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INTERACTION DUE TO MECHANICAL SOURCES IN MICROPOLAR CUBIC CRYSTAL

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The response of a micropolar cubic crystal due to various sources has been investigated. The eigen-value approach after applying Laplace and Fourier transforms has been employed to solve the problem. The integral transforms have been inverted by using a numerical technique to obtain the displacement, microrotation and stress components in the physical domain. The results of normal displacement, normal force stress and tangential couple stress have been compared for a micropolar cubic crystal and micropolar isotropic solid and illustrated graphically.

Key words: micropolar cubic crystal, eigen-value, Laplace and Fourier transforms, microrotation.

1. INTRODUCTION

The classical theory of elasticity does not explain certain discrepancies that occur in the case of problems involving elastic vibrations of high frequency and short wave length, that is, vibrations due to the generation of ultrasonic waves. The reason lies in the microstructure of the material which exerts a special influence at high frequencies and short wave lengths.

An attempt was made to eliminate these discrepancies by suggesting that the transmission of interaction between two particles of a body through an elementary area lying within the material was affected not solely by the action of a force vector but also by a moment (couple) vector. This led to the existence of couple stress in elasticity. Polycrystalline materials, materials with fibrous or coarse grain structure come in this category. The analysis of such materials requires incorporating the theories of oriented media. For this reason, micropolar theories were developed by Eringen (1966a, b) for elastic solids and fluids.

Following various methods, the elastic fields of various loadings, inclusion and inhomogeneity problems, and interaction energy of point defects and dislocation arrangement have been discussed extensively. Generally, all materials have elastic anisotropic properties and this implies that the mechanical behavior of an engineering material is characterized by the direction dependence. However, the three dimensional study for an anisotropic material is much more complicated to obtain than the isotropic one, due to the large number of elastic constants involved in the calculation. In recent years, the elastodynamic response of anisotropic continuum has received the attention of several researchers. In particular, transversly isotropic and orthotropic materials, which may not be distinguished from each other in plane strain and plane stress, have been more regularly studied. The orthotropic material has the symmetry of its elastic properties with respect to two orthogonal planes. The tetragonal material is a particular type of orthotropic material that has the same properties along two axes and different properties along the third axis and a cubic material is a

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tetragonal material that is invariant to an additional change of coordinates. Kumar and Choudhary(2002a, b, 2003) discussed different types of problems in a micropolar orthotropic continua.

A wide class of crystals such as W, Si, Cu, Ni, Fe, Au, Al etc., which are frequently used substances, belong to cubic materials. Cubic materials have nine planes of symmetry whose normals are on the three coordinate axes and on the coordinate planes making an angle $\pi/4$ with the coordinate axes. With the chosen coordinate system along the crystalline directions, the mechanical behavior of a cubic crystal can be characterized by four independent elastic constants A_1, A_2, A_3 and A_4.

To understand the crystal elasticity of a cubic material, Chung and Buessem (1967) presented a convenient method to describe the degree of the elasticity anisotropy in a given cubic crystal. Later, Lie and Koehler (1968) used a Fourier expansion scheme to calculate the stress fields caused by a unit force in a cubic crystal. Steeds (1973) gave a complete discussion on the displacements, stresses and energy factors of the dislocations for two-dimensional anisotropic materials. Boulanger and Hayes (2000) investigated inhomogeneous plane waves in cubic elastic materials. Bertram ***et al***. (2000) discussed generation of discrete isotropic orientation distributions for linear elastic cubic crystals. Kobayashi and Giga (2001) investigated anisotropy and curvature effects for growing crystals. Domanski and Jablonski (2001) studied resonances of nonlinear elastic waves in cubic crystal. Destrade (2001) considered the explicit secular equation for surface acoustic waves in monoclinic elastic crystals. Zhou and Ogawa (2002) investigated elastic solutions for a solid rotating disk with cubic anisotropy. Minagawa *et al*. (1981) discussed the propagation of plane harmonic waves in a cubic micropolar medium. Recently Kumar and Rani (2003) studied time harmonic sources in a thermally conducting cubic crystal. However, no attempt has been made to study source problems in micropolar cubic crystals.

The present investigation is to determine the components of displacement, microrotation and stresses in a micropolar cubic crystal due to various types of sources. The solutions are obtained by using the eigen-value approach after employing the integral transformation technique. The integral transforms are inverted using a numerical method.

2. Problem formulation

We consider a homogeneous micropolar cubic crystal of infinite extent with a Cartesian coordinate system (x, y, z). To analyze the displacements, microrotation and stresses at the interior of the medium due to various sources, the continuum is divided into two half-spaces defined by

i. half space I $|x| < \infty$, $-\infty < y \le 0$, $|z| < \infty$,

ii. half space II $|x| < \infty$, $0 \le y < \infty$, $|z| < \infty$.

If we restrict our analysis to the plane strain parallel to the $xy-$plane with the displacement vector $\boldsymbol{u} = (u_1, u_2, 0)$ and microrotation vector $\mathrm{f} = (0,\ 0,\ \phi_3)$, then the field equations and constitutive relations for such a medium in the absence of body forces and body couples given by Minagawa *et al.* (1981) can be recalled as

$$A_1 \frac{\partial^2 u_1}{\partial x^2} + A_3 \frac{\partial^2 u_1}{\partial y^2} + (A_2 + A_4)\frac{\partial^2 u_2}{\partial x \partial y} + (A_3 - A_4)\frac{\partial \phi_3}{\partial y} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{2.1}$$

$$A_3 \frac{\partial^2 u_2}{\partial x^2} + A_1 \frac{\partial^2 u_2}{\partial y^2} + (A_2 + A_4)\frac{\partial^2 u_1}{\partial x \partial y} - (A_3 - A_4)\frac{\partial \phi_3}{\partial x} = \rho \frac{\partial^2 u_2}{\partial t^2}, \tag{2.2}$$

$$B_3\nabla^2\phi_3+(A_3-A_4)\left(\frac{\partial u_2}{\partial x}-\frac{\partial u_1}{\partial y}\right)-2(A_3-A_4)\phi_3=\rho\, j\,\frac{\partial^2\phi_3}{\partial t^2}, \tag{2.3}$$

$$t_{22}=A_2\frac{\partial u_1}{\partial x}+A_1\frac{\partial u_2}{\partial y}, \tag{2.4}$$

$$t_{21}=A_4\left(\frac{\partial u_2}{\partial x}-\phi_3\right)+A_3\left(\frac{\partial u_1}{\partial y}+\phi_3\right), \tag{2.5}$$

$$m_{23}=B_3\frac{\partial\phi_3}{\partial y}. \tag{2.6}$$

Let us introduce the dimensionless variables defined by the expressions

$$\begin{gathered}x'=\frac{\omega^*}{c_1}x,\qquad y'=\frac{\omega^*}{c_1}y,\qquad u_1'=\frac{\omega^*}{c_1}u_1,\qquad u_2'=\frac{\omega^*}{c_1}u_2,\qquad \phi_3'=\frac{A_1}{A_4}\phi_3,\\ \{t_{22}',t_{21}'\}=\frac{\{t_{22},t_{21}\}}{A_1},\qquad m_{23}'=\frac{c_1}{B_3\omega^*}m_{23},\qquad t'=\omega^* t,\qquad a'=\frac{\omega^*}{c_1}a\end{gathered} \tag{2.7}$$

where

$$\omega^{*2}=\frac{A_4-A_3}{\rho\, j},\qquad c_1^{\,2}=\frac{A_1}{\rho}. \tag{2.8}$$

