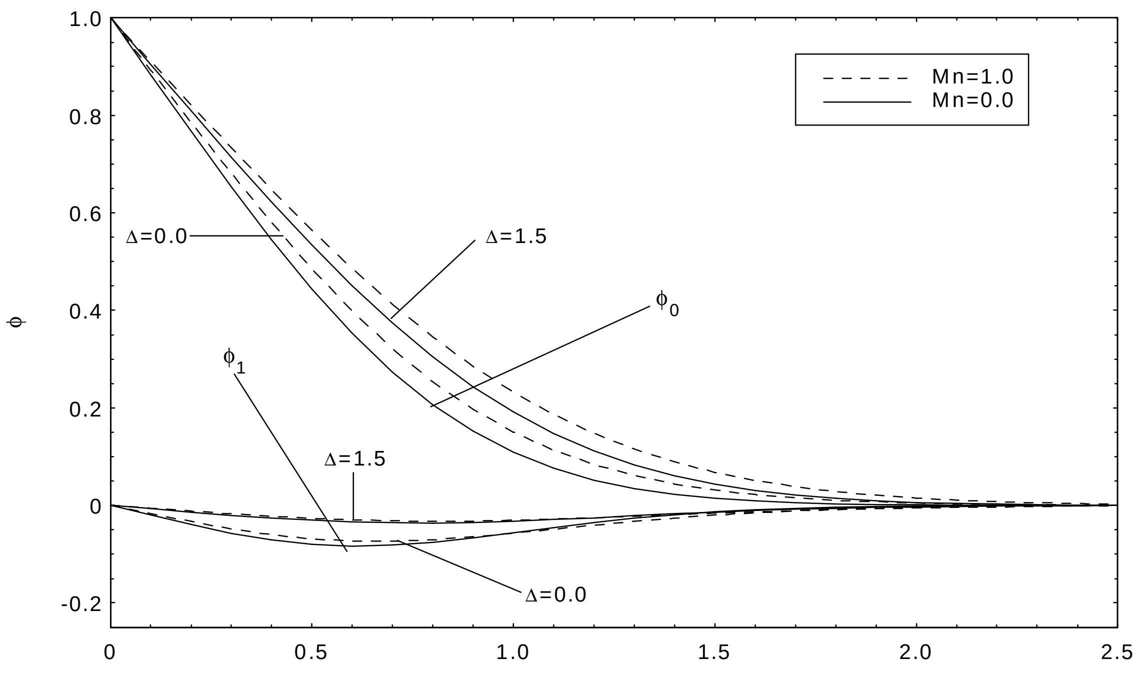


Fig.2. Temperature profiles for isothermal surface with horizontal leading edge.

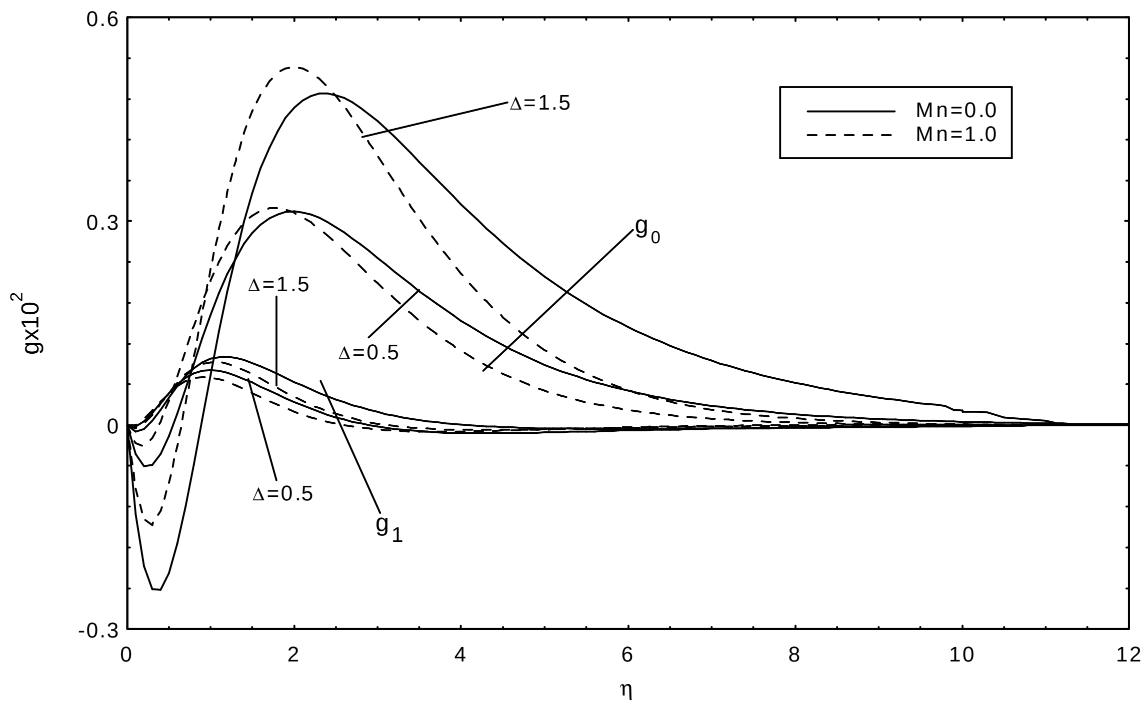


Fig.3. Microrotation profiles for isothermal surface with horizontal leading edge.

