

CALCULATION OF SHIP SINKAGE AND TRIM IN DEEP WATER USING A POTENTIAL BASED PANEL METHOD

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The present work is an extension of Morino's panel method for the calculation of wave-making resistance of ships with special reference to sinkage and trim. The body boundary is linearized about the undisturbed position of the body and the free surface is linearized about the mean water level by the systematic method of perturbation. The surfaces are discretized into flat quadrilateral elements and the influence coefficients are calculated by Morino's analytical formula. Dawson's upstream finite difference operator is used in order to satisfy the radiation condition. The sinkage and trim of a ship are computed by equating the vertical force and pitching moment to the hydrostatic restoring force and moment. The present method has been applied to the Series 60 hull for different Froude numbers and is found to be efficient for evaluating the flow field, wave pattern and wave-making resistance in deep water.

Key words: resistance, Morino's panel method, perturbation method, sinkage, trim, Dawson's finite difference operator.

1. Introduction

A steadily advancing surface ship experiences sinkage and trim notably at high Froude numbers due to the hydrodynamic forces acting on the ship hull. Sinkage and trim effects are also observed in towing tank experiments if the ship model is not fixed to the towing carriage. A ship model free to sink and trim can experience an increase in wave resistance. Large trim changes may also affect the performance of a ship. Sinkage and trim in very shallow water may set an upper limit to the speed at which ships can operate without touching the bottom surface. Therefore, it is of practical importance to include sinkage and trim effects in the calculation of steady ship waves.

Suzuki (1979) developed the Neumann-Kelvin problem as a method of calculating the effect of sinkage and trim on the wave resistance of a ship. Yasukawa (1993) predicted the wave-making resistance taking into account the effect of sinkage and trim by the Rankine source method. The sinkage and trim are computed by equating the vertical force and pitching moment to the hydrostatic restoring force and moment.

Doctors and Day (2000) developed an inviscid linearized near-field solution with the framework of classical thin ship theory for the flow past a vessel with a transom stern. To take into account the effects of transom stern the hollow in the water behind the vessel is represented by an extension to the usual centre plane source distribution employed to model the ship itself.

Yang *et al.* (2000) developed a parallel free surface flow solver based on unstructured grid for steady ship wave resistance problem. The problem is formulated in terms of the Euler or Reynolds-average Navier Stokes (RANS) equation and then extended to incorporate the dynamic sinkage and trim to the steady wave calculation. The overall scheme combines a finite element, equal order, projection type three-dimensional incompressible flow solver with a finite element, two-dimensional advection equation solver for the free surface equation. The importance of the trim and sinkage of a ship are well indicated by Subramani *et al.* (2000). The other researchers who have made important contributions in the field of wave-making resistance

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of ship taking into account the effects of sinkage and trim are Tuck (1967), Bessho and Sakuma (1992), Gourlay and Tuck (2001), Azcueta (2002) etc.

The objective of the present paper is to continue the development of a more efficient model to predict steady ship waves by Morino's panel method while the Kelvin classical linearized free surface conditions are employed. Due to the significant effects of sinkage and trim on the hydrodynamic performance of a ship, dynamic sinkage and trim are incorporated in the calculations.

2. Mathematical modelling of the problem

Let us consider two co-ordinate systems of which $x' y' z'$ is fixed with respect to the ship and $x y z$ is a steady moving frame of reference with a forward speed U in the direction of the positive x -axis as shown in Fig.1.

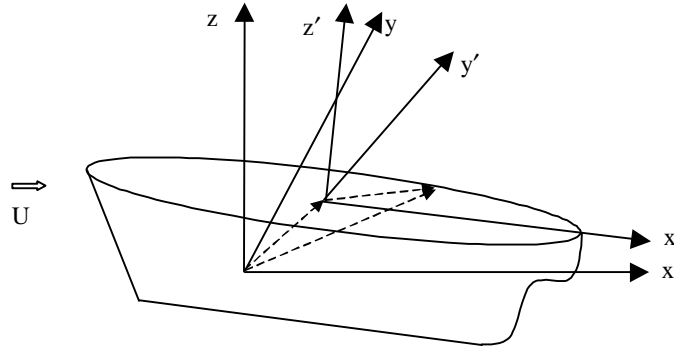


Fig.1. Definition sketch of the co-ordinate system.