Using formulae (2.7), the system of Eqs (2.1)-(2.3) reduces to (dropping the primes)

$$A_1\frac{\partial^2 u_1}{\partial x^2}+A_3\frac{\partial^2 u_1}{\partial y^2}+(A_2+A_4)\frac{\partial^2 u_2}{\partial x\partial y}+\frac{A_4(A_3-A_4)}{A_1}\frac{\partial\phi_3}{\partial y}=\rho c_1^{\,2}\frac{\partial^2 u_1}{\partial t^2}, \tag{2.9}$$

$$A_3\frac{\partial^2 u_2}{\partial x^2}+A_1\frac{\partial^2 u_2}{\partial y^2}+(A_2+A_4)\frac{\partial^2 u_1}{\partial x\partial y}-\frac{A_4(A_3-A_4)}{A_1}\frac{\partial\phi_3}{\partial x}=\rho c_1^{\,2}\frac{\partial^2 u_2}{\partial t^2}, \tag{2.10}$$

$$B_3\frac{A_4\omega^{*2}}{A_1c_1^{\,2}}\nabla^2\phi_3+(A_3-A_4)\left(\frac{\partial u_2}{\partial x}-\frac{\partial u_1}{\partial y}\right)-2\frac{A_4(A_3-A_4)}{A_1}\phi_3=\rho j\omega^{*2}\frac{A_4}{A_1}\frac{\partial^2\phi_3}{\partial t^2}. \tag{2.11}$$

The initial conditions are given by

$$\begin{gathered}u_n(x,y,0)=\dot{u}_n(x,y,0)=0;\qquad n=1,2,\\ \phi_3(x,y,0)=\dot{\phi}_3(x,y,0)=0.\end{gathered} \tag{2.12}$$

Applying the Laplace transform with respect to time 't' defined by

$$\left\{\bar{u}_n(x, y, p), \bar{\phi}_3(x, y, p)\right\} = \int_0^\infty \left\{u_n(x, y, t), \phi_3(x, y, t)\right\} e^{-pt}\, dt\ , \qquad n = 1, 2\,, \tag{2.13}$$

and then the Fourier transform with respect to 'x' defined by

$$\left\{\tilde{u}_n(\xi, y, p), \tilde{\phi}_3(\xi, y, p)\right\} = \int_{-\infty}^{\infty} \left\{\bar{u}_n(x, y, p), \bar{\phi}_3(x, y, p)\right\} e^{i\xi x} dx\,, \qquad n = 1, 2 \tag{2.14}$$

on Eqs (2.9)-(2.11) and with the help of initial conditions (2.12), we obtain

$$D^2\, \tilde{u}_1 = b_{11}\, \tilde{u}_1 + a_{12} D \tilde{u}_2 + a_{13}\, D\tilde{\phi}_3\,, \tag{2.15}$$

$$D^2\, \tilde{u}_2 = b_{22}\, \tilde{u}_2 + a_{21} D \tilde{u}_1 + b_{23}\, \tilde{\phi}_3\,, \tag{2.16}$$

$$D^2 \tilde{\phi}_3 = b_{33}\, \tilde{\phi}_3 + a_{31} D \tilde{u}_1 + b_{32}\, \tilde{u}_2 \tag{2.17}$$

where

$$\begin{gathered}
b_{11} = \frac{\rho c_1^2 p^2 + \xi^2 A_1}{A_3}, \qquad b_{22} = \frac{\rho c_1^2 p^2 + \xi^2 A_3}{A_1}, \qquad b_{23} = -\frac{i\xi A_4 (A_3 - A_4)}{{A_1}^2}, \\
b_{32} = \frac{i\xi c_1^2 A_1 (A_3 - A_4)}{\omega^{*2} A_4 B_3}, \qquad b_{33} = \frac{1}{B_3}\left[\xi^2 B_3 + \rho\, j {c_1}^2 p^2 + 2(A_3 - A_4)\frac{c_1^2}{\omega^{*2}}\right], \\
a_{13} = -\frac{A_4 (A_3 - A_4)}{A_1 A_3}, \qquad a_{21} = \frac{i\xi (A_2 + A_4)}{A_1}, \qquad a_{31} = \frac{A_1 (A_3 - A_4) c_1^2}{A_4 B_3 \omega^{*2}}, \\
a_{12} = \frac{i\xi (A_2 + A_4)}{A_3}, \qquad D = \frac{d}{dy}.
\end{gathered} \tag{2.18}$$

Equations (2.15)-(2.17) may be written as

$$D\, W(\xi, y, p) = A(\xi, p)\ W(\xi, y, p) \tag{2.19}$$

where

$$\begin{gathered}
W = \begin{pmatrix} V \\ DV \end{pmatrix}, \qquad A = \begin{pmatrix} O & I \\ A_1^* & A_2^* \end{pmatrix}, \qquad V = \begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{\phi}_3 \end{pmatrix}, \\
A_1^* = \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \end{pmatrix}, \qquad A_2^* = \begin{pmatrix} 0 & a_{12} & a_{13} \\ a_{21} & 0 & 0 \\ a_{31} & 0 & 0 \end{pmatrix}.
\end{gathered} \tag{2.20}$$

O and I are respectively zero and the identity matrix of order 3.

To solve Eq.(2.19), we assume

$$W(\xi,\ y,\ p)=X(\xi,\ p)\,e^{qy}\,, \tag{2.21}$$

which leads to the eigen value problem. The characteristic equation corresponding to matrix A is given by

$$\det\left[A-qI\right]=0\,, \tag{2.22}$$

which on expansion provides us with

$$q^6+\lambda_1 q^4+\lambda_2 q^2+\lambda_3=0 \tag{2.23}$$

where

$$\lambda_1=-\left(a_{12}a_{21}+a_{13}a_{31}+b_{11}+b_{22}+b_{33}\right),$$

$$\lambda_2=a_{12}\left(a_{21}b_{33}-b_{23}a_{31}\right)+a_{13}\left(b_{22}a_{31}-a_{21}b_{32}\right)+b_{22}b_{33}-b_{23}b_{32}+b_{11}\left(b_{22}+b_{33}\right), \tag{2.24}$$

$$\lambda_3=b_{11}\left(b_{23}b_{32}-b_{22}b_{33}\right).$$

The eigen values of matrix A are the characteristic roots of Eq.(2.23). The vectors $X(\xi,\ p)$ corresponding to the eigen values q_s can be determined by solving the homogeneous equation

$$\left[A-qI\right]X(\xi,\ p)=0\,. \tag{2.25}$$

The set of eigen vectors $X_s(\xi,\ p),\ s=1,2.........6$ may be obtained as

$$X_s(\xi,p)=\begin{pmatrix} X_{g1}(\xi,\ p) \\ \\ X_{g2}(\xi,\ p)\end{pmatrix} \tag{2.26}$$

where

$$X_{g1}(\xi,p)=\begin{pmatrix} q_g \\ a_g \\ b_g\end{pmatrix},\qquad X_{g2}(\xi,p)=\begin{pmatrix} q_g^2 \\ a_g q_g \\ b_g q_g\end{pmatrix},\qquad q=q_g\,;\qquad g=1,\,2,\,3\,, \tag{2.27}$$