Figures 4-12 show the effect of non-zero γ_f on the velocity, temperature and angular velocity profiles for the uniform heat flux surface, the flow above a horizontal line thermal source and the flow above a horizontal line thermal source on a vertical adiabatic surface respectively.

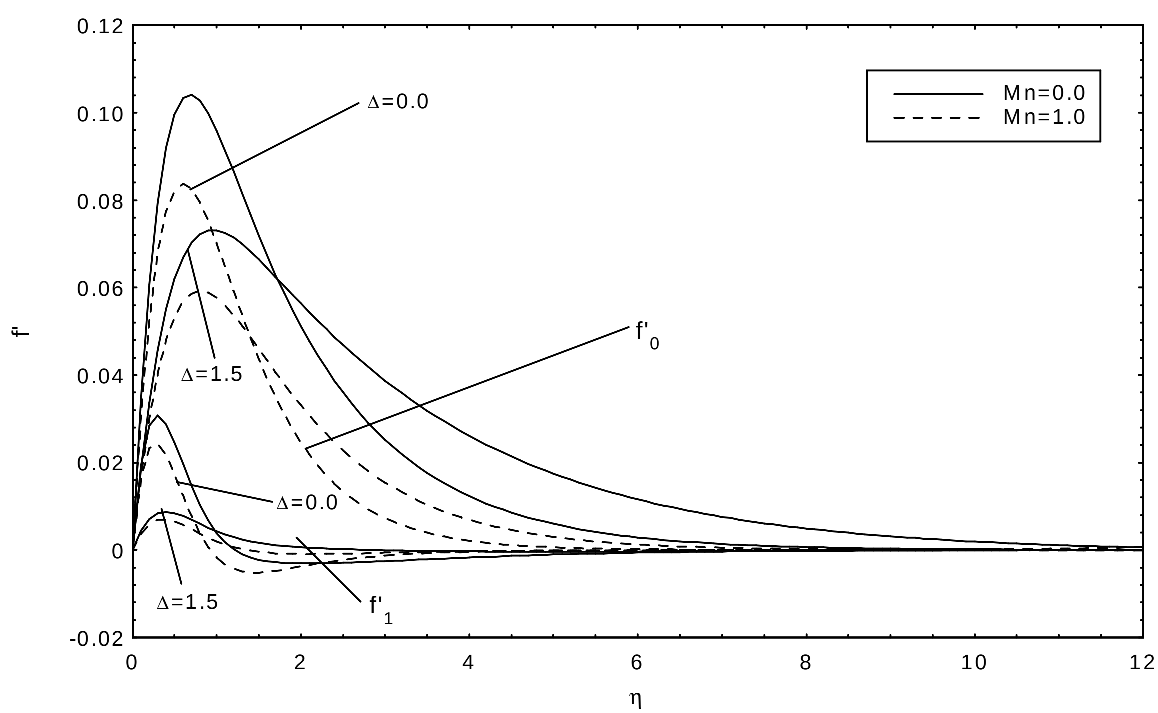


Fig.4. Velocity profiles for constant flux surface with horizontal leading edge.

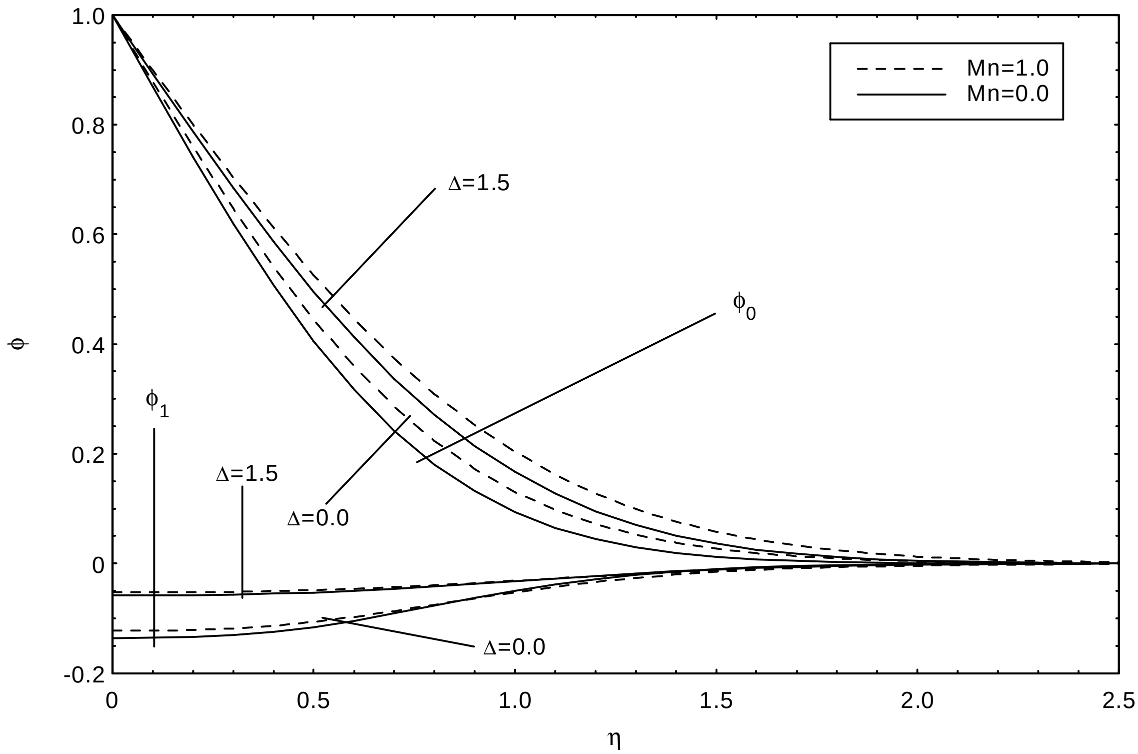


Fig.5. Temperature profiles for constant flux surface with horizontal leading edge.