The z -axis is vertically upwards and the y -axis extends to starboard. The origin of the co-ordinate system is located amidship on the calm water surface. It is assumed that the fluid is incompressible and inviscid and the flow irrotational. The total velocity potential function Φ and wave elevation ζ can be expressed as

$$\Phi = Ux + \phi + \delta\phi = Ux + \sum_{n=1}^{\infty} \epsilon^n \phi_n + \delta(s\phi_s + t\phi_t), \quad (2.1)$$

$$\zeta = \sum_{n=1}^n \epsilon^n \zeta_n + \delta(s\zeta_s + t\zeta_t)$$

where, ϵ and δ are two small parameters, ϕ is perturbation potential due to uniform flow, ϕ_s is steady potential due to unit sinkage, ϕ_t is steady potential due to unit trim, s is the sinkage (positive upward) and t is the trim angle (trim by the stern is positive). It is assumed that ϕ , ϕ and ζ are very small compared to free stream potential. If ϕ is considered to be of second order, Eq.(2.1) can now be expressed as

$$\Phi = Ux + \epsilon\phi_1 + \epsilon^2(\phi_2 + s\phi_s + t\phi_t), \quad (2.2)$$

$$\zeta = \epsilon\zeta_1 + \epsilon^2(\zeta_2 + s\zeta_s + t\zeta_t).$$

The velocity potential Φ satisfies the Laplace equation

$$\nabla^2 \Phi = 0, \quad (2.3)$$

in the fluid domain V . The fluid domain V is bounded by the hull surface S_H , free surface S_F and a surface of large hemisphere S_R in the lower half space. Now the problem can be constructed by specifying the boundary conditions as follows:

(a) *Hull boundary condition*: The hull boundary condition simply expresses the fact that the flow must be tangential to the hull surface, i.e., the normal component of the velocity must be zero.

$$\varepsilon: \nabla \phi_1 \cdot \mathbf{n} = -U n_x,$$

$$\varepsilon^2: \nabla \phi_2 \cdot \mathbf{n} = 0.$$

Due to sinkage and trim (2.4)

$$\varepsilon^2: \nabla \phi_s \cdot \mathbf{n} = m_3$$

$$\varepsilon^2: \nabla \phi_t \cdot \mathbf{n} = m_5$$

The m -terms are defined by Ogilvie and Tuck (1969) as

$$m_1 \mathbf{i} + m_2 \mathbf{j} + m_3 \mathbf{k} = -(\mathbf{n} \cdot \nabla) \mathbf{W},$$

$$m_4 \mathbf{i} + m_5 \mathbf{j} + m_6 \mathbf{k} = -(\mathbf{n} \cdot \nabla)(\mathbf{x} \times \mathbf{W}),$$

in which $\mathbf{n} = n_x \mathbf{i} + n_y \mathbf{j} + n_z \mathbf{k}$ denotes the unit normal vector on the surface and is positive into the fluid. \mathbf{W} is the fluid velocity due to the steady forward motion of the vessel in the ship fixed coordinate frame.

(b) *Free surface condition*: The kinematic and dynamic boundary conditions on the free surface can be respectively written as

$$\Phi_x \zeta_x + \Phi_y \zeta_y - \Phi_z = 0 \quad \text{on} \quad z = \zeta, \quad (2.5)$$

$$g\zeta + \frac{1}{2}(\nabla \Phi \cdot \nabla \Phi - U^2) = 0 \quad \text{on} \quad z = \zeta. \quad (2.6)$$

Combining Eqs (2.4) and (2.5) we get

$$\nabla \Phi \cdot \nabla \left[\frac{1}{2}(\nabla \Phi \cdot \nabla \Phi) \right] + g\Phi_z = 0 \quad \text{on} \quad z = \zeta. \quad (2.7)$$

The free surface boundary condition Eq.(2.7) is nonlinear in nature and should be satisfied on the true surface, which is unknown and can be linearized as a part of the solution using the perturbation method. Substituting Eq.(2.2) into Eq.(2.7) and expanding the potential Φ in a Taylor series about the mean free

surface $z=0$ the following free surface boundary conditions for the first and second order approximations can be obtained as (see Maruo, 1966)

$$\begin{aligned}\varepsilon: \quad \phi_{Ixx} + K_0 \phi_{Iz} &= 0, \\ \varepsilon^2: \quad \phi_{2xx} + K_0 \phi_{2z} &= f(\phi_I), \\ f(\phi_I) &= -\frac{1}{U} \frac{\partial}{\partial x} (\phi_{Ix}^2 + \phi_{Iy}^2 + \phi_{Iz}^2) - \zeta_I \frac{\partial}{\partial z} (\phi_{Ixx} + K_0 \phi_{Iz}) \quad \text{on} \quad z=0\end{aligned}\quad (2.8)$$

With respect to sinkage and trim

$$\begin{aligned}\varepsilon^2: \quad \phi_{sxx} + K_0 \phi_{sz} &= 0, \\ \varepsilon^2: \quad \phi_{txx} + K_0 \phi_{tz} &= 0\end{aligned}$$

where K_0 is the wave number defined by $K_0 = \frac{g}{U^2}$.

