Technical note

ELASTIC STABILITY OF SQUARE STIFFENED PLATES WITH CUTOUTS UNDER BIAXIAL LOADING

A.K.L. SRIVASTAVA*
Applied Mechanics Department
N. I. T. Jamshedpur- 831014, INDIA
e-mail: aklsriv_nitjsr@yahoo.com

P.K. DATTA
Aerospace Engineering Department
I. I. T. Kharagpur-721302, INDIA
e-mail: pkdatta@aero.iitkgp.ernet.in

A finite element method for the buckling loads on a longitudinally square stiffened plate with square cutouts is investigated under various combinations of biaxial loading at the plate boundary. The forces are assumed to act in the plane of the undeformed middle surface of the plate. The characteristic equations for the natural frequencies, buckling loads and their corresponding mode shapes are obtained from the equation of motion. The buckling load parameter for various modes of the stiffened plate with square cutouts subjected to in-plane biaxial loads, has been determined for various edge conditions. Numerical results are presented for a range of hole to plate size from *0* to *0.8*. In the structural modeling, the plate and the stiffeners are treated as separate elements where the compatibility between these two types of elements is maintained. The present approach is more flexible than any other finite element modeling in that the mesh division is independent of the location of the stiffeners.

Key words: finite element method, buckling, vibration, cutout, buckling load parameter.

1. Introduction

In aerospace structures, cutouts are commonly found as access ports for mechanical and electrical systems, or simply to reduce weight. Cutouts in aerospace, civil, mechanical and marine structures are inevitable mainly for practical considerations. In addition, the designers often need to incorporate cutouts or openings in a structure to serve as doors and windows. The buckling and vibration analysis of structures with cutouts poses a tremendous challenge and must be properly understood in the structural design. The instability effects are improved with the provision of stiffeners.

An eight noded isoparametric stiffened plate-bending element for the free vibration analysis of a stiffened plate has been presented by Mukherjee and Mukhopadhyay (1988). Here the stiffener can be positioned anywhere within the plate element and need not necessarily be placed on the nodal lines.

Olson and Hazell (1977) have presented a critical study on a clamped integrally stiffened plate by the finite element method. The mode shapes and frequencies have been determined experimentally using the real time holographic technique. Buckling and vibration characteristics of a stiffened plate subjected to in-plane partial edge loading have been studied by Srivastava *et al.* (2002). In the formulation, the stiffener can be positioned anywhere within the plate element and follow the plate beam idealization approach.

The numerical method for computing the natural frequencies of rectangular plates with cutouts can be broadly classified into three categories, namely finite element and finite difference methods, a series type analytical method and the semi analytical approach based on the Rayleigh-Ritz method.

Numerical results obtained by the finite element method have been reported by Ali and Atwal (1987), Shastry and Rao (1977), Reddy (1982) and Laura *et al.* (1986). Paramsivam (1973a) used a finite

* To whom correspondence should be addressed

difference approach in analyzing the effects of openings on the fundamental frequencies of plates with simply supported and clamped boundary conditions. A finite element analysis of clamped thin plates with different cutout sizes, along with experiments was carried out by Monahan ***et al***. (1970). Mundkur ***et al***. (1994) studied the vibration of square plates with square cutouts by using boundary characteristics orthogonal polynomials satisfying the boundary conditions. Chang and Chiang (1988) studied the vibration of the rectangular plate with an interior cutout by using the finite element method. Lee ***et al***. (1990) predicted the natural frequencies of rectangular plates with an arbitrarily located rectangular cutout.