$$X_{R1}(\xi,\ p)=\begin{pmatrix} -q_R \\ a_R \\ b_R\end{pmatrix},\qquad X_{R2}(\xi,\ p)=\begin{pmatrix} {q_R}^2 \\ -a_R q_R \\ -b_R q_R\end{pmatrix},\qquad R=g+3\,,\qquad q=-q_g\,,\qquad g=1,2,3\,, \tag{2.28}$$

and

$$a_g = \frac{b_{11}b_{23} - q_g^2\left(b_{23} + a_{21}a_{13}\right)}{\nabla_g},$$

$$b_g = \frac{q_g^2 a_{31} + a_g b_{32}}{q_g^2 - b_{33}}, \tag{2.29}$$

$$\nabla_g = q_g^2 a_{13} + a_{12}b_{23} - b_{22}a_{13}.$$

The solution of Eq.(2.19) is given by

$$W(\xi,\ y,\ p) = \sum_{s=1}^{3}\left[\ D_s X_s(\xi, p)\exp(q_s y) + D_{s+3}X_{s+3}(\xi, p)\exp(-q_s y)\ \right] \tag{2.30}$$

where $D_\Xi\,(\Xi = 1, 2 \text{..........} 6)$ are arbitrary constants.

Equation (2.30) represents the solution to the general problem in case of micropolar cubic crystals and can be applied to a class of problems in the domain of Laplace and Fourier transforms.

3. Applications

We consider an infinite micropolar cubic crystal in which an arbitrary normal/tangential load is acting at the origin of the Cartesian coordinate system. Mathematically, the boundary conditions at the interface of two half-spaces $y = 0$ are given by

$$\begin{gathered} u_1\left(x, 0^+, t\right) - u_1\left(x, 0^-, t\right) = 0, \qquad u_2\left(x, 0^+,\ t\right) - u_2\left(x, 0^-, t\right) = 0, \\ \phi_3\left(x, 0^+, t\right) - \phi_3\left(x, 0^-,\ t\right) = 0, \qquad m_{23}\left(x, 0^+, t\right) - m_{23}\left(x, 0^-, t\right) = 0, \\ t_{22}\left(x, 0^+, t\right) - t_{22}\left(x, 0^-,\ t\right) = -F_2(x,\ t), \qquad t_{21}\left(x, 0^+,\ t\right) - t_{21}\left(x, 0^-,\ t\right) = -F_1(x,\ t), \end{gathered} \tag{3.1}$$

3.1. Case I: continous load in normal direction

For a continous load in normal direction

$$F_1(x, t) = 0, \qquad F_2(x, t) = \psi_2(x)H(t). \tag{3.2}$$

3.2. Case II: continous load in tangential direction

In this case

$$F_1(x, t) = \psi_1(x)H(t), \qquad F_2(x, t) = 0. \tag{3.3}$$

Using formulae (2.7), (3.2) and applying Laplace and Fourier transforms defined by Eqs (2.13) and (2.14) on Eqs (3.1), after suppressing the primes, and with the help of Eq.(2.30), we obtain the transformed components of displacement, microrotation and stresses as

$$\tilde{u}_1 = q_1 D_1 e^{q_1 y} + q_2 D_2 e^{q_2 y} + q_3 D_3 e^{q_3 y} - \left(q_1 D_4 e^{-q_1 y} + q_2 D_5 e^{-q_2 y} + q_3 D_6 e^{-q_3 y} \right), \tag{3.4}$$

$$\tilde{u}_2 = a_1 D_1 e^{q_1 y} + a_2 D_2 e^{q_2 y} + a_3 D_3 e^{q_3 y} + a_1 D_4 e^{-q_1 y} + a_2 D_5 e^{-q_2 y} + a_3 D_6 e^{-q_3 y}, \tag{3.5}$$

$$\tilde{\phi}_3 = b_1 D_1 e^{q_1 y} + b_2 D_2 e^{q_2 y} + b_3 D_3 e^{q_3 y} + b_1 D_4 e^{-q_1 y} + b_2 D_5 e^{-q_2 y} + b_3 D_6 e^{-q_3 y}, \tag{3.6}$$

$$\begin{aligned} \tilde{m}_{23} = \frac{A_4}{A_1} \Big[& b_1 q_1 D_1 e^{q_1 y} + b_2 q_2 D_2 e^{q_2 y} + b_3 q_3 D_3 e^{q_3 y} + \\ & - \left(b_1 q_1 D_4 e^{-q_1 y} + b_2 q_2 D_5 e^{-q_2 y} + b_3 q_3 D_6 e^{-q_3 y} \right) \Big], \end{aligned} \tag{3.7}$$

$$\tilde{t}_{21} = r_1 D_1 e^{q_1 y} + r_2 D_2 e^{q_2 y} + r_3 D_3 e^{q_3 y} - \left(r_1 D_4 e^{-q_1 y} + r_2 D_5 e^{-q_2 y} + r_3 D_6 e^{-q_3 y} \right), \tag{3.8}$$

$$\tilde{t}_{22} = s_1 D_1 e^{q_1 y} + s_2 D_2 e^{q_2 y} + s_3 D_3 e^{q_3 y} + s_1 D_4 e^{-q_1 y} + s_2 D_5 e^{-q_2 y} + s_3 D_6 e^{-q_3 y} \tag{3.9}$$

where

$$\begin{aligned} & r_n = \frac{1}{A_1} \left[-i\xi a_n A_4 + q_n^2 A_3 + \frac{(A_3 - A_4)}{A_1} b_n A_4 \right], \\ & s_n = q_n \left[-i\xi \frac{A_2}{A_1} + a_n \right], \qquad n = 1, 2, 3, \end{aligned} \tag{3.10}$$

and

$$\begin{aligned} & D_1 = D_4 = \frac{4F_0 G_2 \tilde{\psi}_2(\xi)}{p\Delta} q_2 q_3 (b_2 - b_3), \\ & D_2 = D_5 = \frac{4F_0 G_2 \tilde{\psi}_2(\xi)}{p\Delta} q_1 q_3 (b_3 - b_1), \\ & D_3 = D_6 = \frac{4F_0 G_2 \tilde{\psi}_2(\xi)}{p\Delta} q_1 q_2 (b_1 - b_2) \end{aligned} \tag{3.11}$$

where

$$\begin{aligned} & \Delta = 8G_1 G_2, \quad G_1 = s_1 q_2 q_3 (b_3 - b_2) - s_2 q_1 q_3 (b_3 - b_1) + s_3 q_1 q_2 (b_2 - b_1), \\ & G_2 = a_1 (r_2 b_3 - r_3 b_2) - a_2 (r_1 b_3 - r_3 b_1) + a_3 (r_1 b_2 - r_2 b_1). \end{aligned} \tag{3.12}$$