(c) *Radiation condition*: It is necessary to impose a condition to ensure that the free surface waves vanish upstream of the disturbance.

3. The boundary element method

Applying Green's second identity, Laplace's equation can be transformed into an integral equation as (see Curle and Davies, 1968)

$$4\pi E\phi(p) = \sum_{j=1}^{N_H} \int_{S_H} \phi(q) \frac{\partial G}{\partial n_q} dS - \sum_{j=1}^{N_H} \int_{S_H} \frac{\partial \phi(q)}{\partial n_q} G dS - \sum_{j=1}^{N_F} \int_{S_F} \frac{\partial \phi(q)}{\partial n_q} G dS \quad (3.1)$$

where,

$$E = \begin{cases} 1/2 & \text{on } S_H \\ 1 & \text{on } S_F \end{cases} \quad (3.2)$$

Green's function G can be approximated as (see Faltinsen, 1993)

$$G = \frac{1}{R(p; q)} + \frac{1}{R'(p; q)} \quad (3.3)$$

where, R is the position vector between the field point p and the point of singularity q on the surface and R' is its image. The integral over the surface S_R must be zero as the radius of the hemisphere increases infinitely. The integral over the element in Eq.(3.1) is calculated by Morino's analytical expression (see Suciu and Morino, 1976) based on the assumption of a quadrilateral hyperboloid element. After satisfying the boundary conditions as stated in Eqs (2.4) and (2.8), the integral Eq.(3.1) can be written into a matrix form as

$$[A]x = [B]$$

where, $[A]$ and $[B]$ are the matrices built up by the Green's function and its derivatives, and x is the column matrix formed by the strength of the sources and dipoles respectively. The second derivatives of velocity potentials in the left side of free surface condition (2.8) are computed by Dawson's upstream finite difference operator (see Dawson, 1977) in order to satisfy the radiation condition. The matrix of linear system of equations is solved by the LU decomposition method as described by Press *et al.* (1999).

4. Hydrodynamic forces on a ship

In order to calculate the forces and moments on the ship, the pressure must be evaluated on the actual instantaneous position of the ship hull. We choose to express the pressure at a point of the hull surface S_H , in terms of the pressure at the corresponding point of S_0 , the undisturbed position of the hull. The fluid pressure acting on the instantaneous wetted surface S_H during oscillatory motions of the ship can be written by Bernoulli's equation

$$p - p_\infty = \frac{1}{2}\rho(U^2 - \nabla\Phi \cdot \nabla\Phi) - \rho gz. \quad (4.1)$$

The pressure at any point on the surface S_H can be expressed in terms of the pressure at the corresponding point on the surface S_0 by a Taylor series expansion. Thus

$$[p - p_\infty]_{S_H} = \left[I + (\alpha \cdot \nabla) + \frac{1}{2}(\alpha \cdot \nabla)^2 + \dots \right] [p - p_\infty]_{S_0}. \quad (4.2)$$

Now the total fluid velocity vector on the instantaneous wetted surface for the second order approximation S_H can be obtained as

$$\begin{aligned} \nabla\Phi &= W + \varepsilon^2(\nabla\phi_2 + \nabla\phi), \\ W &= Ui + \varepsilon\nabla\phi_1. \end{aligned} \quad (4.3)$$

It is assumed that the oscillatory motions of the ship are so small that the second order terms of the unsteady components may be neglected, then the linearized form of the pressure on the wetted surface S_H becomes

$$\begin{aligned} p - p_\infty &= \frac{1}{2}\rho(U^2 - \nabla\Phi \cdot \nabla\Phi) + \frac{1}{2}\rho(\alpha \cdot \nabla)(U^2 - \nabla\Phi \cdot \nabla\Phi) - \rho gz = \\ &= -\rho \left\{ \frac{1}{2}(W^2 - U^2) + W \cdot \nabla\phi_2 \right\} - \rho gz - \rho \{ s(W \cdot \nabla\phi_s) + t(W \cdot \nabla\phi_t) \} + \\ &\quad - \left\{ tz' \frac{\partial}{\partial x} + (s - tx') \frac{\partial}{\partial z} \right\} \frac{1}{2}\rho W^2 + O(\alpha^2). \end{aligned} \quad (4.4)$$

