UNSTEADY MHD FLOW IN THE PRESENCE OF RADIATION

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A magnetohydrodynamic unsteady free convection flow in the presence of radiation is studied. The fluid is considered to be a gray, absorbing-emitting radiating but non-scattering medium. The differential equations governing the problem are solved exactly. The effects of radiation and suction parameters on the temperature and velocity fields are given.

1. Introduction

A free convection flow past different types of bodies is studied because of its wide application in geophysical and cosmic sciences, industrial areas and aerodynamics. A free convection flow past a vertical plate at normal temperature has been studied extensively under different physical conditions by many authors and many of these have been referred to in Gebhart *et al*. (1988). In the case of high temperatures, radiation effects are quite significant. Studies of the interaction of thermal radiation and free convection were made by Ali *et al*. (1984), Hossain *et al*. (1998), Hossain *et al*. (1999) and Ghaly (2002) in the case of steady flow.

The present analysis deals with the magnetohydrodynamic unsteady free convection flow over an infinite vertical plate in the presence of radiation. The fluid is considered to be a gray, absorbing-emitting radiating but non-scattering medium.

2. Analysis

Consider the unsteady two-dimensional free convection flow of an electrically conducting, viscous and incompressible fluid bounded by an infinite vertical porous plate. A magnetic field of constant density is applied perpendicular to the plate. The fluid is a gray, emitting and absorbing radiating, but non-scattering medium, and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. All the fluid properties are assumed constant except that the influence of the density variation with temperature is considered only in the body force term. The *x*-axis is taken along the plate in the upward direction and the *y*-axis normal to the plate. The radiative heat flux in the *x*-direction is assumed negligible in comparison with that in the *y*-direction.

The governing equations for the two-dimensional MHD unsteady free convection flow for an incompressible fluid, in the presence of radiation, are written within the boundary layer as follows:
continuity equation

$$\frac{\partial v}{\partial y} = 0\,, \tag{2.1}$$

momentum equation

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$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-g-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\sigma B_0^2}{\rho}u\ , \tag{2.2}$$

energy equation

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{2.3}$$

where u and v are the components of the velocity parallel and perpendicular to the plate respectively, t is the time, ρ is the density of the fluid, ν is the kinematic viscosity, p is the pressure, g is the acceleration due to gravity, σ is the electrical conductivity, B_0 is the magnetic induction, T is the temperature, c_p is the specific heat at constant pressure, k is the thermal conductivity and q_r is the radiative heat flux.

The radiative heat flux term, by using the Rosseland approximation, is given by

$$q_r=-\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial y} \tag{2.4}$$

where σ^* is the Stefan-Boltzmann constant and κ^* is the mean absorption coefficient.

When the temperature of the fluid and the temperature away from the plate have a difference which is proportional to t^m, the boundary conditions are

$$\left.\begin{array}{ll} u(0,t)=0, & u(\infty,t)\to 0 \\ T(0,t)=T_\infty+Lt^m, & T(\infty,t)\to T_\infty \end{array}\right\} \tag{2.5}$$

where m, L are constants and T_∞ is the constant temperature away from the plate.

For variable suction of the plate proportional to $t^{-1/2}$, Eq.(2.1) yields

$$\upsilon=-a\left(\frac{\nu}{t}\right)^{1/2} \tag{2.6}$$

where a is the suction parameter.

Away from the plate Eq.(2.2) becomes

$$\frac{\partial p}{\partial x}=-\rho_\infty g \tag{2.7}$$

where ρ_∞ is the density away from the plate.

The state equation is

$$\rho_\infty-\rho=\beta\rho\left(T-T_\infty\right) \tag{2.8}$$

where β is the volumetric coefficient of thermal expansion.

On eliminating $\frac{\partial p}{\partial x}$ between Eqs (2.2) and (2.7) and taking into account Eqs (2.6) and (2.8), Eq.(2.2) is reduced to

$$\frac{\partial u}{\partial t}-a\left(\frac{\nu}{t}\right)^{1/2}\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\theta-\frac{\sigma B_0^2}{\rho}u \tag{2.9}$$

where $\theta = T - T_\infty$.