Using formulae (2.7), (3.3) and applying Laplace and Fourier transforms defined by Eqs (2.13) and (2.14) on Eqs (3.1), after suppressing the primes, and with the help of Eq.(2.30), the transformed components of displacement, microrotation and stresses in the case of tangential load are again given by Eqs (3.4)-(3.9) where

$$D_1 = D_4 = \frac{4F_0G_1\tilde{\psi}_1(\xi)}{p\Delta}(a_2b_3 - a_3b_2),$$

$$D_2 = D_5 = \frac{4F_0G_1\tilde{\psi}_1(\xi)}{p\Delta}(a_3b_1 - a_1b_3), \tag{3.13}$$

$$D_3 = D_6 = \frac{4F_0G_1\tilde{\psi}_1(\xi)}{p\Delta}(a_1b_2 - a_2b_1).$$

3.3. Particular cases

3.3.1. Concentrated force

In order to determine displacements, microrotation and stresses due to a concentrated force in normal/tangential direction described as Dirac delta function, $\{\psi_1(x), \psi_2(x)\} = \delta(x)$ must be used. The Fourier transform of $\psi_1(x)$ and $\psi_2(x)$ with respect to the pair (x, ξ) will be

$$\tilde{\psi}_n(\xi) = 1, \qquad (n = 1, 2). \tag{3.14}$$

3.3.2. Uniformly distributed force

The solution due to a uniformly distributed force in normal/tangential direction is obtained by setting

$$\{\psi_1(x), \psi_2(x)\} = \begin{bmatrix} 1 & \text{if} & |x| \le a, \\ \\ 0 & \text{if} & |x| > a, \end{bmatrix}$$

in Eqs (3.1). The Fourier transform with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width 2a applied at the origin of the coordinate system $(x = y = 0)$ in a dimensionless form after suppressing the primes becomes

$$\{\tilde{\psi}_1(\xi), \tilde{\psi}_2(\xi)\} = \left[2\sin\left(\frac{\xi c_1 a}{\omega^*}\right)\Big/\xi\right], \qquad \xi \ne 0. \tag{3.15}$$

3.3.2. Linearly distributed force

The solution due to a linearly distributed force in normal/tangential direction is obtained by substituting

$$\{\psi_1(x), \psi_2(x)\} = \begin{bmatrix} 1 - \frac{|x|}{a} & \text{if} & |x| \le a, \\ \\ 0 & \text{if} & |x| > a, \end{bmatrix}$$

in Eqs (3.1). The Fourier transform of $\psi_n(x)$, $(n=1,2)$ in a dimensionless form after suppressing the primes is

$$\left\{\tilde{\psi}_1(\xi),\tilde{\psi}_2(\xi)\right\}=\frac{2\left[1-\cos\left(\frac{\xi c_1 a}{\omega^*}\right)\right]}{\frac{\xi^2 c_1 a}{\omega^*}}. \tag{3.16}$$

The expressions for the components of displacement, microrotation, force stress and couple stress are obtained as in Eqs (3.4)-(3.9), by replacing $\tilde{\psi}_n(\xi)$, $(n=1,2)$ by 1, $\left[2\sin\left(\frac{\xi c_1 a}{\omega^*}\right)\Big/\xi\right]$ and $\frac{2\left[1-\cos\left(\frac{\xi c_1 a}{\omega^*}\right)\right]}{\frac{\xi^2 c_1 a}{\omega^*}}$ in the case of a concentrated force, uniformly distributed force and linearly distributed force respectively in Eq.(3.11) for load in normal direction and in Eq.(3.13) for load in tangential direction.

3.4. Case IV: moving force

The boundary conditions for a force, moving along the x-axis with uniform velocity U at the interface of two half-spaces $y=0$ are given by

$$\begin{aligned}
&u_1(x,0^+,t)-u_1(x,0^-,t)=0, && u_2(x,0^+,t)-u_2(x,0^-,t)=0,\\
&\phi_3(x,0^+,t)-\phi_3(x,0^-,t)=0, && m_{23}(x,0^+,t)-m_{23}(x,0^-,t)=0,\\
&t_{21}(x,0^+,t)-t_{21}(x,0^-,t)=0, && t_{22}(x,0^+,t)-t_{22}(x,0^-,t)=-F\,H(t)\delta(x-Ut).
\end{aligned} \tag{3.17}$$

Using Eq.(2.7) and then applying Laplace and Fourier transforms from Eqs (2.13) and (2.14) on Eqs (3.17), after suppressing the primes, and with the help of Eq.(2.30), we obtain the transformed components of displacement, microrotation and stresses as given by Eqs (3.4)-(3.9) where

$$\begin{aligned}
&D_1=D_4=\frac{4F_0G_2}{(p-i\xi U)\Delta}q_2q_3(b_2-b_3), \qquad D_2=D_5=\frac{4F_0G_2}{(p-i\xi U)\Delta}q_1q_3(b_3-b_1),\\
&D_3=D_6=\frac{4F_0G_2}{(p-i\xi U)\Delta}q_1q_2(b_1-b_2).
\end{aligned} \tag{3.18}$$

3.5. Case V: moving couple

The boundary conditions for a couple, with its axis parallel to the *z*-axis and moving along the *x*-axis with constant velocity U are given by

$$\begin{aligned}
&u_1(x,0^+,t)-u_1(x,0^-,t)=0, && u_2(x,0^+,t)-u_2(x,0^-,t)=0,\\
&\phi_3(x,0^+,t)-\phi_3(x,0^-,t)=0, && m_{23}(x,0^+,t)-m_{23}(x,0^-,t)=-F\,H(t)\delta(x-Ut),
\end{aligned} \tag{3.19}$$

$$t_{21}\left(x,0^{+},t\right)-t_{21}\left(x,0^{-},t\right)=0\,, \qquad t_{22}\left(x,0^{+},t\right)-t_{22}\left(x,0^{-},t\right)=0\,.$$

Using Eq.(2.7) and then applying Laplace and Fourier transforms from Eqs (2.13) and (2.14) on Eqs (3.19), after suppressing the primes, and with the help of Eq.(2.30), we obtain the transformed components of displacement, microrotation and stresses the same as given by Eqs (3.4)-(3.9) with the changed values of $D_m(m=1,2.........6)$ as

$$D_1=D_4=\frac{4i\xi F_0G_2}{(p-i\xi U)\Delta}(q_3s_2-q_2s_3), \qquad D_2=D_5=\frac{4i\xi F_0G_2}{(p-i\xi U)\Delta}(q_1s_3-q_3s_1),$$
$$D_3=D_6=\frac{4i\xi F_0G_2}{(p-i\xi U)\Delta}(q_2s_1-q_1s_2). \tag{3.20}$$

For all the cases discussed above the transformed displacement, microrotation, force stress and couple stress components for the region $-\infty<y\le 0$, are obtained by inserting $D_4=D_5=D_6=0$ in Eqs (3.4)-(3.9).
Similarly, for the region $0\le y<\infty$, the components are obtained by inserting $D_1=D_2=D_3=0$ in Eqs (3.4)-(3.9).

3.6. Special case

Taking $A_1=\lambda+2\mu+K$, $A_2=\lambda$, $A_3=\mu+K$, $A_4=\mu$, $B_3=\gamma$, in Eqs (3.4)-(3.9) with Eqs (3.11), (3.13), (3.18) and (3.20) we obtain the corresponding expressions in a micropolar isotropic medium for the load in normal direction, load in tangential direction, moving force and moving couple respectively. The results for the concentrated force, uniformly distributed force and linearly distributed force are obtained by using (3.14)-(3.16) after substituting (3.11) and (3.13) for the load in normal direction and load in tangential direction respectively. These results tally with the one obtained when we solve the problem in a micropolar isotropic medium.