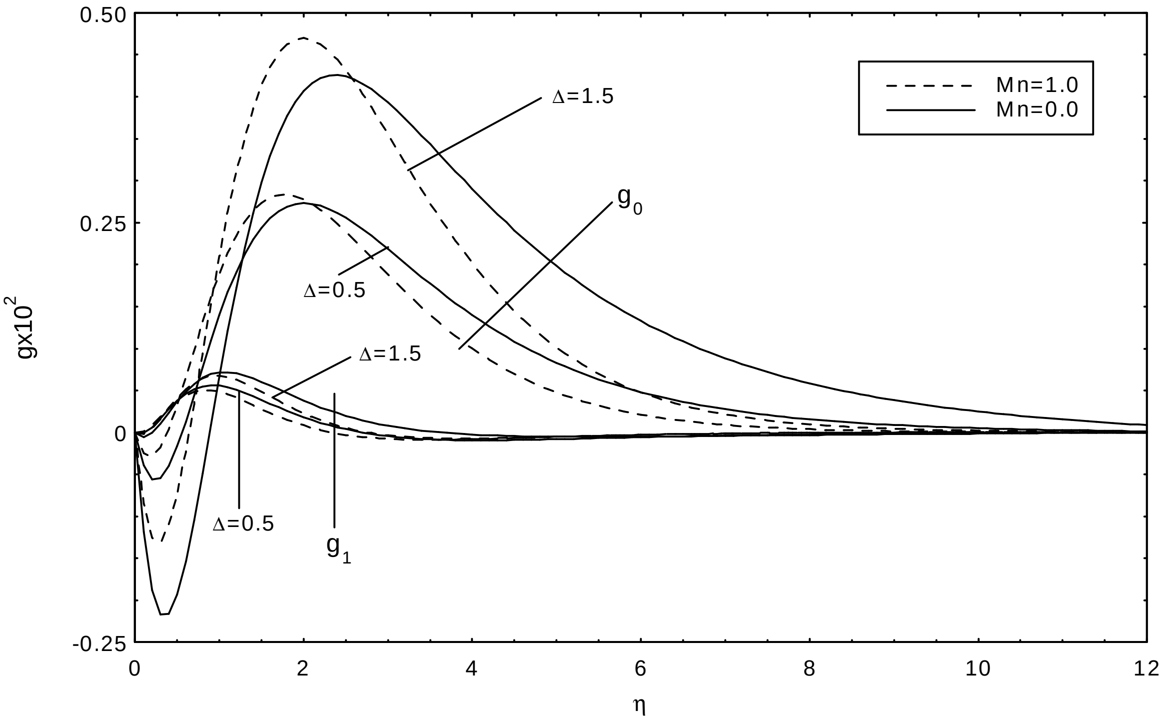
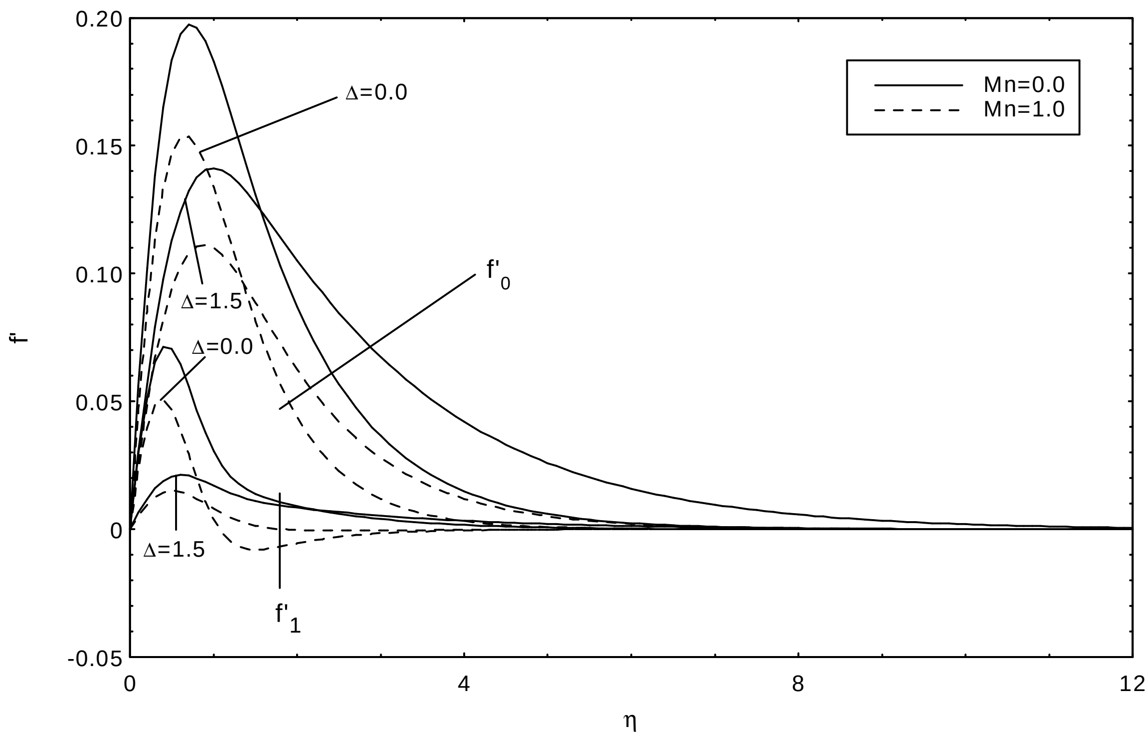


Fig.6. Microrotation profiles for constant flux surface with horizontal leading edge.