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 . \tag{2.10}$$

By using Eqs (2.4), (2.6) and (2.10), Eq.(2.3) becomes

$$\frac{\partial \theta}{\partial t}-a\left(\frac{\nu}{t}\right)^{1/2}\frac{\partial \theta}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 \theta}{\partial y^2}+\frac{16\sigma T_\infty^3}{3\rho c_p \kappa^*}\frac{\partial^2 \theta}{\partial y^2} . \tag{2.11}$$

The boundary conditions (2.5) now become

$$\left.\begin{array}{ll} u(0,t)=0, & u(\infty,t)\to 0 \\ \theta(0,t)=Lt^m, & \theta(\infty,t)\to T_\infty \end{array}\right\} . \tag{2.12}$$

We define the velocity and temperature as

$$u(\eta,t)=L\beta g t^{m+1}\left[u_0(\eta)+Mu_1(\eta)+M^2u_2(\eta)+\ldots\right], \tag{2.13}$$

$$\theta = Lt^m f(\eta) \tag{2.14}$$

where $\eta=\frac{1}{2}y(\nu t)^{-1/2}$ and $M=\frac{\sigma B_0^2 t}{\rho}$ (magnetic parameter).

For small values of M, the substitution of the expressions (2.13) and (2.14) into Eqs (2.9) and (2.11) and the comparison of the coefficients of like powers of M give the following differential equations

$$(3N+4)f''+6N\mathrm{P}(\eta+a)f'-12N\mathrm{P}m\,f=0 , \tag{2.15}$$

$$u_0''+2(\eta+a)u_0'-4(m+1)u_0=-4f , \tag{2.16}$$

$$u_1''+2(\eta+a)u_1'-4(m+2)u_1=4u_0 , \tag{2.17}$$

$$u_2''+2(\eta+a)u_2'-4(m+3)u_2=4u_2 \tag{2.18}$$

where $N=\dfrac{\kappa^{*}k}{4\sigma^{*}T_{\infty}^{3}}$ is the radiation parameter, $\mathrm{P}=\dfrac{\rho\nu c_{p}}{k}$ is the Prandtl number and a dash denotes differentiation with respect to η.

The corresponding boundary conditions (2.12) become

$$f(0)=1,\qquad f(\infty)\to 0, \tag{2.19}$$

$$u_{r}(0,t)=0,\qquad u_{r}(\infty,t)\to 0,\qquad r=0,1,2,\ldots. \tag{2.20}$$

The solution of Eq.(2.15) satisfied by the boundary conditions (2.19) is

$$f(\eta)=\frac{Hh_{2m}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m}\left(\left(\sqrt{2C}\right)a\right)} \tag{2.21}$$

where $C=\dfrac{3N\mathrm{P}}{3N+4}$, $\xi=\eta+a$ and $Hh(\ )$ is defined by Jeffreys and Jeffreys (1972).

The solutions of Eqs (2.16)-(2.18) satisfied by the boundary conditions (2.20) are for $C\neq 1$

$$u_{0}(\eta)=\frac{2}{(C-1)}\frac{Hh_{2m+2}\left(\left(\sqrt{2C}\right)a\right)}{Hh_{2m}\left(\left(\sqrt{2C}\right)a\right)}\left[\frac{Hh_{2m+2}\left(\left(\sqrt{2}\right)\xi\right)}{Hh_{2m+2}\left(\left(\sqrt{2}\right)a\right)}-\frac{Hh_{2m+2}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m+2}\left(\left(\sqrt{2C}\right)a\right)}\right], \tag{2.22}$$

$$\begin{aligned}u_{1}(\eta)&=\frac{4}{(C-1)^{2}}\frac{Hh_{2m+4}\left(\left(\sqrt{2C}\right)a\right)}{Hh_{2m}\left(\left(\sqrt{2C}\right)a\right)}\left[\frac{Hh_{2m+4}\left(\left(\sqrt{2}\right)\xi\right)}{Hh_{2m+4}\left(\left(\sqrt{2}\right)a\right)}-\frac{Hh_{2m+4}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m+4}\left(\left(\sqrt{2C}\right)a\right)}\right]+\\&+\frac{2}{(C-1)}\frac{Hh_{2m+2}\left(\left(\sqrt{2C}\right)a\right)}{Hh_{2m}\left(\left(\sqrt{2C}\right)a\right)}\left[\frac{Hh_{2m+4}\left(\left(\sqrt{2}\right)\xi\right)}{Hh_{2m+4}\left(\left(\sqrt{2}\right)a\right)}-\frac{Hh_{2m+2}\left(\left(\sqrt{2}\right)\xi\right)}{Hh_{2m+2}\left(\left(\sqrt{2}\right)a\right)}\right],\end{aligned} \tag{2.23}$$

