EFFECT OF COMPRESSIBILITY AND SUSPENDED PARTICLES ON THERMAL CONVECTION IN A WALTERS' B' ELASTICO-VISCOUS FLUID IN HYDROMAGNETICS

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A layer of a compressible, electrically conducting Walters' B' elastico-viscous fluid permeated with suspended particles heated from below in the presence of a magnetic field is considered. For the case of stationary convection, the Walters' (model B') elastico-viscous fluid behaves like a Newtonian fluid and the compressibility, magnetic field are found to have stabilizing effects, whereas the suspended particles have a destabilizing effect on the thermal convection. The presence of each – viscoelasticity, magnetic field and suspended particles introduces oscillatory modes in the system which were non-existent in their absence.

Key words: thermal convection, elastico-viscous fluid, compressibility.

1. Introduction

A detailed account of thermal convection in a Newtonian fluid layer in the presence of a magnetic field has been given by Chandrasekhar (1981). Chandra (1938) observed a contradiction between the theory for the onset of convection in fluids heated from below and his experiment. He performed the experiment in an air layer and found that the instability depended on the depth of the layer. A Bénard-type cellular convection with the fluid descending at the cell centre was observed when the predicted gradients were imposed for layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurred at much lower gradients than predicted, if the layer depth was less than 7 mm it was called columnar instability. An aerosol to mark the flow pattern was added. Motivated by interest in fluid-particle mixtures and columnar instability, Scanlon and Segel (1973) studied the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced, solely because the heat capacity of the pure gas was supplemented by that of the particles. Sharma et al. (1976) considered the effect of suspended particles on the onset of Bénard convection in hydromagnetics. Bhatia and Steiner (1972) have studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation and have found that the rotation has a destabilizing effect, in contrast to the stabilizing effect on an ordinary (Newtonian) viscous fluid. Bhatia and Steiner (1973) have also studied the thermal instability of a Maxwellian viscoelastic fluid in the presence of a magnetic field, while the thermal convection in an Oldroydian viscoelastic fluid in hydromagnetics has been considered by Sharma et al. (1976).

The fluids have been considered to be Newtonian or viscoelastic (Maxwellian or Oldroydian) in all the above studies. There are many elastico-viscous fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such fluid is Walters' (model B') elastico-viscous fluid having relevance and importance in chemical technology and industry. Walters (1962) reported that the mixture of

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polymethyl methacrylate and pyridine at $25^{\circ}C$ containing 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters' (model B') elastico-viscous fluid. Polymers are used in the manufacture of spacecrafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastic engineering equipments, contact lens etc. Walters' (model B') elastico-viscous fluid forms the basis for the manufacture of many such important and useful products. Sharma *et al.* (1999) have considered the thermosolutal instability of Walters' (model B') rotating fluid in porous medium whereas the Rayleigh-Taylor instability of Walters' B' elastico-viscous fluid through porous medium has been studied by Sharma *et al.* (2002). In thermal and thermosolutal convection problems, the Boussinesq approximation has been used, which is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become more complicated. Spiegel and Veronis (1960) have simplified the set of equations governing the flow of compressible fluids under the assumption that the depth of the fluid layer is much smaller than the scale height as defined by them, if only motions of infinitesimal amplitude are considered. Sharma (1977) has studied the thermal instability in compressible fluids in the presence of rotation and a magnetic field.

Keeping in mind the importance of non-Newtonian fluids, compressibility and suspended particles in chemical technology, industry and geophysical fluid dynamics, the present paper attempts to study the thermal convection in an electrically conducting, compressible, Walters' (moldel B') elastico-viscous fluid permeated with suspended particles in the presence of a uniform magnetic field.

2. Perturbation equations

Consider an infinite, horizontal, electrically conducting, compressible, Walters' (moldel B') elasticoviscous fluid layer of thickness *d*, permeated with suspended particles, bounded by the planes z = 0 and z = d. This fluid particle layer is heated from below so that a uniform temperature gradient $\beta = (|dT/dz|)$ is maintained and the layer is acted on by the gravity field g(0, 0, -g) and a uniform vertical magnetic field H(0, 0, H).