4. Inversion of the transform

The transformed displacements and stresses are functions of *y*, the parameters of Laplace and Fourier transforms p and ξ respectively, and hence are of the form $\tilde{f}(\xi,y,p)$. To get the function in the physical domain, first we invert the Fourier transform using

$$\bar{f}(x,y,p)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\xi x}\tilde{f}(\xi,y,p)d\xi$$

or (4.1)

$$\bar{f}(x,y,p)=\frac{1}{\pi}\int_{0}^{\infty}\{\cos(\xi x)f_e-i\sin(\xi x)f_o\}d\xi$$

where f_e and f_o are even and odd parts of the function $\tilde{f}(\xi, y, p)$ respectively. Thus, expressions (3.4)-(3.9) give us the transform $\overline{f}(x, y, p)$ of the function $f(x, y, t)$.

Following Honig and Hirdes (1984) the Laplace transform function $\overline{f}(x, y, p)$ can be inverted to $f(x, y, t)$.

The last step is to evaluate the integral in Eq.(4.1). The method for evaluating this integral (Press *et al*., 1986) involves the use of Rhomberg's integration with adaptive step size. This, also uses the results from successive refinement of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

5. Numerical results and discussions

For numerical computations, we take the following values of relevant parameters for a micropolar cubic crystal as

$$A_1 = 13.97 \times 10^{10}\ dyne/cm^2, \qquad A_3 = 3.2 \times 10^{10}\ dyne/cm^2,$$

$$A_2 = 13.75 \times 10^{10}\ dyne/cm^2, \qquad A_4 = 2.2 \times 10^{10}\ dyne/cm^2, \qquad B_3 = 0.056 \times 10^{10}\ dynes.$$

For the comparison with a micropolar isotropic solid, following Gauthier (1982), we take the following values of relevant parameters for the case of an aluminium epoxy composite as

$$\rho = 2.19\ gm/cm^3, \qquad \lambda = 7.59 \times 10^{10}\ dyne/cm^2, \qquad \mu = 1.89 \times 10^{10}\ dyne/cm^2,$$

$$K = 0.0149 \times 10^{10}\ dyne/cm^2, \qquad \gamma = 0.0268 \times 10^{10}\ dyne, \qquad j = 0.00196\ cm^2.$$

The values of normal force stress t_{22} and tangential couple stress m_{23} for a micropolar cubic crystal (MCC) and micropolar isotropic solid (MIS) have been studied at $t = 0.1$, 0.2 and 0.5 and the variations of these components with distance *x* have been shown by (a) solid line (———) for MCC and dashed line (--------) for MIS at $t = 0.1$, (b) solid line with centered symbol (x—x—x) for MCC and dashed line with centered symbol (x---x---x) for MIS at $t = 0.2$ and (c) solid line with centered symbol (o—o—o) for MCC and dashed line with centered symbol (o---o---o) for MIS at $t = 0.5$. These variations are shown in Figs 1-16. A comparison between a micropolar cubic crystal and micropolar isotropic solid is shown. All the results are for one value of dimensionless width $a=1.0$ and for $U < c_1$. Computations are carried out for $y=1.0$ in the range $0 \le x \le 10.0$.

6. Discussions for various cases

6.1. Case I: load in normal direction

6.1.1. Concentrated force

The values of normal force stress and tangential couple stress for MCC lie in a short range as compared to the values for MIS. Hence the variations of all the quantities for MIS are more oscillatory in nature in comparison to the variations for MCC. Very close to the point of application of the source, the values of tangential couple stress for both MCC and MIS decrease with an increase in time. The variations of the quantities converge to zero with an increase in horizontal distance *x*. These variations of normal force stress and tangential couple stress are shown in Figs 1 and 2 respectively.

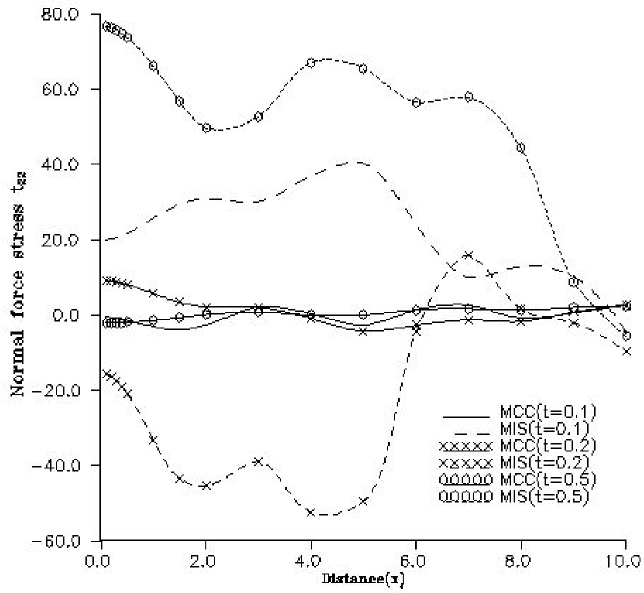

Fig.1. Variation of normal force stress t_{22} with distance *x*. Concentrated force in normal direction.

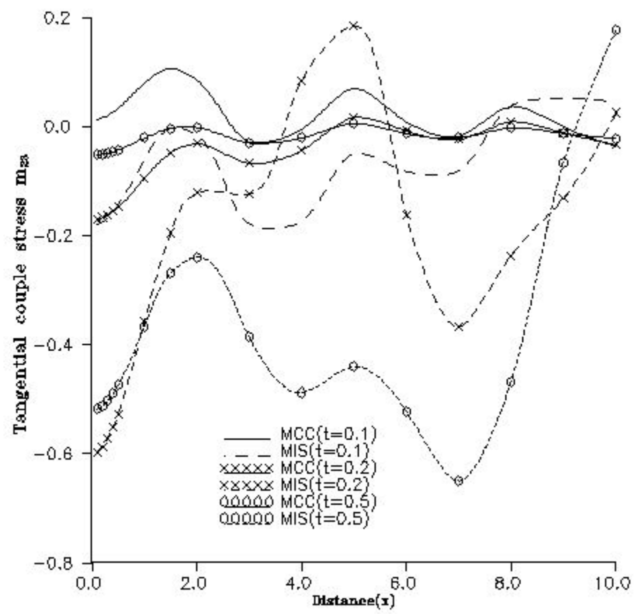

Fig.2. Variation of tangential couple stress m_{23} with distance *x*. Concentrated force in normal direction.

6.1.2. Distributed force (uniformly and linearly)

When a uniformly distributed force is applied, the variations of normal force stress and tangential couple stress are more oscillatory in nature as compared to the variations obtained on application of a linearly distributed force. Also the values of the quantities obtained in the case of a uniformly distributed force are large for MIS and hence to show the comparison of both the solids, the values of normal force stress and tangential couple stress for MIS have been demagnified by *100* (for uniformly distributed force) but the values obtained in the case of a linearly distributed force are of comparable magnitude. It is also observed that the variations of normal force stress and tangential couple stress in the case of a linearly distributed force follow a linear law (to the best approximation). While the variations of normal force stress and tangential couple stress in the case of a uniformly distributed force are shown in Figs 3-4, the variations for a linearly distributed force are depicted in Figs 5-6 respectively.