The hydrodynamic forces ($k = 1, 2, 3$ indicates surge, sway and heave) and moments ($k = 4, 5, 6$ indicates rolling, pitching and yawing) in the k -th direction can be represented as (see Yasukawa, 1993)

$$F_k = -\int (p - p_\infty) n_k dS \approx F_k^0 + sF_k^s + tF_k^t,$$

$$F_k^0 = \rho \int \left\{ \frac{1}{2} (W^2 - U^2) + W \cdot \nabla \phi_2 \right\} n_k dS,$$

$$F_k^s = \rho \int \left\{ W \cdot \nabla \phi_s + \frac{1}{2} \frac{\partial}{\partial z} W^2 \right\} n_k dS,$$

$$F_k^t = \rho \int \left\{ W \cdot \nabla \phi_t + \frac{1}{2} \left(z' \frac{\partial}{\partial x} - x' \frac{\partial}{\partial z} \right) W^2 \right\} n_k dS,$$
(4.5)

$$n_4 i + n_5 j + n_6 k = r \times n \quad \text{and} \quad r = x' i + y' j + z' k.$$

5. Sinkage and trim

Suppose the ship responds to these forces and experiences a sinkage s defined as the downward vertical displacement at $x=0$ and trim t defined as the bow up angle of rotation about $y=0$. Now the following equations are obtained from the static equilibrium of forces

$$F_3 = \rho g \int_{-L/2}^{L/2} (s - tx) f_w(x) dx = s \rho g \int_{-L/2}^{L/2} f_w(x) dx - t \rho g \int_{-L/2}^{L/2} x f_w(x) dx,$$
(5.1)

$$F_5 = -\rho g \int_{-L/2}^{L/2} (s - tx) x f_w(x) dx = -s \rho g \int_{-L/2}^{L/2} x f_w(x) dx + t \rho g \int_{-L/2}^{L/2} x^2 f_w(x) dx.$$
(5.2)

$f_w(x)$ is the width of the waterplane area at the still water level and L denotes the length of the ship. Combining Eqs (4.5), (5.1) and (5.2) we get

$$s(F_3^s - H_0) + t(F_3^t + H_1) = -F_3^0,$$
(5.3)

$$s(F_5^s + H_1) + t(F_5^t - H_2) = -F_5^0,$$
(5.4)

$$H_0 = \rho g \int_{-L/2}^{L/2} f_w(x) dx; \quad H_1 = \rho g \int_{-L/2}^{L/2} x f_w(x) dx \quad \text{and} \quad H_2 = \rho g \int_{-L/2}^{L/2} x^2 f_w(x) dx.$$

Solving Eqs (5.3) and (5.4) we shall get the value of sinkage s and trim t .

6. Wave profile and resistance

The linearized equation of wave profile for the first and second order approximation can be obtained as

$$\zeta_I = -\frac{U}{g}\phi_{Ix}, \quad (6.1)$$

$$\zeta_2 = -\frac{U}{g}\phi_{2x} - \frac{U}{g}\zeta_I\phi_{Ixz} - \frac{I}{2g}(\phi_{Ix}^2 + \phi_{Iy}^2 + \phi_{Iz}^2), \quad (6.2)$$

$$\zeta_s = -\frac{U}{g}\phi_{sx}, \quad (6.3)$$

$$\zeta_t = -\frac{U}{g}\phi_{tx}. \quad (6.4)$$

After calculating the fluid velocity $\nabla\Phi$ at the control points on the hull surface the pressure coefficient can be evaluated as

$$C_p = 1 - \left(\frac{\nabla\Phi}{U} \right)^2.$$

The first order velocity potential $\nabla\Phi$ for fixed sinkage and trim is given by

$$\nabla\Phi = U + \nabla\phi_I,$$

and the second order velocity potentials $\nabla\Phi$ for fixed and free sinkage and trim are given respectively as

$$\nabla\Phi = U + \nabla\phi_I + \nabla\phi_2,$$

$$\nabla\Phi = U + \nabla\phi_I + \nabla\phi_2 + s\phi_s + t\phi_t.$$

Now including the waterline integral the wave-making coefficient can be obtained as