Spiegel and Veronis (1960) defined f as any of the state variables (pressure (p), density (ρ) or temperature (T)) and expressed these in the form

$$f(x, y, z, t) = f_m + f_0(z) + f'(x, y, z, t)$$
(2.1)

where f_m is the constant space average of f, f_0 is the variation in the absence of motion and f' is the fluctuation resulting from the motion.

The initial state is, therefore, a state in which the density, pressure, temperature and velocity in the fluid are given by

$$\rho = \rho(z), \quad p = p(z), \quad T = T(z), \quad v = 0,$$
(2.2)

respectively, where

$$T(z) = T_0 - \beta z ,$$

$$p(z) = p_m - g \int_0^d (\rho_m + \rho_0) dz ,$$

$$\rho(z) = \rho_m [I - \alpha_m (T - T_m) + K_m (p - p_m)],$$
(2.3)

$$\alpha_m = -\left(\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)_m \ (=\alpha, \text{ say}) \qquad \qquad K_m = \left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)_m.$$

The linearized hydromagnetic perturbation equations for thermal convection in a compressible, Walters' B' elastico-viscous fluid permeated with suspended particles (Chandrasekhar, 1981; Walters, 1962; Spiegel and Veronis, 1960 and Sharma, 1977) are

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{1}{\rho_m} \nabla \delta \boldsymbol{p} + \boldsymbol{g} \frac{\delta \boldsymbol{p}}{\rho_m} + \left(\boldsymbol{v} - \boldsymbol{v}' \frac{\partial}{\partial t} \right) \nabla^2 \boldsymbol{v} + \frac{\mu_e}{4\pi\rho_m} (\nabla \times \boldsymbol{h}) \times \boldsymbol{H} + \frac{KN_0}{\rho_m} (\boldsymbol{v}_d - \boldsymbol{v}), \tag{2.4}$$

$$\nabla \mathbf{v} = 0 , \qquad (2.5)$$

$$m N_0 \frac{\partial \mathbf{v}_d}{\partial t} = K N_0 \left(\mathbf{v} - \mathbf{v}_d \right), \tag{2.6}$$

$$\frac{\partial M}{\partial t} + \nabla \mathbf{v}_d = 0 , \qquad (2.7)$$

$$\nabla \boldsymbol{h} = 0, \qquad (2.8)$$

$$\frac{\partial \boldsymbol{h}}{\partial t} = (\boldsymbol{H}\nabla)\boldsymbol{v} + \eta\nabla^2\boldsymbol{h}, \qquad (2.9)$$

$$(1+h)\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{c_p}\right)(w+hs) + \kappa \nabla^2 \theta.$$
(2.10)

Here $\mathbf{v}(u, v, w)$, $\mathbf{v}_d(l, r, s)$, N, $\mathbf{h}(h_x, h_y, h_z)$, $\delta \rho$, δp and θ denote, respectively, the perturbations in fluid velocity (0, 0, 0) suspended particle velocities (0, 0, 0) suspended particles number density N_0 , magnetic field $\mathbf{H}(0, 0, H)$ density ρ , pressure p and temperature $T \cdot v, v', \kappa, \mu_e$ and η stand for the kinematic viscosity, kinematic viscoelasticity, thermal diffusivity, magnetic permeability and electrical resistivity respectively. Here $\mathbf{v}_d(\bar{x}, t)$, $N(\bar{x}, t)$ denote velocity, number density of suspended particles.

 $K = 6\pi\mu\epsilon'$, where ϵ' is the particle radius, K is Stokes' drag coefficient and $\bar{x} = (x, y, z)$. $M = \frac{N}{N_0}$ and