Studies on vibration characteristics of stiffened plates with cutouts are scanty in the literature. The free vibration characteristics of unstiffened and longitudinally stiffened square panels with symmetrically square cutouts are investigated by Sivasubramonian ***et al.*** (1999) using the finite element method. Lam and Hung (1990) studied the vibrations of plates with stiffened openings using orthogonal polynomials and the partitioning method. Natural frequencies of simply supported and fully clamped plates with stiffened openings were presented. Paramsivam and Sridhar Rao (1973b) modified the grid framework model suitably to obtain the natural frequencies of a square plate with stiffened square openings. Recently, dynamic instability of stiffened plates subjected to harmonic in-plane uniform edge loading has been studied by Srivastava ***et al.*** (2003) considering and neglecting in-plane displacements. The element matrices of the stiffened plate element consist of the contribution of the plate and that of the stiffener. The contribution of the beam element is reflected in all nodes of the plate element, which contains the stiffener. Further Srivastava ***et al.*** (2003) extended their work to study the principal dynamic instability behaviour of stiffened plates subjected to non-uniform harmonic in-plane edge loading. The plate skin and the stiffeners are modelled as separate elements but the compatibility between them is maintained. The present paper deals with the effects of various parameters such as the size and location of cutouts, aspect ratios of the plate and cutout, different boundary conditions and stiffener parameters on buckling and vibration characteristics of rectangular stiffened plates with cutouts. The finite element method is applied to analyze the vibration and the buckling behaviour of stiffened plates with cutouts subjected to in-plane uniform biaxial edge loading at the plate boundary. A nine-nodded isoparametric quadratic element with its ability to accommodate curved boundaries is selected for the modelling of the stiffened plate element with cutouts. The main elegance of the formulation lies in the treatment of stiffeners in which the stiffener can be placed anywhere within the plate element which helps to increase considerably the amount of flexibility in the mesh generation.

2. Finite element formulation

The formulation is based on Mindlin's plate theory, which will allow for the incorporation of shear deformation. The element matrices of the stiffened plate element consist of the contribution of the plate and that of the stiffeners. The effect of in-plane deformations is taken into account in addition to the deformations due to bending. A nine-noded isoparametric quadratic element with five degrees of freedom (u, v, w, θ_x, and θ_y) per node is employed in the present analysis. The element matrices of the stiffened plate element consist of the contribution of the plate and that of the stiffener. It reveals that the contribution of the beam element is reflected in all 9 nodes of the plate element, which contains the stiffener. The contribution of the stiffener to a particular node depends on the proximity of the stiffener to that node. For a given edge loading and boundary conditions, the static equation, i.e., $[K]\ \{\delta\}=\{F\}$ is solved to get the stresses. The geometric stiffness matrix is now constructed with the known stresses. The overall elastic stiffness matrix, geometric stiffness matrix and mass matrix are generated from the assembly of those element matrices and stored in a single array where the variable bandwidth profile storage scheme is used. The solution of eigenvalues is performed by the simultaneous iteration technique proposed by Corr and Jenning (1976).

The elastic stiffness matrix $[K_p]$, geometric stiffness matrix $[K_{Gp}]$ and mass matrix $[M_p]$ of the plate element may be expressed as follows

$$\left[K_p\right]=\int_{-1}^{+1}\int_{-1}^{+1}\left[B_p\right]^T\left[D_p\right]\left[B_p\right]\left|J_p\right|d\xi\, d\eta\,, \tag{2.1}$$

$$\left[K_{Gp}\right]=\int_{-1}^{+1}\int_{-1}^{+1}\left[B_{Gp}\right]^T\left[\sigma_p\right]\left[B_{Gp}\right]\left|J_p\right|d\xi\, d\eta\,, \tag{2.2}$$

$$\left[M_p\right]=\int_{-1}^{+1}\int_{-1}^{+1}[N]^T\left[m_p\right][N]\left|J_p\right|d\xi\, d\eta\,. \tag{2.3}$$