Velocity profiles for adiabatic surface with concentrated heat source along the horizontal leading edge, Fig.7. a wall plume.

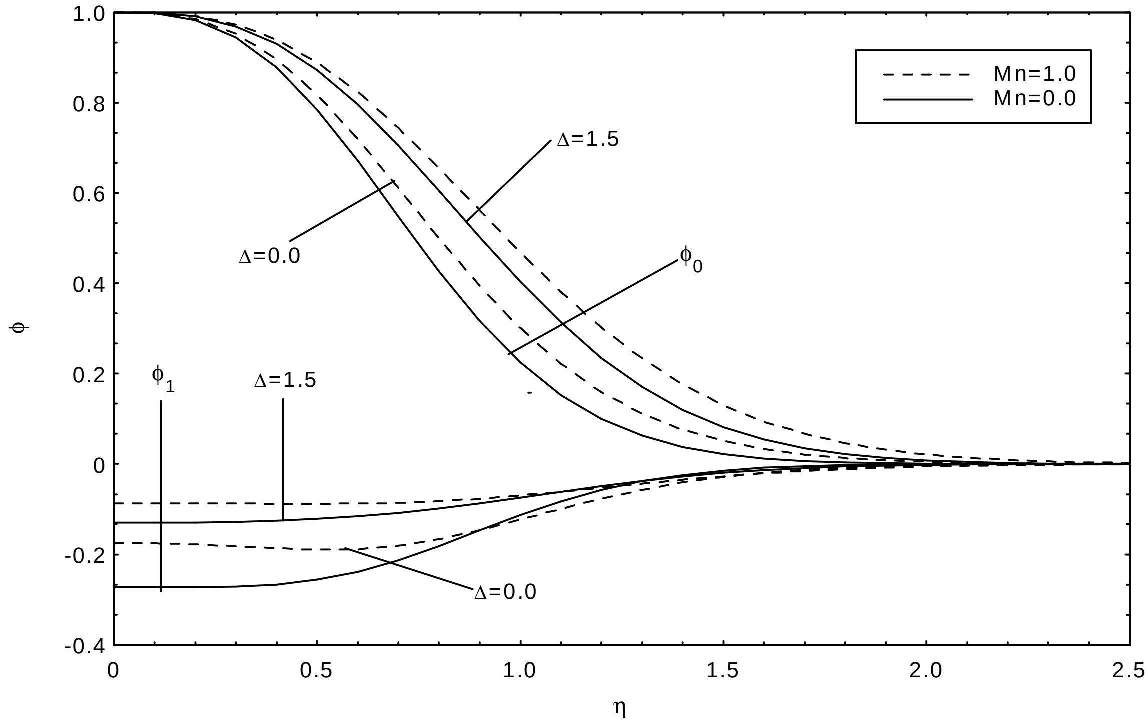


Fig.8. Temperature profiles for adiabatic surface with concentrated heat source along the horizontal leading edge, a wall plume.