 $h = \frac{mN_0 c_{pt}}{\rho_m c_v}$. c_p and c_v are the specific heat of the fluid at constant pressure and volume, respectively. c_{pt}

is the specific heat of the particles. The distances between particles are assumed to be so large compared with their diameter that interparticle reactions need not be accounted for. The effects of pressure, gravity and the magnetic field on the suspended particles, assuming large distances apart, are negligibly small and therefore ignored. Under the above assumptions, Eqs (2.6) and (2.7) represent the equations of motion and continuity for the particles, mN being the mass of particles per unit volume. The equation of state is

$$\rho = \rho_m \left[I - \alpha (T - T_0) \right] \tag{2.11}$$

where α is the coefficient of thermal expansion, as density variations arise mainly due to temperature variations. Therefore the change in density $\delta\rho$ caused by the perturbation θ in temperature is given by

$$\delta \rho = -\rho_m \alpha \theta \,. \tag{2.12}$$

Writing the scalar components of Eq.(2.4), after elimination of v_d with the help of Eq.(2.6), eliminating $u, v, h_x, h_y, \delta p$ between them by using Eqs (2.5), (2.8) and (2.12), we obtain

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+I\right)\left[\frac{\partial}{\partial t}\nabla^{2}w-g\alpha\left(\frac{\partial^{2}\theta}{\partial x^{2}}+\frac{\partial^{2}\theta}{\partial y^{2}}\right)-\frac{\mu_{e}H}{4\pi\rho_{m}}\frac{\partial}{\delta z}\nabla^{2}h_{z}\right]+ \\
+\frac{KN_{0}}{\rho_{m}}\frac{m}{K}\frac{\partial}{\partial t}\nabla^{2}w=\left(\frac{m}{K}\frac{\partial}{\partial t}+I\right)\left(v-v'\frac{\partial}{\partial t}\right)\nabla^{4}w.$$
(2.13)

The z-component of Eq.(2.9) yields

$$\left(\frac{\partial}{\partial t} - \eta \nabla^2\right) h_z = H \frac{\partial w}{\partial z}.$$
(2.14)

Equation (2.10), on substituting for s in terms of w with the help of Eq.(2.6), yields

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+I\right)\left[\left(I+h\right)\frac{\partial}{\partial t}-\kappa\nabla^{2}\right]\theta = \left(\frac{G-I}{G}\right)\left(\frac{m}{K}\frac{\partial}{\partial t}+H\right)w.$$
(2.15)

Equations (2.13)-(2.15) yield three perturbation equations in w, θ and h_z .

3. Dispersion relation

Analyze the perturbations into normal modes by seeking solutions in the form

$$[w, \theta, h_z] = [W(z), \Theta(z), K(z)] \exp(ik_x x + ik_y y + nt)$$
(3.1)

where *n* is, in general, a complex constant and $k = (k_x^2 + k_y^2)^{\frac{1}{2}}$ is the resultant wave number of the disturbance.

Equations (2.13)-(2.15) in a dimensionless form, using expression (3.1), become

$$(l+p_{I}\sigma\tau)\left(D^{2}-a^{2}\right)\left[(l-F\sigma)\left(D^{2}-a^{2}\right)-\sigma\right]W-f\sigma\left(D^{2}-a^{2}\right)W+ (l+p_{I}\sigma\tau)\frac{\mu_{e}Hd}{4\pi\rho_{m}\nu}\left(D^{2}-a^{2}\right)DK = (l+p_{I}\sigma\tau)\frac{g\alpha d^{2}}{\nu}a^{2}\Theta,$$
(3.2)

$$\left(D^2 - a^2 - p_2 \sigma\right) K = -\frac{Hd}{\eta} DW, \qquad (3.3)$$

$$(1+p_1\sigma\tau)\left(D^2-a^2-Hp_1\sigma\right)\Theta = -\frac{\beta d^2}{\kappa}\left(\frac{G-1}{G}\right)(H+p_1\sigma\tau)W$$
(3.4)

where $p_1 = \frac{v}{\kappa}$ is the Prandtl number, $p_2 = \frac{v}{\eta}$ is the magnetic Prandtl number, $F = \frac{v'}{d^2}$ is the dimensionless kinematic viscoelasticity, $G = \frac{c_p \beta}{g}$ is the dimensionless compressibility, $\sigma = \frac{nd^2}{v}$, a = kd, H = l + h, $h = \frac{mN_0c_{pl}}{\rho_m c_v}$, $f = \frac{mN_0}{\rho_m}$, $\tau = \frac{m\kappa}{Kd^2}$. Here we have put $x^* = \frac{x}{d}$, $y^* = \frac{y}{d}$, $z^* = \frac{z}{d}$ and $D = \frac{d}{dz^*}$. Stars (*)

have not been written hereafter, for convenience.