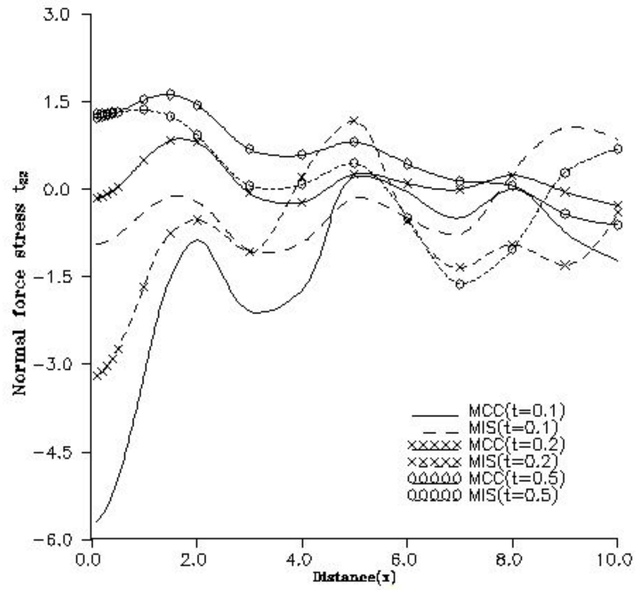

Fig.3. Variation of normal force stress t_{22} with distance *x*. Uniformly distributed force in normal direction.

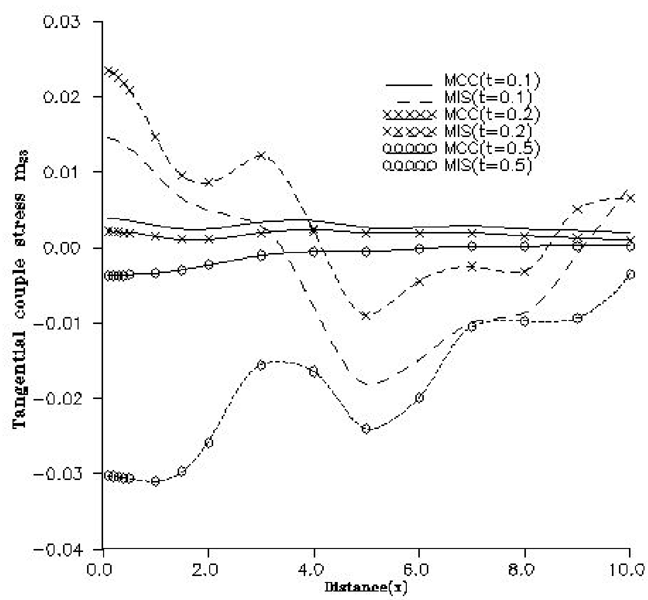

Fig.4. Variation of tangential couple stress m_{23} with distance *x*. Uniformly distributed force in normal direction.

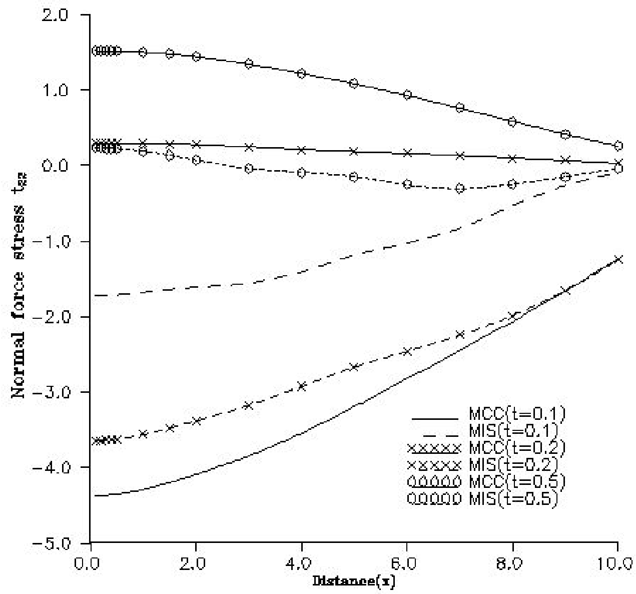

Fig.5. Variation of normal force stress t_{22} with distance *x*. Linearly distributed force in normal direction.

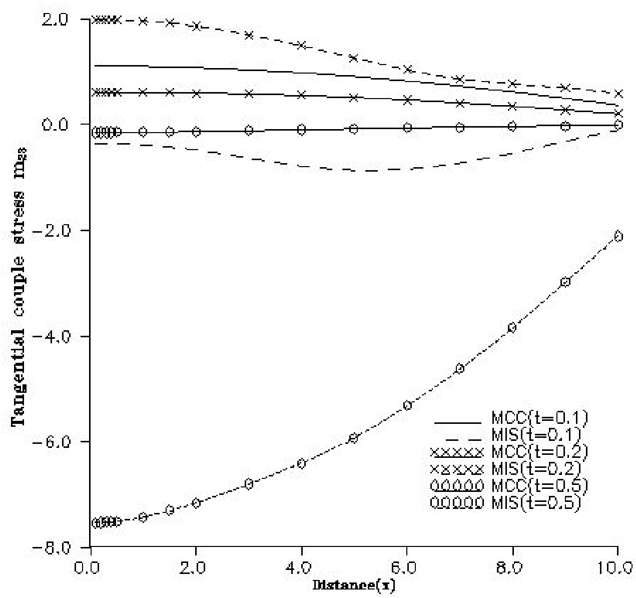

Fig.6. Variation of tangential couple stress m_{23} with distance *x*. Linearly distributed force in normal direction.

6.2. Case II: load in tangential direction

6.2.1. Concentrated force

The variations of all the quantities being oscillatory are quite smooth in nature. It is observed that the values of normal force stress and tangential couple stress, very close to the point of application of the source, decrease with an increase in time. The values of tangential couple stress for MCC are less in magnitude as compared to the values for MIS and hence to compare the variations of both solids, the values of tangential couple stress for MCC have been magnified by *10*. These variations of normal force stress and tangential couple stress are shown in Figs 7-8 respectively.

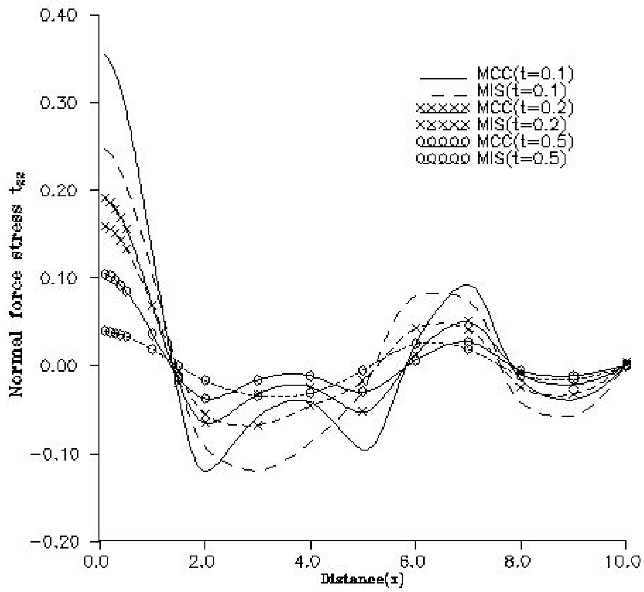

Fig.7. Variation of normal force stress t_{22} with distance *x*. Concentrated force in tangential direction.