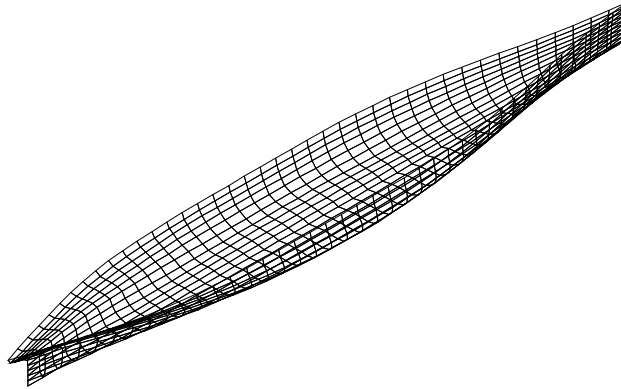
$$C_w = -\frac{\sum_{i=1}^{N_H} C_p n_x \Delta S}{\sum_{i=1}^{N_H} \Delta S} - \frac{\rho g \oint_{WL} \zeta^2 n_x dL}{\rho S U^2} \quad (6.5)$$

where ΔS denotes the area of a panel on the hull surface.

7. Results and discussion

To calculate the wave resistance of a ship-like body, the method has been tested for the Series 60 hull. Since the body is symmetric one-half of the computational domain is used for numerical treatment. The panels from 0.5 ship length upstream to 1.5 ship length downstream cover the free surface domain. The transverse extension of the free surface is about 1.05 ship length. The number of panels on the hull and free surface are taken 40×10 and 70×15 respectively as shown in Fig.2. A three-point upstream difference operator is used in both longitudinal and transverse direction to advect disturbances in the downstream direction. The method employs a clustering of panels on the free surface.

a) hull surface



b) Free surface

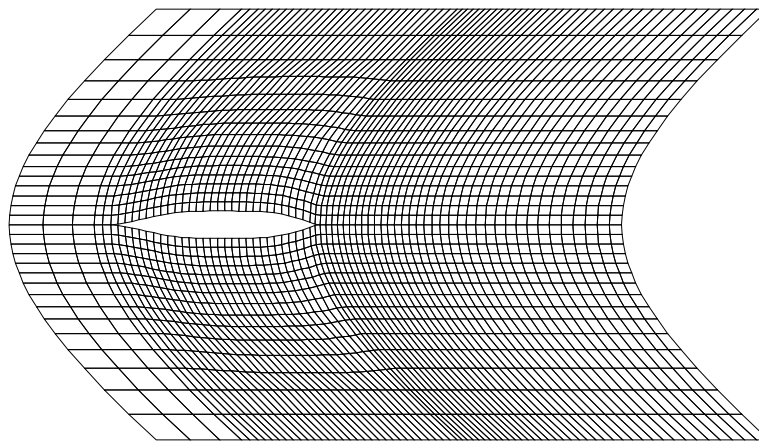


Fig.2. Distribution of panels on the body and free surface.

Figure 3 presents a comparison of the measured and calculated wave profiles based on the second order approximation with free sinkage and trim (abbreviated as *2nd, free*) at different speeds. The main differences are found at the bow and stern region and are likely to have been caused by the following reasons: The wave profiles are taken from the free surface elevations at the panels next to the body, not at the actual hull surface, which resulted in some error especially near the bow and the stern. Another important fact is that the wave profile near the bow and stern region is strongly influenced by the nonlinear terms and the linearized free surface condition may not simulate the exact boundary condition properly.

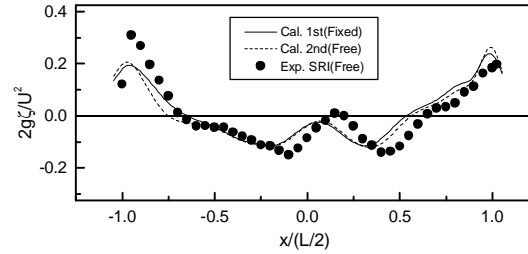
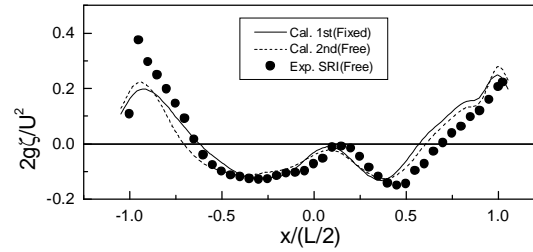
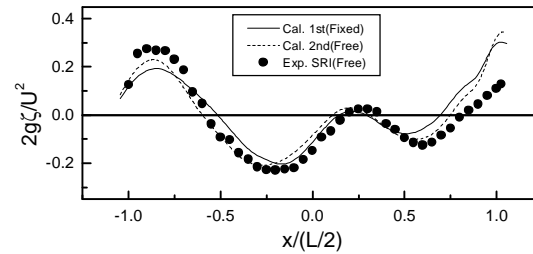
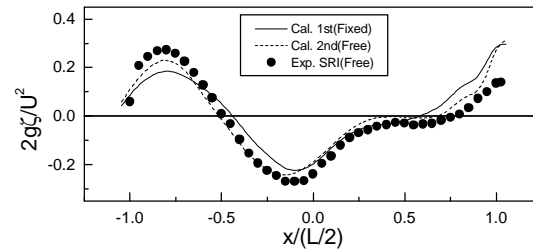
a) Wave profile at $F_n = 0.22$ b) Wave profile at $F_n = 0.25$ c) Wave profile at $F_n = 0.30$ d) Wave profile at $F_n = 0.34$ 

Fig.3. Calculated and measured wave profiles for Series 60 hull.