Eliminating Θ and K between Eqs (3.2)-(3.4), we obtain

$$(l + p_{I}\sigma\tau)(D^{2} - a^{2})(D^{2} - a^{2} - p_{2}\sigma)(D^{2} - a^{2} - Hp_{I}\sigma)[(l - F\sigma)(D^{2} - a^{2}) - \sigma] W + - f\sigma(D^{2} - a^{2} - p_{2}\sigma)(D^{2} - a^{2} - Hp_{I}\sigma)(D^{2} - a^{2})W + - Q(l + p_{I}\sigma\tau)(D^{2} - a^{2} - Hp_{I}\sigma)(D^{2} - a^{2})D^{2}W = = -Ra^{2}(D^{2} - a^{2} - p_{2}\sigma)(\frac{G - l}{G})(H + p_{I}\sigma\tau)W$$
(3.5)

where $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_m v\eta}$ is the Chandrasekhar number and $R = \frac{g\alpha\beta d^4}{v\kappa}$ is the Rayleigh number.

Consider the case in which both the boundaries are free, the medium adjoining the fluid is perfectly conducting and temperatures at the boundaries are kept fixed. The boundary conditions, appropriate for the problem, are

$$W = D^2 W = 0$$
, $K = 0$, $\Theta = 0$ and l . (3.6)

The solution of Eq.(3.5) characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{3.7}$$

where W_0 is constant. Substituting Eq.(3.7) in Eq.(3.5), we get

$$R_{I} = \frac{G}{G-I} \Big((l+x)(l+x+ip_{2}\sigma_{1})(l+x+ip_{1}H\sigma_{1}) \Big[(l+ip_{1}\pi^{2}\tau\sigma_{1}) \times \Big] \\ \times \Big\{ (l-i\pi^{2}F\sigma_{1})(l+x) + i\sigma_{1} \Big\} + i\pi^{2}f\sigma_{1} \Big] + Q_{1}(l+x)(l+ip_{1}\pi^{2}\tau\sigma_{1}) \times \\ \times (l+x+ip_{1}H\sigma_{1}) \Big[x \Big(H+ip_{1}\pi^{2}\tau\sigma_{1} \Big) (l+x+ip_{2}\sigma_{1}) \Big]^{-1} \Big)$$
(3.8)

where $Q_I = \frac{Q}{\pi^2}$, $R_I = \frac{R}{\pi^4}$, $i\sigma_I = \frac{\sigma}{\pi^2}$ (where σ can be complex).

4. The stationary convection

For stationary convection, Eq.(3.8) reduces to

$$R_{I} = \left(\frac{G}{G-I}\right) \left(\frac{I+x}{xH}\right) \left((I+x)^{2} + Q_{I}\right).$$

$$(4.1)$$

For stationary convection, Eq.(4.1) implies that the compressible Walters' elastico-viscous fluid (model B') behaves like an ordinary Newtonian viscous fluid. Equation (4.1) yields

$$\frac{dR_I}{dH} = -\left(\frac{G}{G-I}\right)\left(1+x\right)^2 + Q_I\left(\frac{1+x}{xH^2}\right),\tag{4.2}$$

$$\frac{dR_I}{dQ_I} = \left(\frac{G}{G-I}\right) \left(\frac{1+x}{xH}\right). \tag{4.3}$$

It follows from Eqs (4.2) and (4.3) that the suspended particles have a destabilizing effect, whereas the magnetic field has a stabilizing effect on the thermal convection in a compressible Walters' B' elastico-viscous fluid permeated with suspended particles, for the stationary convection.

For fixed Q_1 and H, let G (accounting for the compressibility effects) be also kept fixed in Eq.(4.1). Then we find that

$$\overline{R}_{c} = \left(\frac{G}{G-I}\right) R_{c} \tag{4.4}$$

where \overline{R}_c and R_c denote, respectively, the critical Rayleigh numbers in the presence and absence of compressibility. The effect of compressibility is thus to postpone the onset of thermal convection in compressible Walters' B' elastico-viscous fluid, for the stationary convection. G > I is relevant here as G < I and G = I correspond to the negative and infinite Rayleigh numbers which are not appropriate in the present problem. The compressibility, therefore, has a stabilizing effect on the thermal convection.