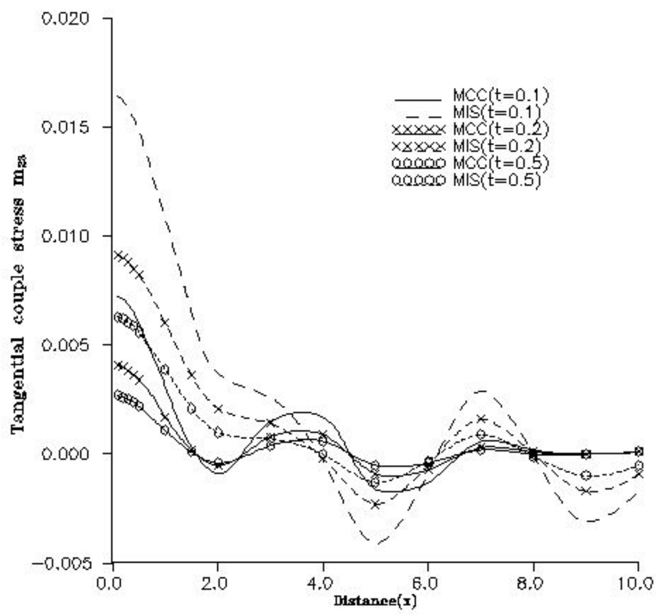

Fig.8. Variation of tangential couple stress m_{23} with distance *x*. Concentrated force in tangential direction.

6.2.2. Distributed force (uniformly and linearly)

The variations of normal force stress and tangential couple stress being oscillatory are similar in nature to the difference in magnitudes in the case of a uniformly distributed force. The values of both the quantities are small for MCC in comparison to the values for MIS. The variation of normal force stress is shown in Fig.9, whereas the variation of tangential couple stress is shown in Fig.10. When a linearly distributed force is applied on the surface of a solid the values of normal force stress for both MCC and MIS are of comparable magnitude, whereas the values of tangential couple stress for MIS lie in a short range as compared to the values for MCC. These variations of normal force stress and tangential couple stress are shown in Figs 11 and 12 respectively.

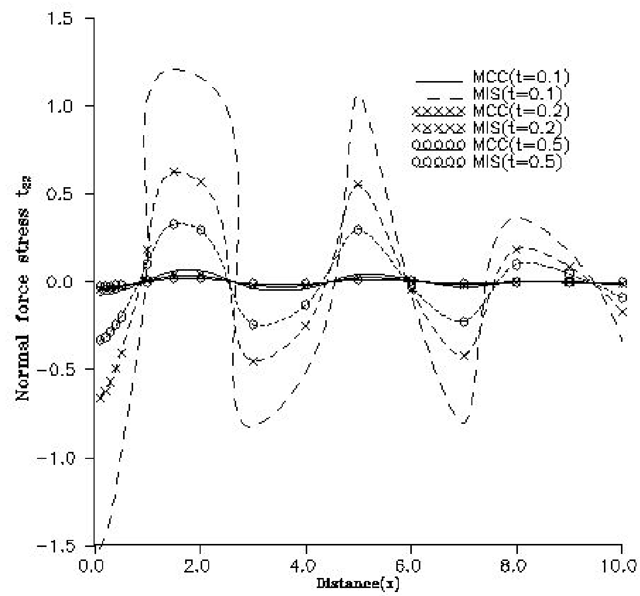

Fig.9. Variation of normal force stress t_{22} with distance *x*. Uniformly distributed force in tangential direction.

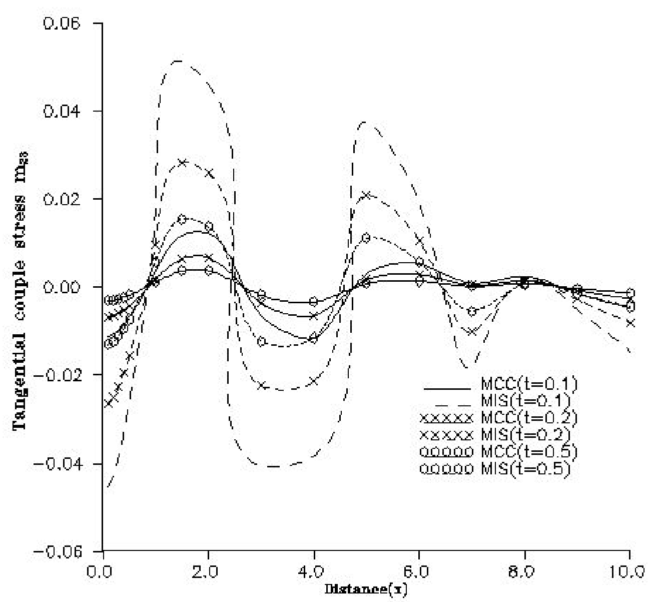

Fig.10. Variation of tangential couple stress m_{23} with distance *x*. Uniformly distributed force in tangential direction.

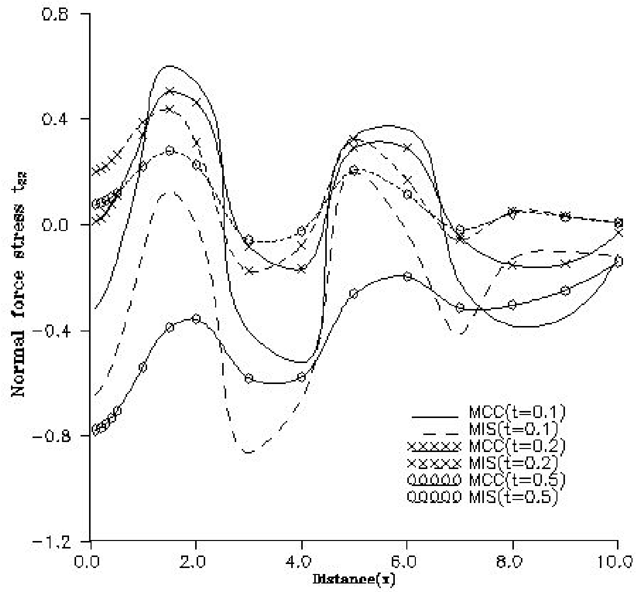

Fig.11. Variation of normal force stress t_{22} with distance x. Linearly distributed force in tangential direction.

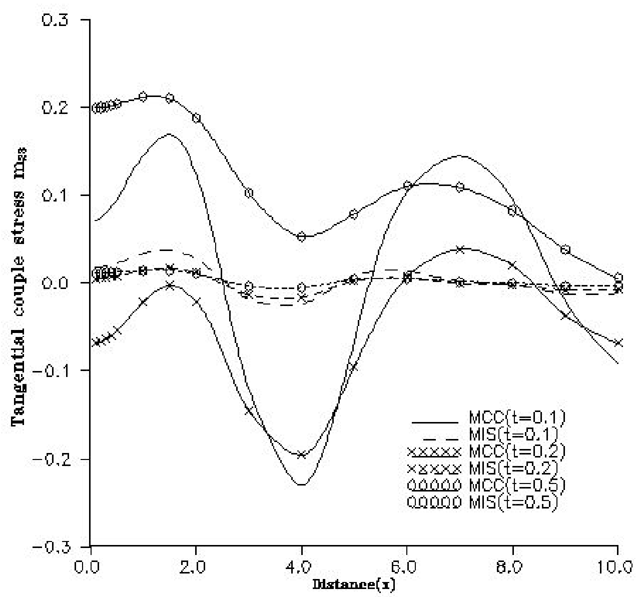

Fig.12. Variation of tangential couple stress m_{23} with distance x. Linearly distributed force in tangential direction.