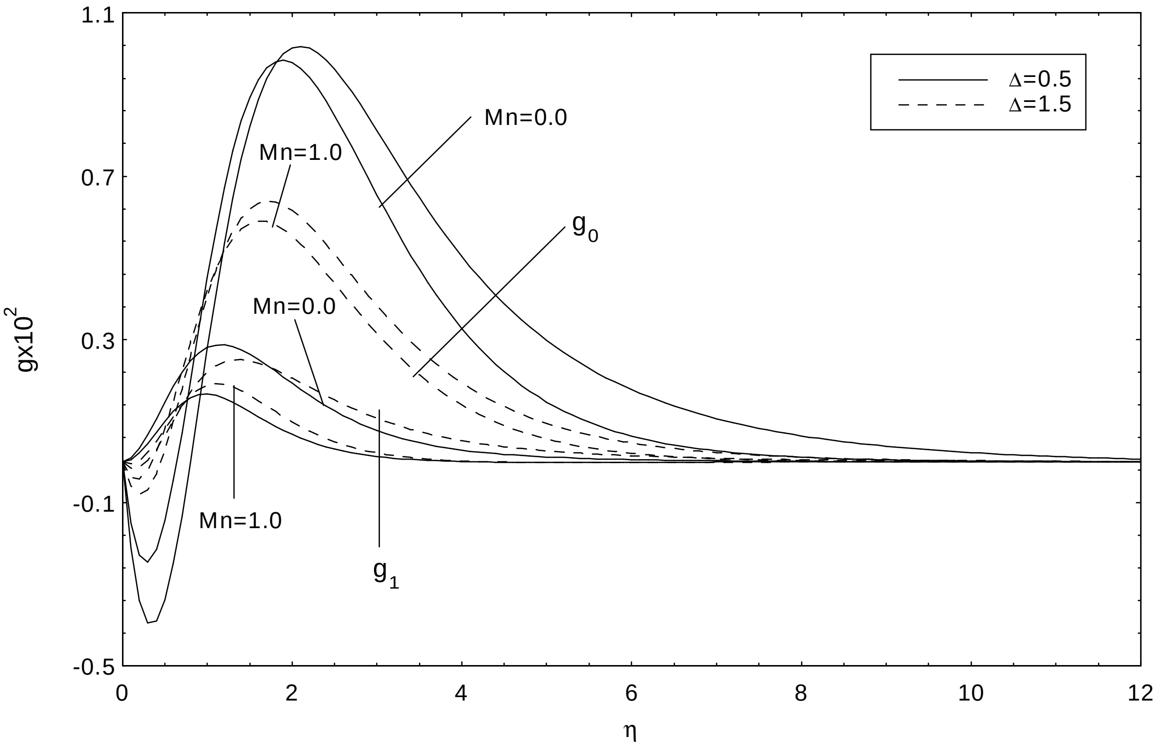


Fig.9. Microrotation profiles for adiabatic surface with concentrated heat source along the horizontal leading edge, a wall plume.

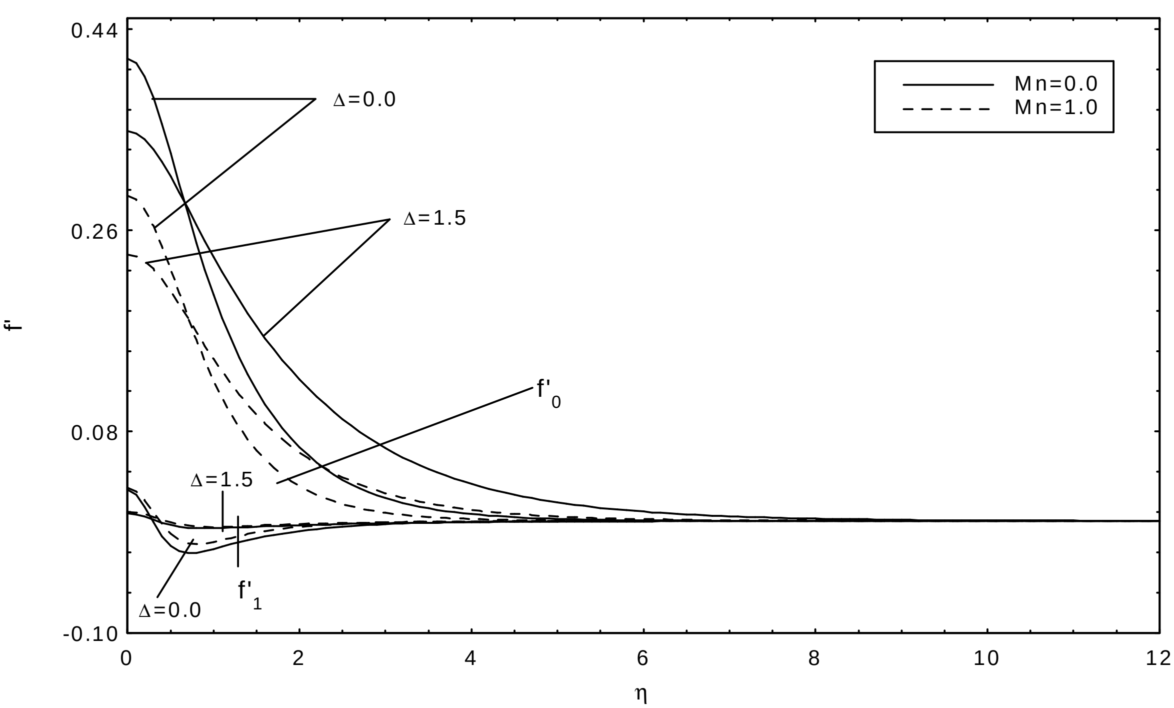


Fig.10. Velocity profiles for unbounded plane plume, rising from a horizontal thermal source at *x=0*.