The elastic stiffness matrix $[K_S]$, geometric stiffness matrix $[K_{GS}]$ and mass matrix $[M_S]$ of a stiffener element placed anywhere within a plate element and oriented in the direction of x may be expressed, in a manner similar to that of the plate element as follows

$$[K_S]=\int_{-1}^{+1}[B_S]^T[D_S][B_S]\left|J_S\right|d\xi\,, \tag{2.4}$$

$$[K_{GS}]=\int_{-1}^{+1}[B_{GS}]^T[\sigma_S][B_{GS}]\left|J_S\right|d\xi\,, \tag{2.5}$$

$$[M_S]=\int_{-1}^{+1}[N]^T[m_S][N]\left|J_S\right|d\xi\,, \tag{2.6}$$

$$[B_P]=\left[\,[B_P]_1\quad [B_P]_2\quad \mathbf{K}\quad [B_P]_r\quad \mathbf{K}\quad [B_P]_9\right], \tag{2.7}$$

$$[B_{GP}]=\left[\,[B_{GP}]_1\quad [B_{GP}]_2\quad \mathbf{K}\quad [B_{GP}]_r\quad \mathbf{K}\quad [B_{GP}]_9\,\right], \tag{2.8}$$

$$[B_S]=\left[[B_S]_1\quad [B_S]_2\quad \mathbf{K}\quad [B_S]_r\quad \mathbf{K}\quad [B_S]_9\right], \tag{2.9}$$

$$[B_{GS}]=\left[[B_{GS}]_1\quad [B_{GS}]_2\quad \mathbf{K}\quad [B_{GS}]_r\quad \mathbf{K}\quad [B_{GS}]_9\right]. \tag{2.10}$$

The different matrices in the above equations may be written as follows

$$[B_S]_r=\begin{bmatrix} \frac{\partial N_r}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_r}{\partial x} \\ 0 & 0 & \frac{\partial N_r}{\partial x} & -N_r & 0 \end{bmatrix}. \tag{2.11}$$

The equation of equilibrium for the stiffened plate subjected to in-plane loads can be written as

$$[M]\{\ddot{q}\} + [[K_b] - P[K_G]]\{q\} = 0. \qquad (2.12)$$

Equation (2.12) can be reduced to the governing equations for buckling and vibration problems.

3. Results and discussion

We have considered here a rectangular plate $(a \times b)$ with stiffeners having a rectangular cutout of size $(g \times d)$ at the center as shown in Fig.1. The plate with the stiffener subjected to in-plane uniform edge loading at the plate boundary and stiffener cross-section are shown in Fig.2. The loading applied is compressive in nature. The length (a) of the stiffened plate considered above is varied keeping its other parameters unchanged.

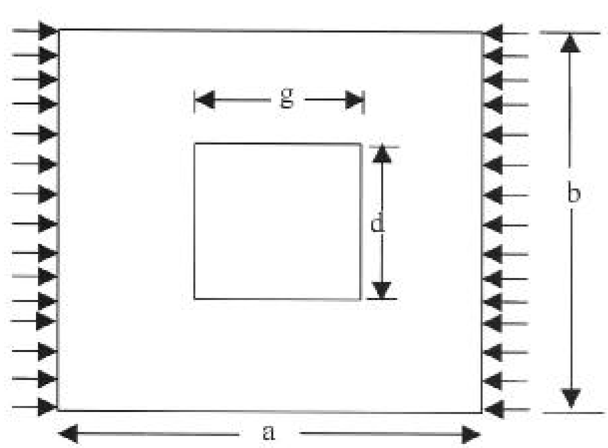

Fig.1. Stiffened plates with cutout under in plane uniform edge loading at plate boundary.

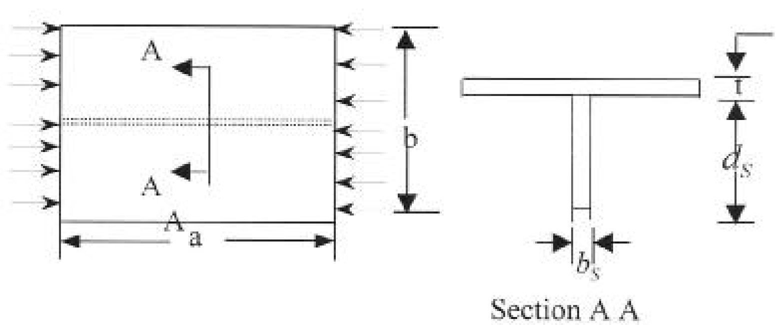

Fig.2. Stiffened plate cross-section.