6.3. Case III: moving force

Similar to the cases discussed above, the values of all the quantities obtained for MCC are smaller as compared to the values obtained for MIS. When the force moves along the interface, the body is deformed to a more extent. This fact is determined by the values of quantities obtained in this case which are much larger than the values obtained in the previous cases. Close to the point of application of the source, the values of tangential couple stress for both MCC and MIS decrease with an increase in time. The values of normal force stress and tangential couple stress for MCC have been magnified by *10* and *100* respectively. Figures 13-14 show the variations of these quantities respectively, in the case of a moving force.

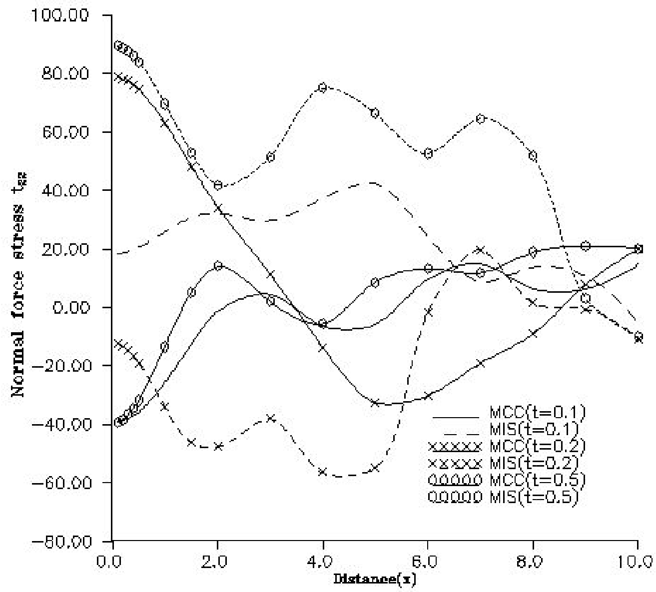

Fig.13. Variation of normal force stress t_{22} with distance x for moving force.

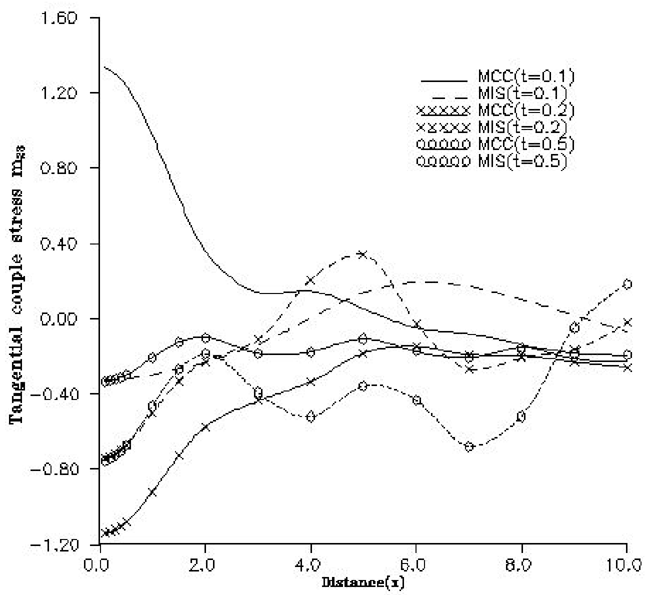

Fig.14. Variation of tangential couple stress m_{23} with distance x for moving force.

6.4. Case IV: moving couple

Since the force acts in two different directions, so the variations obtained in this case are more oscillatory than the variations obtained in previous cases. The oscillations of the quantities increase with an increase in time. While the variations of normal force stress and tangential couple stress (separately) for MCC are quite close to each other for different values of time, the variation for MIS are of comparable magnitude. These variations of normal force stress and tangential couple stress are shown in Figs 15-16 respectively.

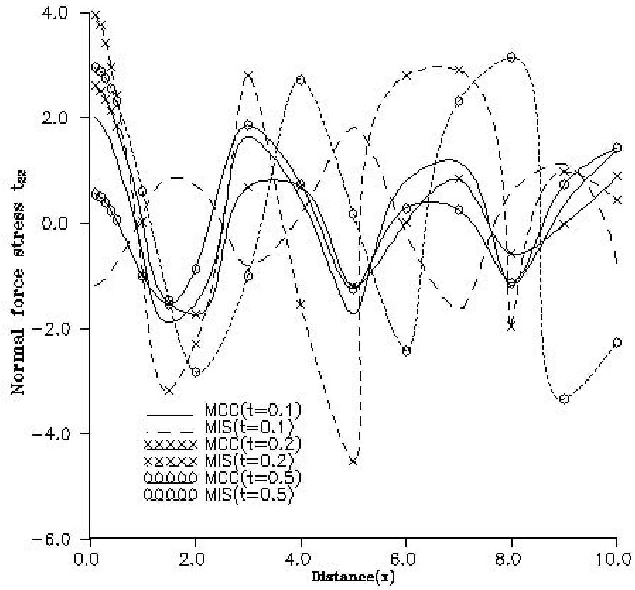

Fig.15. Variation of normal force stress t_{22} with distance x for moving couple.

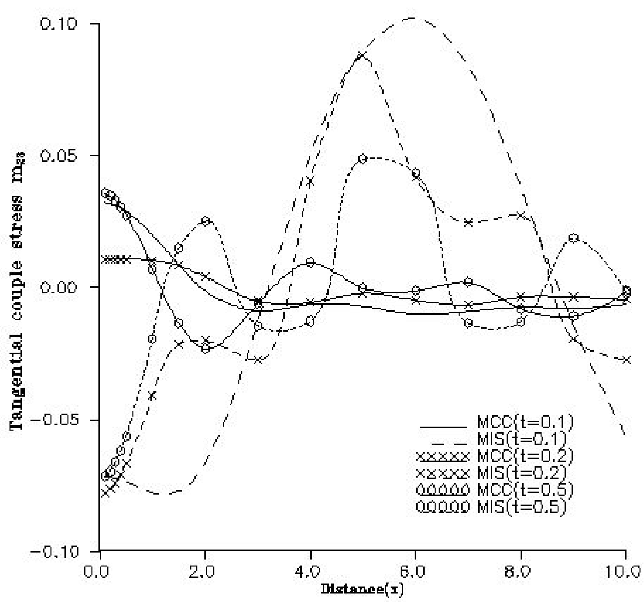

Fig.16. Variation of tangential couple stress m_{23} with distance x for moving force.

7. Conclusion

The properties of a body depend largely on the direction of symmetry. It is observed that the values of all the quantities are less for MCC as compared to the values for MIS. Also the values of tangential couple stress for both MCC and MIS, near the point of application of the source, decrease with an increase in time. For a linearly distributed force the variations of normal force stress and tangential couple stress for both MCC and MIS may be best approximated to a linear law. Also the body is deformed to a greater extent on the application of a moving force/moving couple as compared to other forces applied on the interface.

Nomenclature

A_1, A_2, A_3, A_4, B_3	– characteristic constants of material
$H(t)$	– Heaviside's unit step function
j	– microinertia
m_{23}	– tangential couple stress
$t_{22},\ t_{21}$	– components of force stress
U	– magnitude of moving load velocity
$\boldsymbol{u} = (u_1, u_2, 0)$	– displacement vector
$\delta(\ )$	– Dirac delta function
ρ	– density of solid
$\mathrm{f} = (0, 0, \phi_3)$	– microrotation vector
$\psi_1(x)$	– horizontal load distribution function along x-axis
$\psi_2(x)$	– vertical load distribution function along x-axis
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$	– Laplacian operator

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