5. Stability of the system and oscillatory modes

Multiplying Eq.(3.2) by W^* , the complex conjugate of W, integrating over the range of values of z, and making use of Eqs (3.3) and (3.4) together with the boundary conditions (3.6), we obtain

$$(I+p_{I}\tau\sigma)(I-F\sigma)I_{I}+\sigma(I+F+p_{I}\tau\sigma)I_{2}+\frac{\mu_{e}\eta}{4\pi\rho_{m}\nu}(I+p_{I}\tau\sigma)(I_{3}+p_{2}\sigma^{*}I_{4})+$$

$$-\frac{g\alpha\kappa a^{2}}{\nu\beta}\left(\frac{G}{G-I}\right)\frac{(I+p_{I}\tau\sigma^{*})(I+p_{I}\tau\sigma)}{(H+p_{I}\tau\sigma^{*})}(I_{5}+Hp_{I}\sigma^{*}I_{6})=0$$
(5.1)

where

$$I_{I} = \int_{0}^{1} \left(\left| D^{2}W \right|^{2} + 2a^{2} \left| DW \right|^{2} + a^{4} \left| W \right|^{2} \right) dz,$$

$$I_{2} = \int_{0}^{l} (|DW|^{2} + a^{2}|W|^{2}) dz,$$

$$I_{3} = \int_{0}^{l} (|D^{2}K|^{2} + 2a^{2}|DK|^{2} + a^{4}|K|^{2}) dz,$$

$$I_{4} = \int_{0}^{l} (|DK|^{2} + a^{2}|K|^{2}) dz,$$

$$I_{5} = \int_{0}^{l} (|D\Theta|^{2} + a^{2}|\Theta|^{2}) dz,$$

$$I_{6} = \int_{0}^{l} (|\Theta|^{2}) dz.$$
(5.2)

The integrals $I_1 - I_6$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ and then equating real and imaginary parts of Eq.(5.1), we obtain

$$\begin{cases} l + p_{1}\tau\sigma_{r} - F\sigma_{r} - Fp_{1}\tau\sigma_{r}^{2} + F\sigma_{i}^{2}p_{1}\tau \}I_{1} + \left\{\sigma_{r}(l + f + p_{1}\tau\sigma_{r}) - p_{1}\tau\sigma_{i}^{2}\right\}I_{2} + \\ + \frac{\mu_{e}\eta}{4\pi\rho_{m}\nu} \left[(l + p_{1}\tau\sigma_{r})I_{3} + \left\{p_{2}\sigma_{r}(l + p_{1}\tau\sigma_{r}) + p_{1}p_{2}\tau\sigma_{i}^{2}\right\}I_{4}\right] + \\ - \frac{g\alpha\kappa a^{2}}{\nu\beta} \left(\frac{G}{G-I}\right) \left\{\frac{(l + p_{1}\tau\sigma_{r})^{2} + p_{1}^{2}\tau^{2}\sigma_{i}^{2}}{(H + p_{1}\tau\sigma_{r})^{2} + p_{1}^{2}\tau^{2}\sigma_{i}^{2}}\right\} \times \\ \times \left[(H + p_{1}\tau\sigma_{r})(I_{5} + Hp_{1}\sigma_{r}I_{6}) + p_{1}^{2}\tau H\sigma_{i}^{2}I_{6}\right] = 0 \end{cases}$$
(5.3)

and

$$i\sigma_{i}\{\{I - p_{I}\tau F\sigma_{r} - F(I + p_{I}\tau\sigma_{r})\}I_{I} + \{I + f + p_{I}\tau\sigma_{r}\}I_{2} + \{p_{I}\tau I_{3} - p_{2}I_{4}\} + \frac{g\alpha\kappa a^{2}}{\nu\beta}\left(\frac{G}{G-I}\right)\left\{\frac{(I + p_{I}\tau\sigma_{r})^{2} + p_{I}^{2}\tau^{2}\sigma_{i}^{2}}{(H + p_{I}\tau\sigma_{r})^{2} + p_{I}^{2}\tau^{2}\sigma_{i}^{2}}\right\}\left\{-p_{I}\tau I_{5} + p_{I}H^{2}I_{6}\right\}\right] = 0$$
(5.4)

It may be inferred from Eq.(5.3) that σ_r is positive or negative which means that the system may be stable or unstable. It is clear from Eq.(5.4) that σ_i may be zero or non-zero, meaning that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of viscoelasticity, magnetic field and suspended particles, which were non-existent in their absence.