In Fig.4 the calculated wave-making resistance based on the first and second order approximation with free sinkage and trim in deep water is compared with the experimental results of the Ship Research Institute of Japan (SRI), Ishikawajima-Harima Heavy Industries Co. Ltd. (IHI) and Nippon Kaiji Kyokai (NKK). These experimental results are presented by Takeshi *et al.* (1987). Figures 5 and 6 present a comparison of the calculated sinkage and trim based on second order approximation with the experimental results for the Series 60 hull and agreement is found to be quite satisfactory.

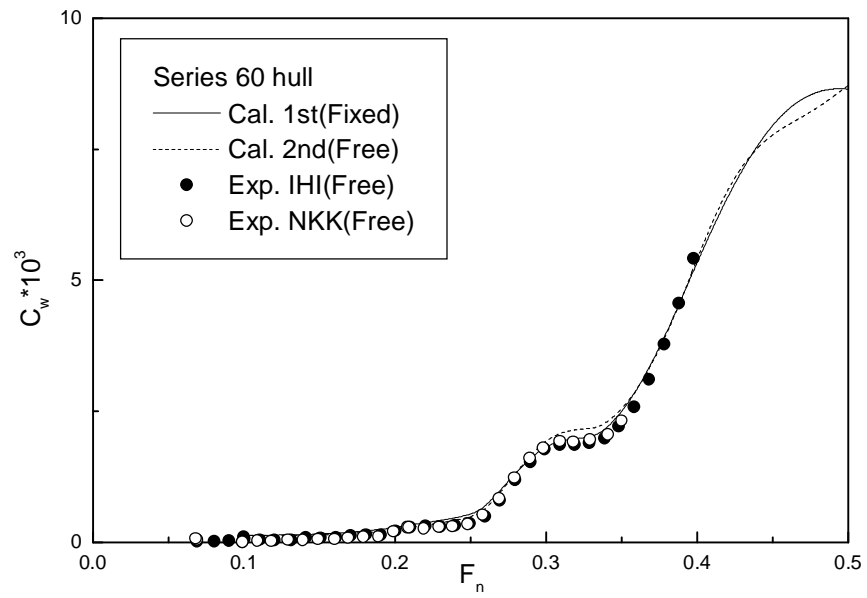


Fig.4. Calculated and measured wave making resistance for Series 60 hull.

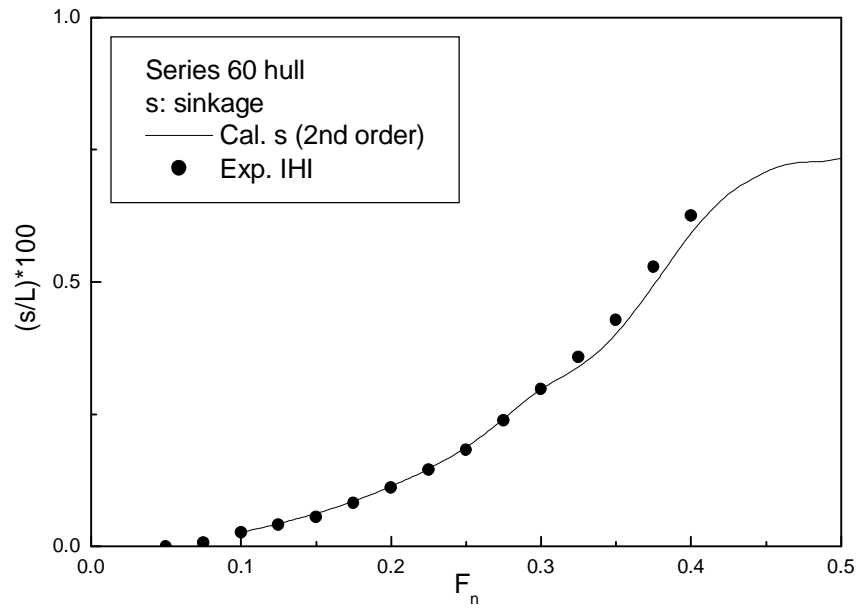


Fig.5. Comparison of calculated and measured sinkage in deep water.