$$\begin{aligned}u_{2}(\eta)&=\frac{8}{(C-1)^{3}}\frac{Hh_{2m+6}\left(\left(\sqrt{2C}\right)a\right)}{Hh_{2m}\left(\left(\sqrt{2C}\right)a\right)}\left[\frac{Hh_{2m+6}\left(\left(\sqrt{2}\right)\xi\right)}{Hh_{2m+6}\left(\left(\sqrt{2}\right)a\right)}-\frac{Hh_{2m+6}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m+6}\left(\left(\sqrt{2C}\right)a\right)}\right]+\\&+\frac{4}{(C-1)^{2}}\frac{Hh_{2m+4}\left(\left(\sqrt{2C}\right)a\right)}{Hh_{2m}\left(\left(\sqrt{2C}\right)a\right)}\left[\frac{Hh_{2m+6}\left(\left(\sqrt{2}\right)\xi\right)}{Hh_{2m+6}\left(\left(\sqrt{2}\right)a\right)}-\frac{Hh_{2m+4}\left(\left(\sqrt{2}\right)\xi\right)}{Hh_{2m+4}\left(\left(\sqrt{2}\right)a\right)}\right]+\\&+\frac{1}{(C-1)}\frac{Hh_{2m+2}\left(\left(\sqrt{2C}\right)a\right)}{Hh_{2m}\left(\left(\sqrt{2C}\right)a\right)}\left[\frac{Hh_{2m+6}\left(\left(\sqrt{2}\right)\xi\right)}{Hh_{2m+6}\left(\left(\sqrt{2}\right)a\right)}-2\frac{Hh_{2m+4}\left(\left(\sqrt{2}\right)\xi\right)}{Hh_{2m+4}\left(\left(\sqrt{2}\right)a\right)}+\frac{Hh_{2m+2}\left(\left(\sqrt{2}\right)\xi\right)}{Hh_{2m+2}\left(\left(\sqrt{2}\right)a\right)}\right].\end{aligned} \tag{2.24}$$

For $\quad C=1$

$$u_{0}(\eta)=\frac{Hh_{2m}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m}\left(\left(\sqrt{2C}\right)a\right)}-\frac{Hh_{2m+2}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m+2}\left(\left(\sqrt{2C}\right)a\right)}, \tag{2.25}$$

$$u_1(\eta)=-\frac{1}{2}\frac{Hh_{2m}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m}\left(\left(\sqrt{2C}\right)a\right)}+\frac{Hh_{2m+2}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m+2}\left(\left(\sqrt{2C}\right)a\right)}-\frac{1}{2}\frac{Hh_{2m+4}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m+4}\left(\left(\sqrt{2C}\right)a\right)}, \tag{2.26}$$

$$\begin{gathered}u_2(\eta)=\frac{1}{6}\frac{Hh_{2m}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m}\left(\left(\sqrt{2C}\right)a\right)}-\frac{Hh_{2m+2}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m+2}\left(\left(\sqrt{2C}\right)a\right)}+\\+\frac{1}{2}\frac{Hh_{2m+4}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m+4}\left(\left(\sqrt{2C}\right)a\right)}-\frac{1}{6}\frac{Hh_{2m+6}\left(\left(\sqrt{2C}\right)\xi\right)}{Hh_{2m+6}\left(\left(\sqrt{2C}\right)a\right)}.\end{gathered} \tag{2.27}$$

3. Results

In order to investigate the effects of various parameters on the problem numerical calculations are carried out for the temperature and velocity fields, when $M = 0.2$.

In Fig.1, we have plotted the temperature profiles showing the effect of the radiation parameter ***N***. It can be seen that the temperature decreases when the radiation parameter increases.