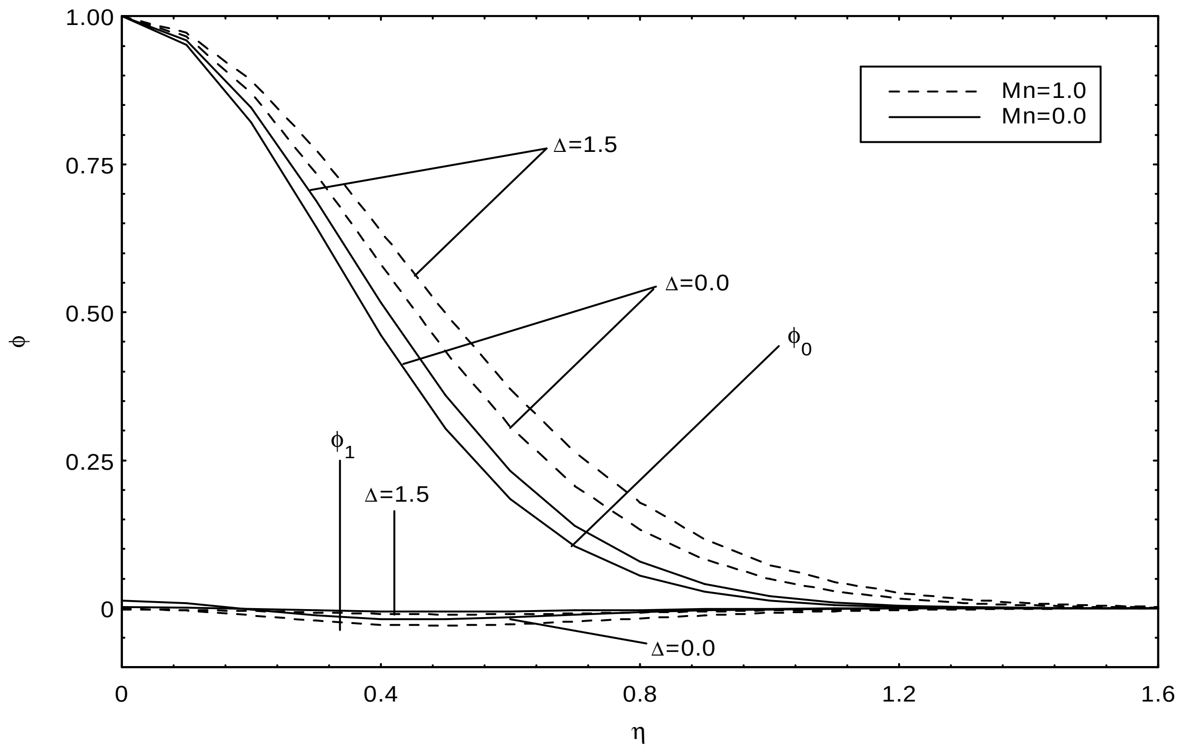


Fig.11. Temperature profiles for unbounded plane plume, rising from a horizontal thermal source at *x=0*.

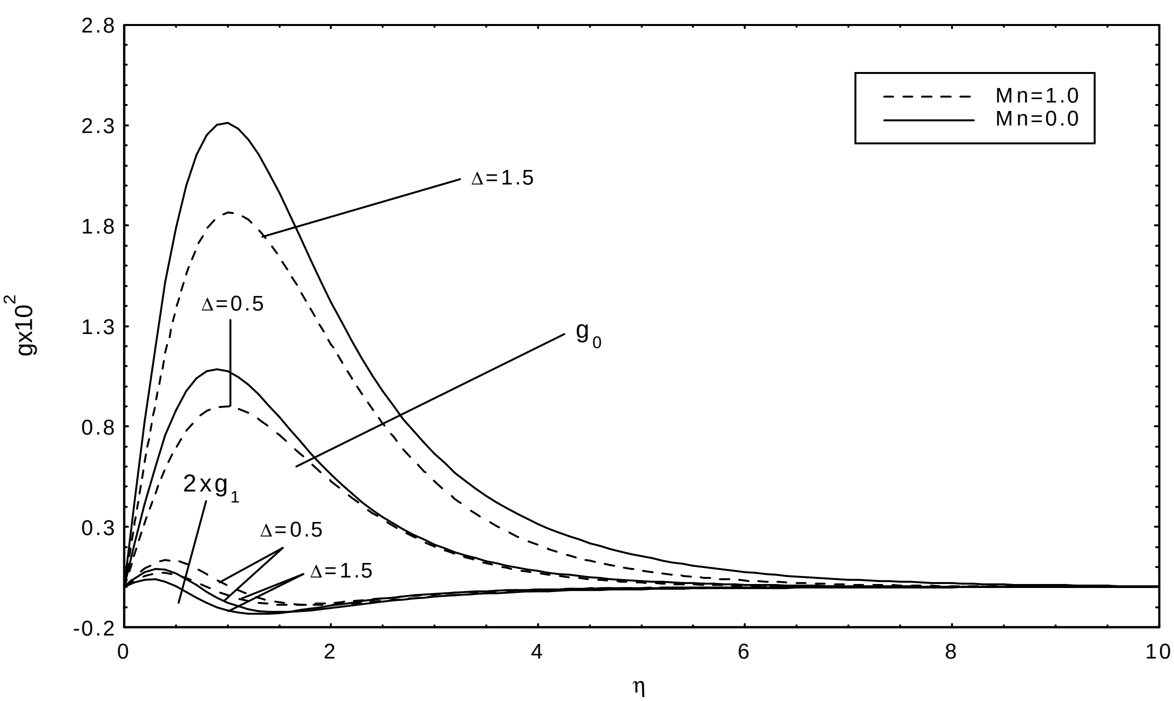
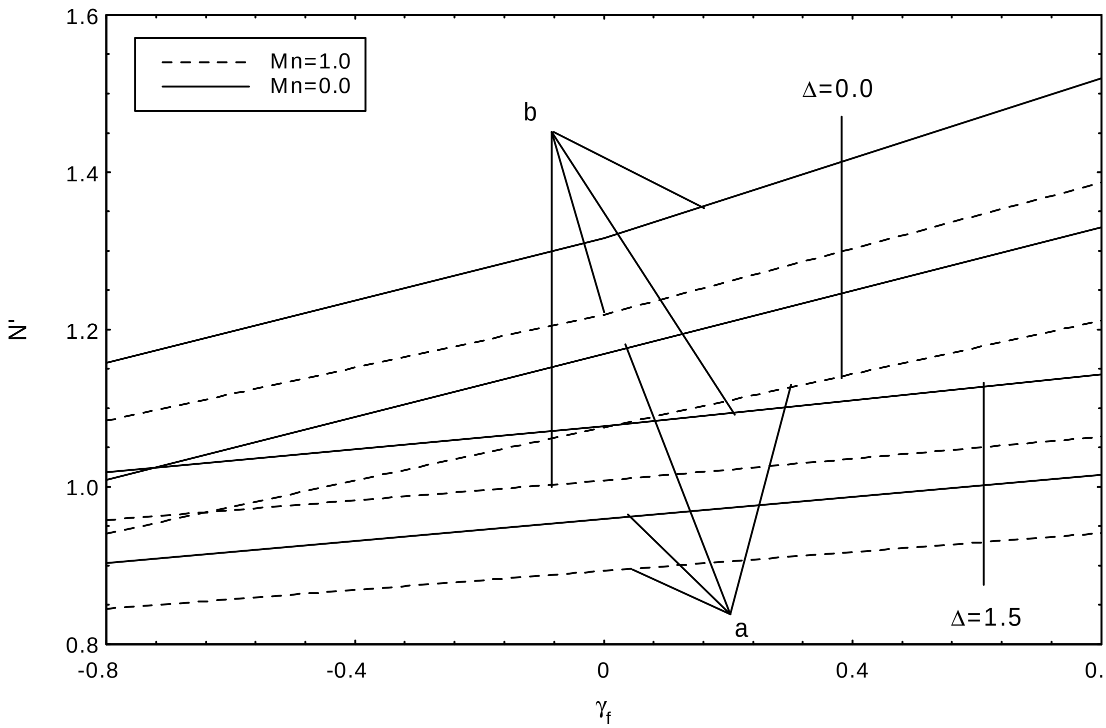