Numerical results are presented for isotropic stiffened plates with cutouts for simply supported and boundary conditions clamped boundary conditions. In the discussion that followed, S, C denote simply supported, clamped respectively. The non-dimensionalisation of different parameters like vibration, buckling for stability analysis is taken as given below.

Frequency parameter $(\omega) = \varpi b^2 \sqrt{\rho t / D}$ and buckling parameter $(\lambda) = N_X\, b^2 / \pi^2 D$ where D is the plate flexural rigidity, $D = E t^3 / 12(1 - \nu^2)$, ρ is the density of the plate material and t is the plate thickness.

Assuming a general case of several longitudinal ribs and denoting by EI_S the flexural rigidity of a stiffener at a distance (D_x) from the edge $y = 0$, the stiffener parameter terms δ and γ are defined as:
$\delta = A_S / bt$ - ratio of cross-sectional area of the stiffener to the plate, where A_S is the area of the stiffener.
$\gamma = EI_S / bD$ - ratio of bending stiffness rigidity of the stiffener to the plate, where I_S is the moment of inertia of the stiffener cross-section about the reference axis. g/a - ratio of cutout to the plate width.

The presence of the cutout in the plate produces stress concentrations and high stress gradients in the neighbourhood of the cutout, which calls for an extra fineness of the mesh in this zone in the finite element discretization. The buckling load parameter for various modes of a rectangular stiffened plate subjected to biaxial edge loading have been determined for various edge conditions.

3.1. Comparison with previous studies

In order to validate the results, linear fundamental frequencies of a simply supported isotropic square plate with various sizes of a rectangular cutout (g/a) are computed and compared with Mundkur *et al.* (1994) in Tab.1. The predicted changes in frequencies for different cutout sizes agree well with the results of Mundkur *et al.* (1994) given in brackets.

Table 1. Comparison of natural frequency parameter.

	Natural frequency parameter (ω)			
	SSSS		CCCC	
g/a	Mundkur *et al.* (1994)	Present	Mundkur *et al.* (1994)	Present
0.167	*20.070*	*19.87*	*37.425*	*36.06*
0.33	*20.9633*	*20.12*	*43.867*	*43.02*
0.5	*24.2434*	*24.24*	*65.715*	*65.27*

3.2. Buckling studies of stiffened plates with a cutout

Numerical results for non-dimensional buckling load parameters are presented for a stiffened plate with central square cutouts having various boundary conditions. The stiffened plates are subjected to uni axial compressive force N_x for the first case, and biaxial loading with $N_x = N_y$ for the second case study. The corresponding values of N_x and N_y are the buckling loads for the mode shape under consideration.

The stiffened plate with a central square cutout is studied by taking different cutout size ratio g/a . The plate is simply supported at its four edges and the data used for its geometry are $a = 100mm$, $b = 100mm$, $t = 1mm$. The stiffener parameter to be used are as follows: $\delta = 0.1$ and $\gamma = 10$. The other data are as: $\nu = 0.30$, $E = 3.0\times10^7\ N/mm^2$, $\rho = 7.8\times10^{-6}\ Kg/mm^3$.

As a first case, numerical results for buckling load parameter for a stiffened square plate having one central stiffener with a square central cutout of different sizes subjected to uniaxial compressive force for various boundary conditions in various modes are presented in Figs 3-6. Figure 3 shows the variation of the buckling load parameter (λ) for a stiffened plate with one central stiffener subjected to uniaxial load for various boundary conditions, (SSSS, CCCC, CCSS, SSCC). It is observed from Fig.3 that buckling load decreases with the increase of cutout sizes for edge conditions SSSS and SSCC. On the other hand, for edge conditions CCCC and CCSS, it tends to increase for $g/a > 0.4$.