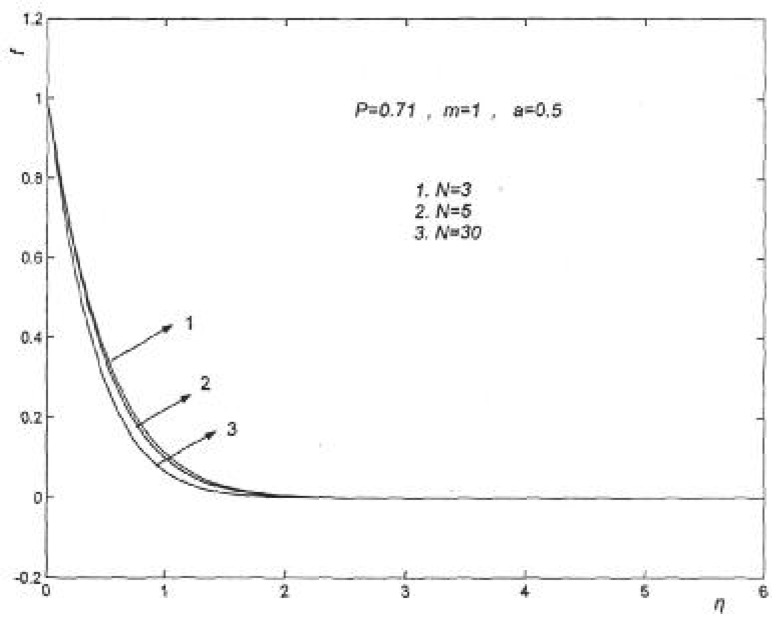

Fig.1. Temperature profiles for various values of the radiation parameter ***N***.

In Fig.2, we have plotted the temperature profiles showing the effect of the suction parameter ***a***. It can be seen that the temperature decreases when the suction parameter increases.

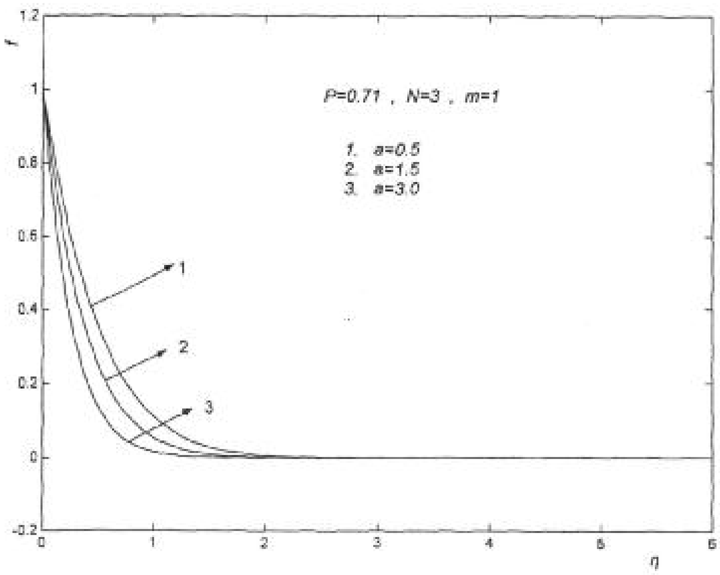

Fig.2. Temperature profiles for various values of the suction parameter ***a***.

In Fig.3, we have plotted the temperature profiles showing the effect of the parameter ***m***. It can be seen that the temperature decreases when the parameter ***m*** increases.