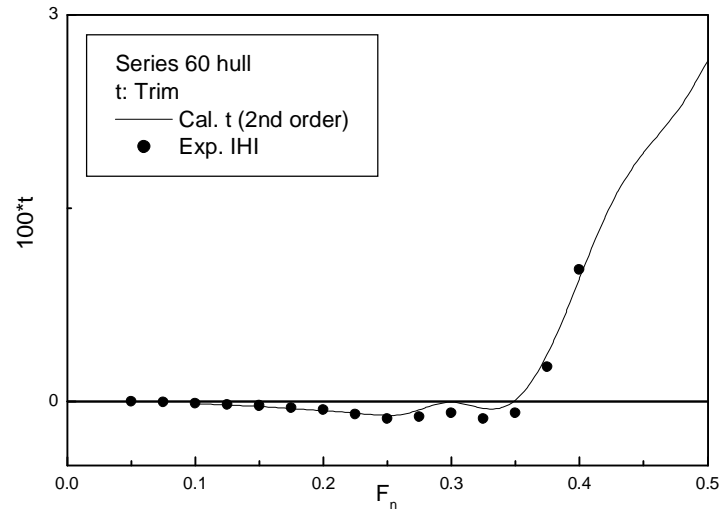
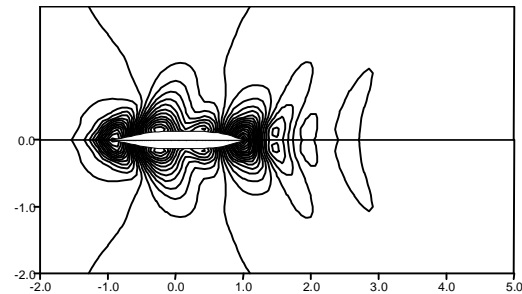


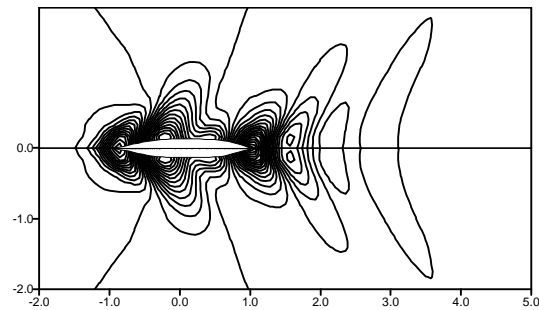
Fig.6. Comparison of calculated and measured trim in deep water.

Figure 7 shows a comparison of the wave pattern for different ship speeds in free to sink and trim condition. As can be seen in these figures, diverging waves are radiating from the bow together with transverse waves following behind the stern of the ship.

a) Wave pattern at $F_n = 0.289$



b) Wave pattern at $F_n = 0.316$



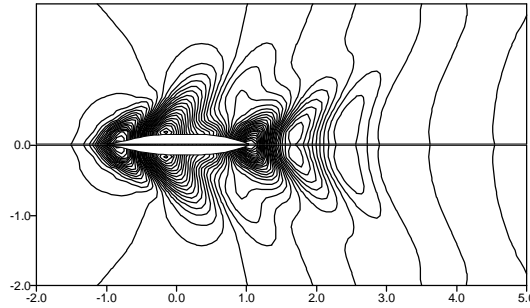
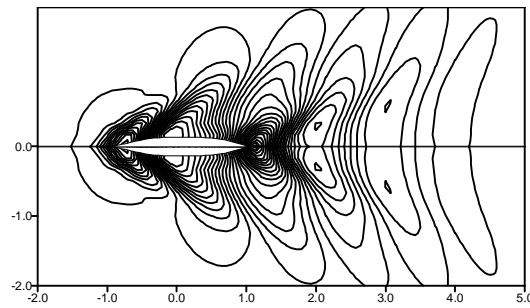
c) Wave pattern at $F_n = 0.35$ d) Wave pattern at $F_n = 0.40$ 

Fig.7. Comparison of wave pattern at different speeds.

8. Conclusions

The present paper numerically investigates the free surface flow around a ship in deep water using Morino's panel method in which the Kelvin classical linearized free surface condition is incorporated. The following conclusions can be drawn from the present numerical analysis:

- The wave-making resistance predicted by the second order solution shows closer agreement with the experiment than that of the first order solution.
- The second order solution significantly improves the wave profiles particularly at the bow and the first trough but after that the difference between the first and second order results seems insignificant.
- The agreement between calculation and measurement tends to become worse with the increase of the Froude number.