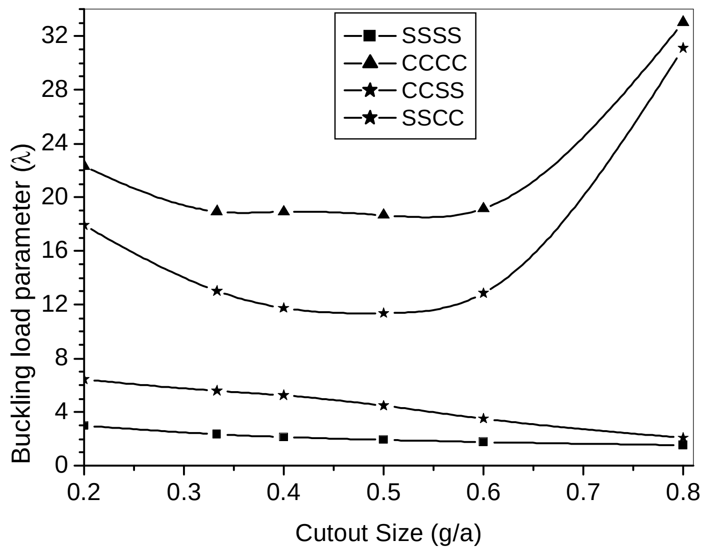

Fig.3. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for uniaxially loaded stiffened plate with one central stiffener ($\delta = 0.1$ and $\gamma = 10$).

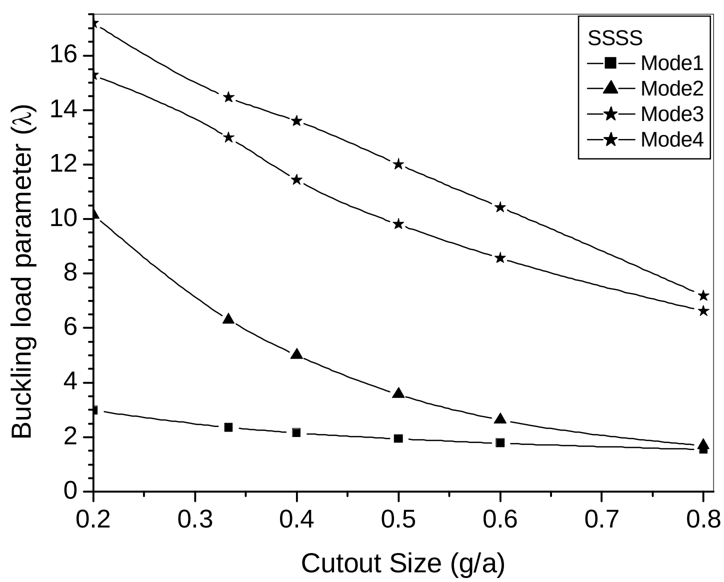

Fig.4. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for uniaxially loaded simply supported stiffened plate with one central stiffener ($\delta = 0.1$ and $\gamma = 10$) in various modes.

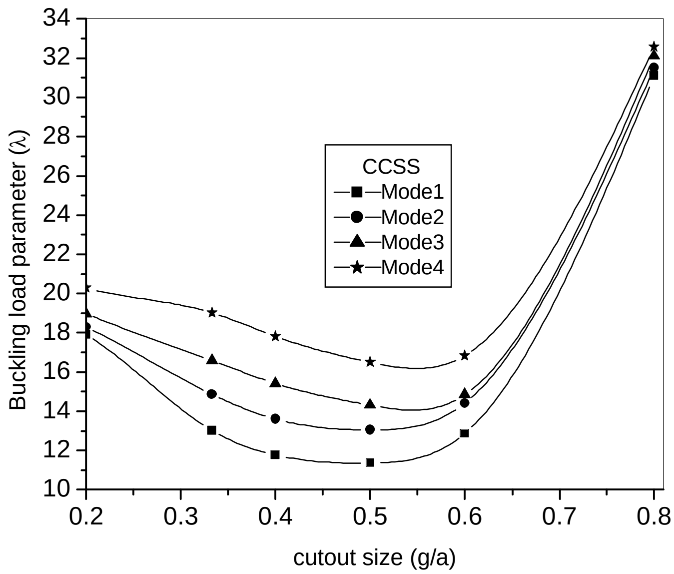

Fig.5. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for uniaxially loaded stiffened plate with one central stiffener ($\delta = 0.1$ and $\gamma = 10$) in various modes. The edges are CCSS.