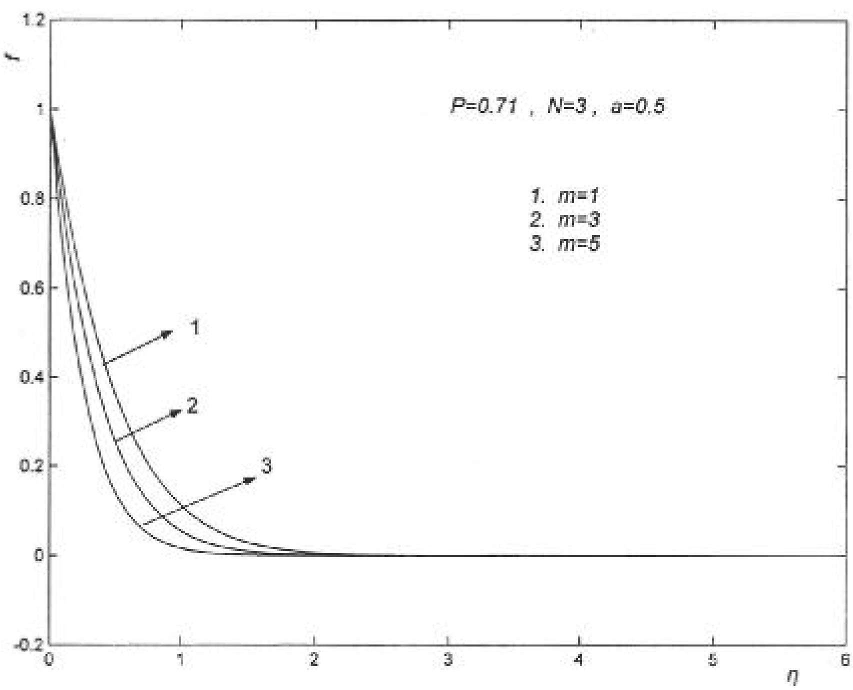

Fig.3. Temperature profiles for various values of the parameter ***m***.

In Fig.4, we have plotted the velocity profiles showing the effect of the radiation parameter ***N***. It can be seen that the velocity decreases when the radiation parameter increases.

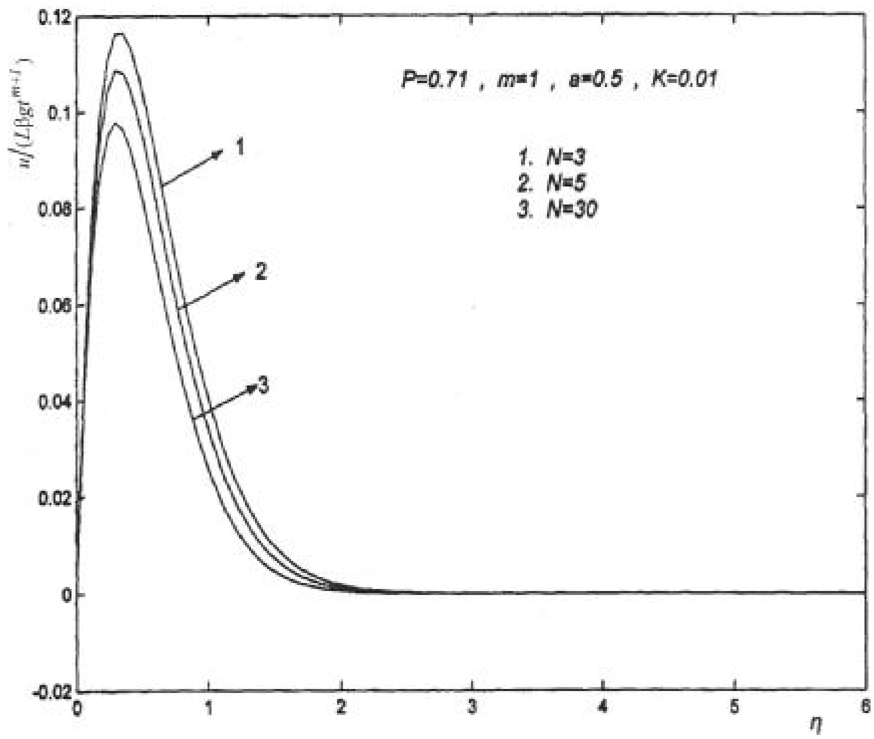

Fig.4. Velocity profiles for various values of the radiation parameter *N*.

In Fig.5, we have plotted the velocity profiles showing the effect of the suction parameter *a*. It can be seen that the velocity decreases when the suction parameter increases.