Nomenclature

- c_p specific heat of fluid at constant pressure
- c_{pt} specific heat of suspended particles
- c_v specific heat of fluid at constant volume
- F dimensionless kinematic viscoelasticity
- G dimensionless compressibility

g(0, 0, -g)	– gravity field
H(0, 0, H)	– magnetic field
$h(h_x, h_y, h_z)$	- perturbation in magnetic field
K	- Stokes' drag coefficient
k	– wave number
k_x, k_y	– wave numbers in x- and y- directions
m	- mass of suspended particles
N	- number density of suspended particles
n	– stability parameter
р	– pressure
p_1	– Prandtl number
p_2	 magnetic Prandtl number
R	– Rayleigh number
R_c	- critical Rayleigh number in the absence of compressibility
Т	– temperature
t	- time coordinate
Q	– Chandrasekhar number
v(u, v, w)	 velocity of fluid
$\mathbf{v}_d(l, r, s)$	- velocity of suspended particles
$\overline{x}(x, y, z)$	– space coordinate
α	- coefficient of thermal expansion
β	 temperature gradient
δρ	- perturbation in pressure
δρ	- perturbation in density
ε'	- radius of suspended particles
η	 electrical resistivity
θ	– perturbation in temperature
κ	 thermal diffusivity
μ	 fluid viscosity
μ_e	 magnetic permeability
ν	 kinematic viscosity
v	 kinematic viscoelasticity
ρ	 – fluid density

References

- Bhatia P.K. and Steiner J.M. (1972): Convective instability in a rotating viscoelastic fluid layer. Z. Angew. Math. Mech., vol.52, pp.321-324.
- Bhatia P.K. and Steiner J.M. (1973): Thermal instability in a viscoelastic fluid layer in hydromagnetics. J. Math. Anal. Appl., vol.41, pp.271-283.

Chandra K. (1938): Instability of fluid heated from below. - Proc. Roy. Soc. (Lon.), vol.A164, pp.231-258.

Chandrasekhar S. (1981): Hydrodymanic and Hydromagnetic Stability. - New York: Dover Publication.

- Scanlon J.W. and Segel L.A. (1973): Some effects of suspended particles on the onset of Bénard convection. Phys. Fluids, vol.16, pp.1573-78.
- Sharma R.C. (1975): *Thermal instability in a viscoelestic fluid in hydromagnetics.* Acta Physica Hungarica, vol.38, pp.293-298.
- Sharma R.C. (1977): Thermal instability in compressible fluids in the presence of rotation and magnetic field. J. Math. Anal. Appl., vol.60, pp.227-235.

- Sharma R.C., Kumar P. and Sharma S. (2002): Rayleigh-Taylor instability of Walters' B' elastico-viscous fluid through porous medium. Int. J. Appl. Mech. Eng., vol.7, pp.433-44.
- Sharma R.C., Prakash K. and Dube S.N. (1976): *Effect of suspended particles on the onset of Bénard convection in hydromagnetics*. Acta Physica Hungarica, vol.40, pp.3-10.
- Sharma R.C., Sunil and Chand S. (1999): *Thermosolutal instability of Walters' rotating fluid (model B') in porous medium.* Arch. Mech., vol.51, pp.181-191.
- Spiegel E.A. and Veronis G. (1960): On the boussinesq approximation for a compressible fluid. Astrophys. J., vol.131, pp.442-444.
- Walters K. (1962): Non-Newtonian effects in some elastico-viscous liquids whose behavior at small rates of shear is characterized by a general linear equation of state. Quart. J. Mech. Appl. Math., vol.15, pp.76-96.

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