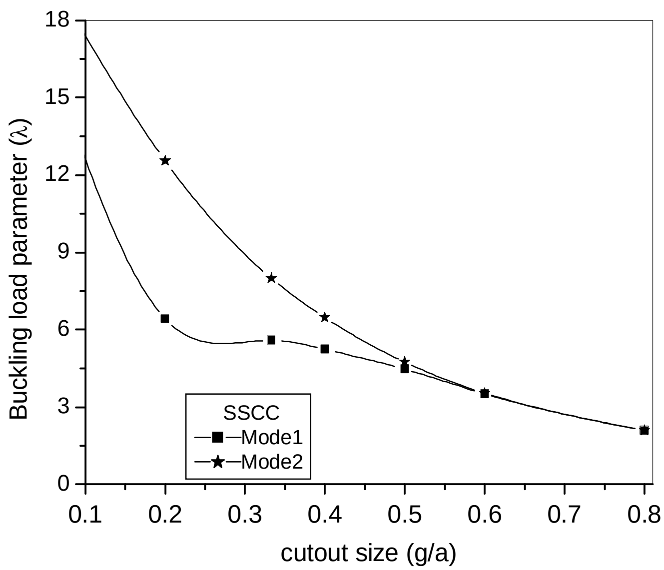

Fig.6. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for uniaxially loaded stiffened plate with one central stiffener ($\delta = 0.1$ and $\gamma = 10$) in various modes. The edges are SSCC.

This variations of the buckling load parameter (λ) with the cutout size for various boundary conditions, (SSSS, CCSS, SSCC) in various modes are shown in Figs 4-6.

As a second case, the effect of bi-axial force on the buckling load parameter for stiffened square plates of the same dimensions as described above with various cutout sizes for various boundary conditions in different modes is analyzed here and the results are presented in Figs 7-8 for unstiffened plates and Figs 9-13 for stiffened plates.

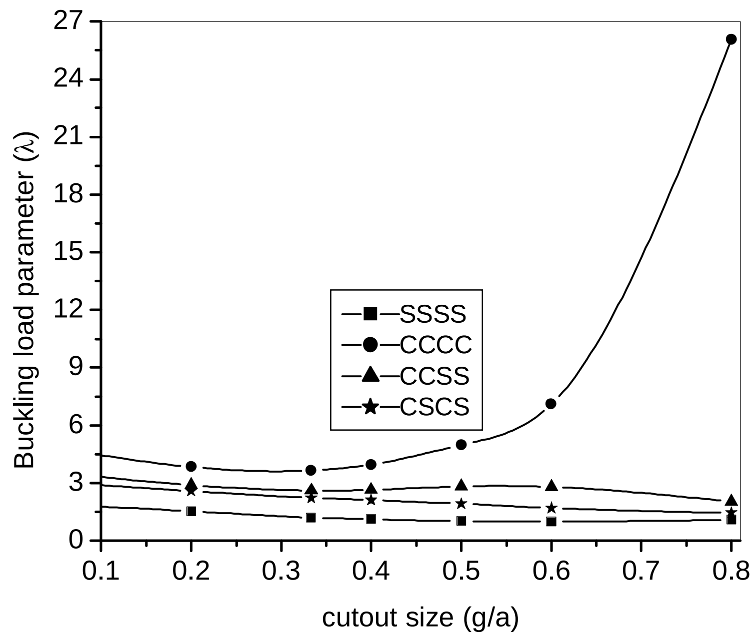

Fig.7. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for biaxially loaded unstiffened plate for different boundary conditions.