Nomenclature

- C_w – wave making co-efficient
 F_n – Froude number
 g – acceleration due to gravity
 G – Green's function
 K_0 – wave number
 L – length of the ship model

- N_H – number of panels on the hull surface
- N_F – number of panels on the free surface
- $R(p; q)$ – position vector between the field point p and the point of singularity q on the surface
- R' – image of R
- S_F – free surface
- S_H – hull surface
- S_R – surface of a large hemishpere
- s – sinkage (positive upward)
- t – trim angle (trim by the stern positive)
- U – uniform velocity in the positive x -direction
- Φ – total velocity potential
- ϵ, δ – perturbation parameter
- ϕ – perturbation velocity potential due to uniform flow
- ϕ_x – velocity of the fluid in the x -direction
- ϕ_y – velocity of the fluid in the y -direction
- ϕ_z – velocity of the fluid in the z -direction
- ϕ_1 – first order perturbation velocity potential
- ϕ_2 – second order contributory part for perturbation velocity potential
- φ – perturbation velocity potential due to sinkage and trim
- φ_s – steady potential due to unit sinkage
- φ_t – steady potential due to unit trim
- ζ – wave elevation
- ζ_1 – first order wave elevation
- ζ_2 – second order contributory part for wave elevation

References

- Azcueta R. (2002): *RANSE simulations for sailing yachts including dynamic sinkage and trim and unsteady motions in waves*. – High Performance Yacht Design Conference, Auckland.
- Bessho M. and Sakuma S. (1992): *Trim and sinkage of two-dimensional shallow draft ships*. – Journal of the Kansai Society of Naval Architects, Japan, No.218.
- Curle N. and Davies H.J. (1968): *Modern Fluid Dynamics*. – D. Van Nostrand Company Ltd., London, vol.1, pp.108-111.
- Dawson C.W. (1977): *A practical computer method for solving ship-wave problems*. – Proceedings of Second International Conference on Numerical Ship Hydrodynamics, pp.30-38.
- Doctors L.J. and Day A.H. (2000): *The squat of a vessel with a transom stern*. – Proceedings of the Fifteenth International Workshop on Water Waves and Floating Bodies (15 IWWF), Israel, pp.40-43.
- Faltinsen O.M. (1993): *Sea Loads on Ships and Offshore Structures*, - Cambridge University Press, pp.110-122.
- Gourlay T.P. and Tuck E.O. (2001): *The maximum sinkage of a ship*. – Journal of Ship Research, vol.45, No.1, pp.50-58.
- Maruo H. (1966): *A note on the higher order theory of thin ships*. – Bulletin of the Faculty of Engineering, vol.15, Yokohama National University, pp.1-21.
- Ogilvie T.F. and Tuck E.O. (1969): *A rational Strip Theory for Ship Motions*, Part 1, Report No. 013. – Department of Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor, Michigan.
- Press W.H., Teukolsky S.A., Vetterling W.T. and Flannery B.P. (1999): *Numerical Recipes in Fortran 77: The Art of Scientific Computing*. – Cambridge University Press, vol.1, pp.35-40.

- Subramani A.K., Paterson E.G. and Stern F. (2000): *CFD calculation of sinkage and trim*. – Journal of Ship Research, vol.44, No.1, pp.59-82.
- Suciu E.O. and Morino L. (1976): *A nonlinear finite element analysis for wings in steady incompressible flows with wake roll-up*. – AIAA Paper, No.76-64, pp.1-10.
- Suzuki K. (1979): *Calculation of ship wave resistance with special reference to sinkage*. – Proceedings of the Workshop on Ship Wave Resistance Computations, vol.2, pp.256-281.
- Takeshi H., Takanori H., Hinatsu M., Tsukada Y. and Fujisawa J. (1987): *ITTC cooperative experiments on a series 60 model at Ship Research Institute- flow measurements and research tests*. – pp.1-47.
- Tuck E.O. (1967): *Sinkage and trim in shallow water of finite width*. – Schiffstechnik, vol.14, pp.92-94.
- Yang C., Lohner R., Noblesse F. and Huang T.T. (2000): *Calculation of ship sinkage and trim using unstructured grids*. – European Congress on Computational Methods in Applied Sciences and Engineering, ECCOMAS 2000.
- Yasukawa H. (1993): *A Rankine panel method to calculate steady wave-making resistance of a ship taking the effect of sinkage and trim into account*. – Transactions of the West Japan Society of Naval Architects, No.86, pp.27-35.

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