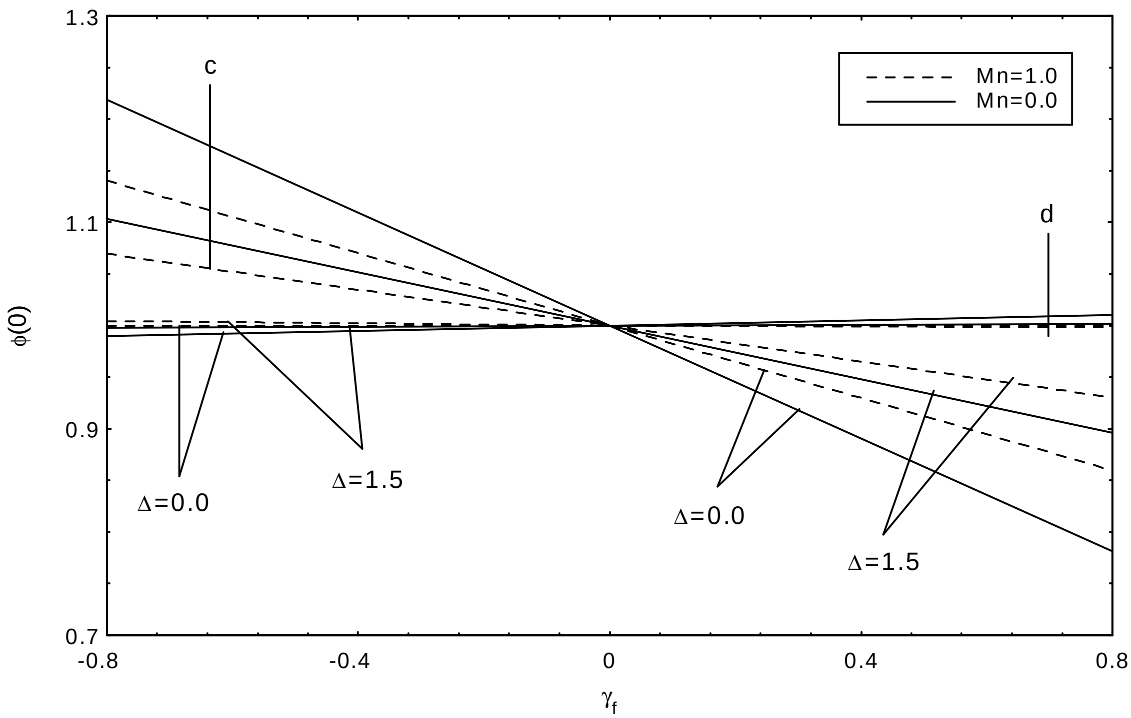
Fig.12. Microrotation profiles for unbounded plane plume, rising from a horizontal thermal source at $x=0$.
Figure 13 shows the values of the heat transfer parameter N' predicted by the perturbation analysis for the isothermal and uniform heat flux surfaces, for the range of γ_f, Mn and Δ For both the isothermal and uniform heat flux surface conditions $\gamma_f < 0$ increases the surface heat transfer while $\gamma_f > 0$ reduces it, also we note that the heat transfer parameter N' increases as R and Δ decrease for the two cases.



The effect of γ_f, R and Δ on heat transfer, for isothermal condition (a) and uniform heat Fig.13. flux (b).

Figure 14 shows the effect of γ_f, Mn and Δ on the centerline velocity for the plane plume, increasing γ_f actually produces a slight decrease in $f'(0, \gamma_f)$, it decreases with Δ and increases with R.

Figure 15 shows the effect of γ_f, Mn and Δ on the centerline temperature for the case of the wall plume and the case of concentrated horizontal source on an adiabatic surface which shows that the centerline temperature decreases with γ_f, also $\phi(0, \gamma_f)$ decreases with Mn and Δ for $\gamma_f < 0$ while the effect is the opposite for $\gamma_f > 0$ in the case of the wall plume, but the opposite behavior for the case of a concentrated horizontal source on an adiabatic surface.



The effect of Δ on transport, $\phi(0, \gamma_f)$ for the adiabatic flows as a function of γ_f (c) plane plume, Fig.14. (d) concentrated horizontal source on an adiabatic surfaces.