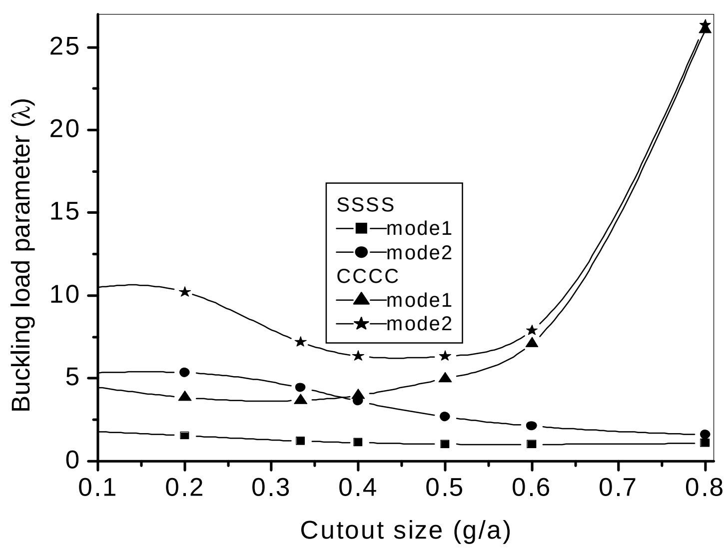

Fig.8. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for biaxially loaded simply supported and clamped unstiffened plate.

It is observed that the variation of the fundamental frequencies with increased in-plane forces is the same as that of uniaxial force in various modes. The curves for the uniaxial and biaxial loadings are identical for normalized compressive forces.

Figure 9 shows the variation of the buckling load parameter (λ) for a stiffened plate with one central stiffener subjected to biaxial load for various boundary conditions, (SSSS, CCCC, CCSS, SSCC). It is observed here that the buckling load decreases with the increase of cutout sizes for SSSS, SSCC and CCSS, but for edge condition CCCC, it increases for the cutout size g/a greater than *0.4*.

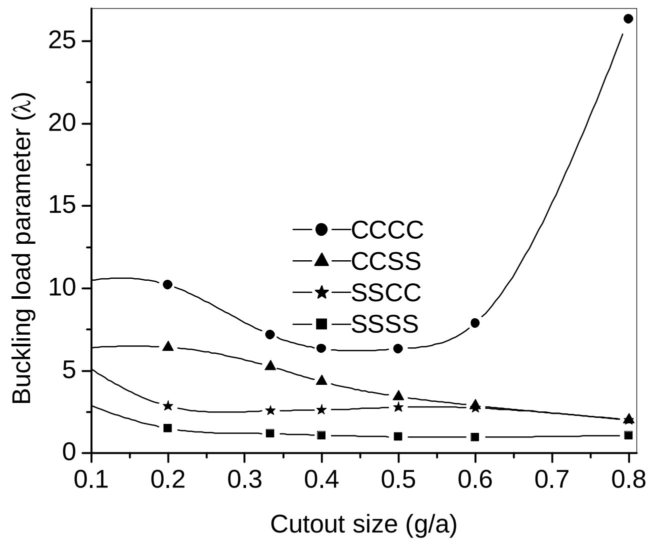

Fig.9. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for biaxially loaded stiffened plate with one central stiffener ($\delta = 0.1$ and $\gamma = 10$).

This variations of the buckling load parameter (λ) with the cutout size for various boundary conditions, (SSSS, CCCC, CCSS, SSCC) in various modes are shown in Figs 10-13.