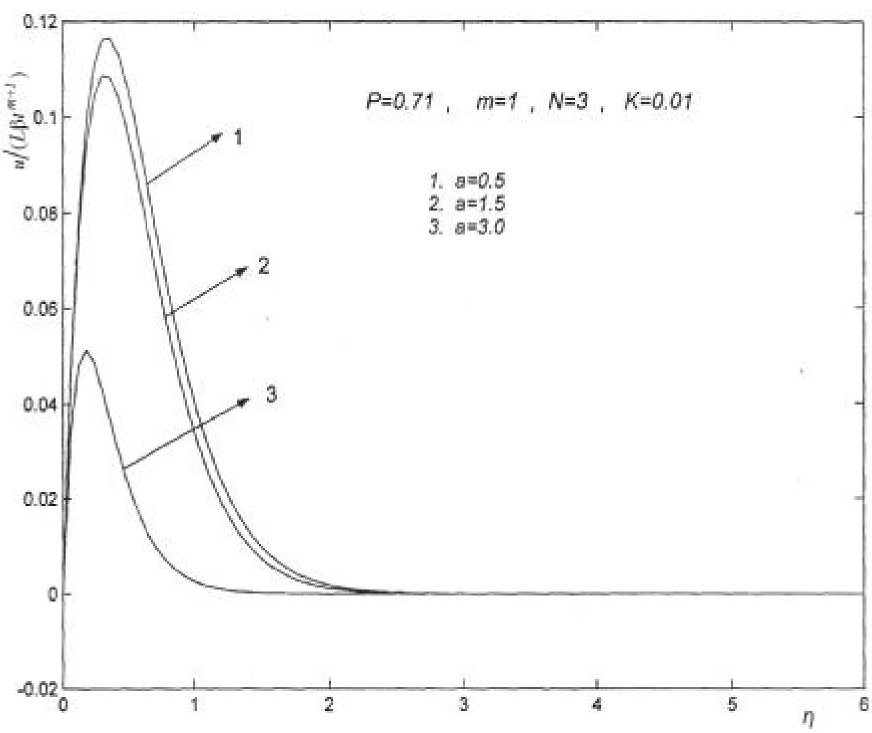

Fig.5. Velocity profiles for various values of the suction parameter *a*.

In Fig.6, we have plotted the velocity profiles showing the effect of the parameter $\boldsymbol{m}$. It can be seen that the velocity decreases when the parameter $\boldsymbol{m}$ increases.

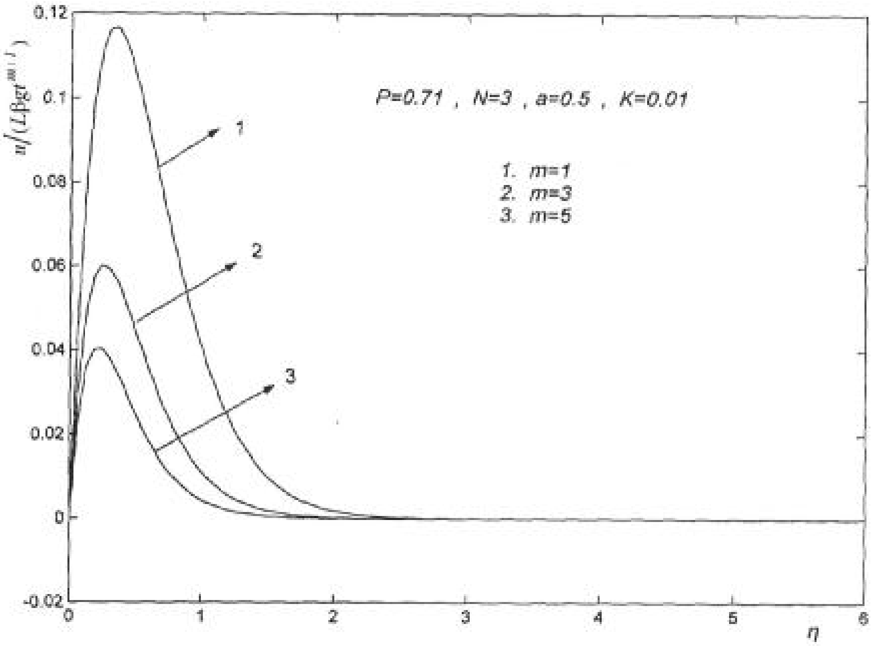

Fig.6. Velocity profiles for various values of the parameter $\boldsymbol{m}$.

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