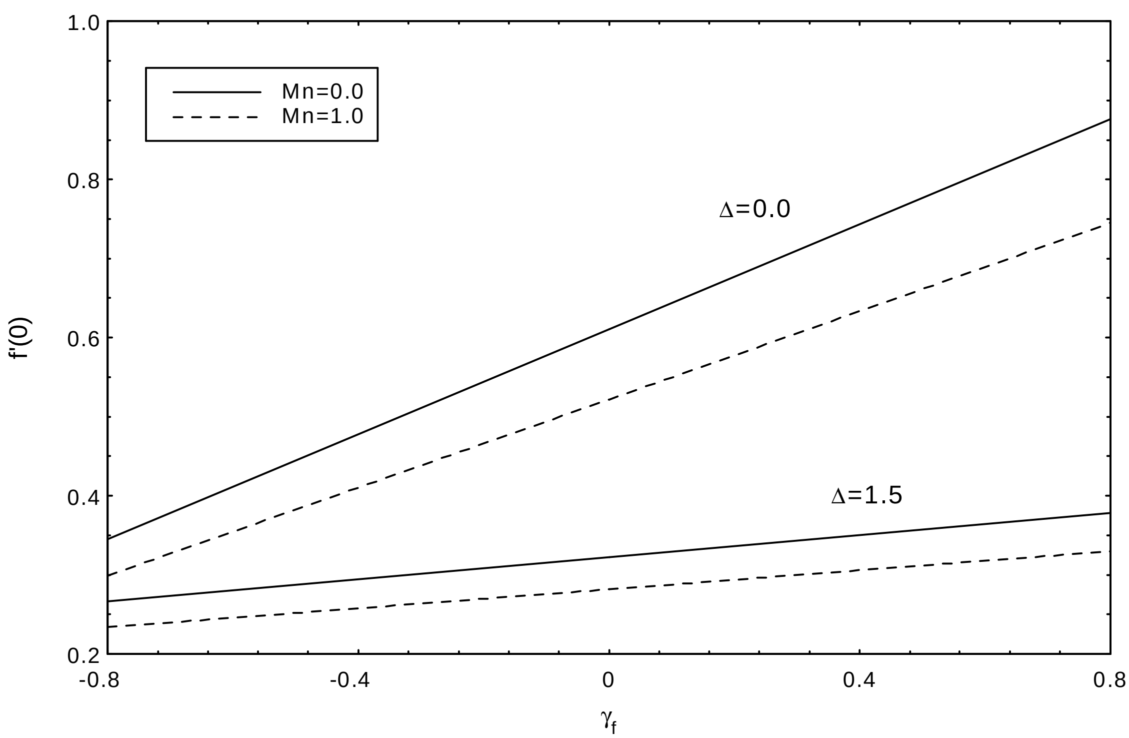


Fig.15. The effect of R and Δ on $f'(0, \gamma_f)$ for the plane plume case as a function of γ_f.

4. Concluding remarks

For several liquids the variation of viscosity with temperature is much greater than the variation of other fluid properties. In this work, we studied the effects of a transverse flow with variable viscosity in micropolar fluids, when the fluid viscosity is assumed to vary as an inverse linear function of temperature. The truncated expansion for μ amounts to a linear variation of viscosity with temperature and is a good approximation for the small values of γ_f required for the present perturbation analysis. Numerical solutions to the governing equations for momentum, angular momentum ,energy and concentration are given. Tabulated values and graphical representations for the velocity, angular velocity, thermal function and Nusselt number are presented for various dimensionless material properties of micropolar fluids. Equations for the surface shear stress, momentum flux and surface heat are also given.

Nomenclature

b, c, d – defined in Eqs (2.8) and (2.9)
B_0 – magnetic field intensity
C_p – specific heat
f – dimensionless velocity
g – dimensionless microrotation
g^* – acceleration due to gravity
Gr – local Grashof number, $\mathrm{Gr} = g^* \beta \rho^2 x^3 (T_0 - T_\infty)/\mu_f^2$
Gr_x^* – local flux Grashof number, $\mathrm{Gr}_x^* = g^* \beta q'' x^4 / k \upsilon^2$
h – local heat transfer coefficient
j – microinertia per unit mass
k – thermal conductivity of fluid
M – momentum flux in x direction
m – mass flow rate per unit width of surface
Mn – magnetic parameter
m_w – local couple stress
N – angular velocity
N – defined in Eq.(2.8)
N' – heat transfer parameter, $\sqrt{2}\,\mathrm{Nu}_x / (\mathrm{Gr}'_x)^{1/4}$
Nu_x – local Nusselt number, $\mathrm{Nu}_x = hx/k$
Q – total heat convected downstream
q'' – surface heat flux
s – defined in Eq.(2.8)
T – temperature
T_e – reference temperature
T_f – film temperature
u – velocity component in x-direction
v – velocity component in y-direction
x – horizontal co-ordinate
y – vertical co-ordinate
β – thermal expansion coefficient
χ – vortex viscosity
γ – spin-gradient viscosity

γ_f – viscosity parameter
η – dimensionless co-ordinate
σ – electrical conductivity
σ_f – Prandtl number
λ – dimensionless material parameter
μ – dynamic viscosity
υ – kinematic viscosity
θ – dimensionless temperature
ψ – stream function
ρ – density of the fluid

Subscripts

w – at the wall
∞ – condition far away from the surface

Superscript

$'$ – differention with respect to η

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