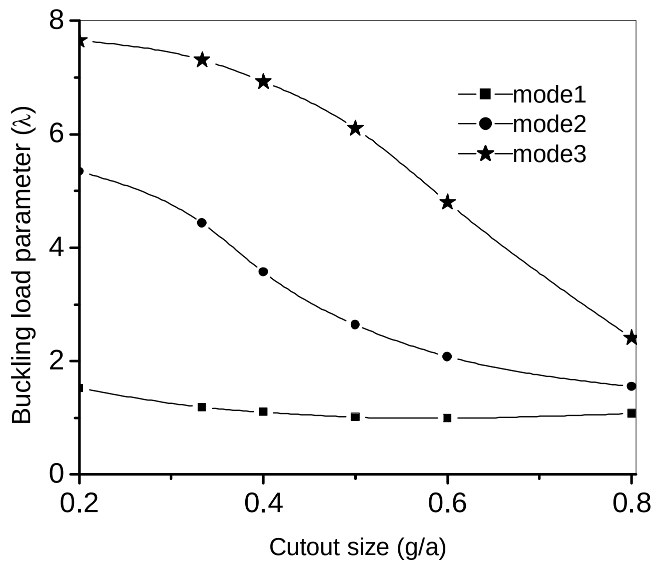

Fig.10. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for biaxially loaded simply supported stiffened plate with one central stiffener ($\delta = 0.1$ and $\gamma = 10$) in various modes.

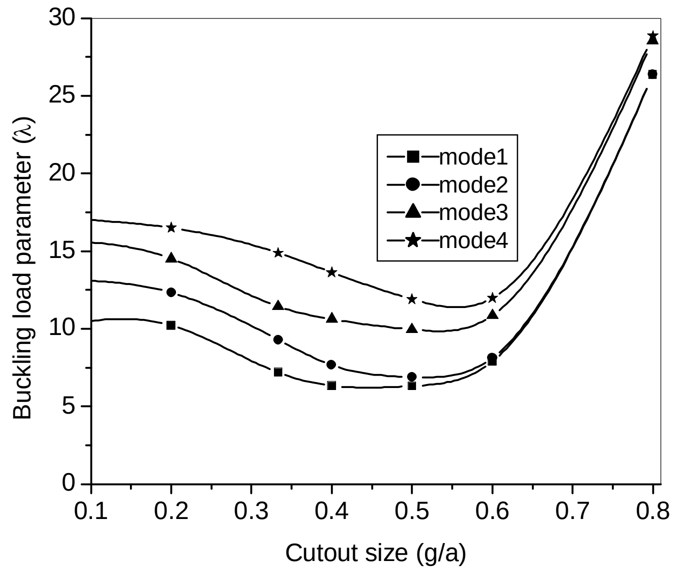

Fig.11. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for biaxially loaded clamped stiffened plate with one central stiffener ($\delta = 0.1$ and $\gamma = 10$) in various modes.

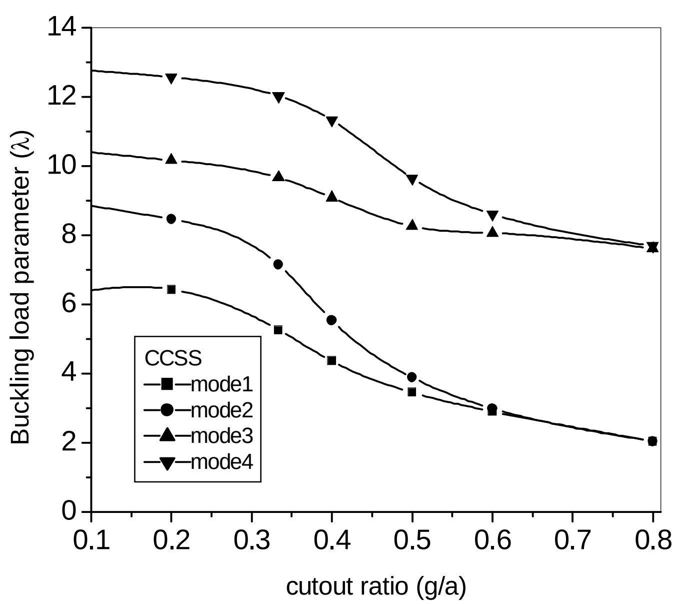

Fig.12. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for biaxially loaded stiffened plate with one central stiffener ($\delta = 0.1$ and $\gamma = 10$) in various modes. The edges are CCSS.

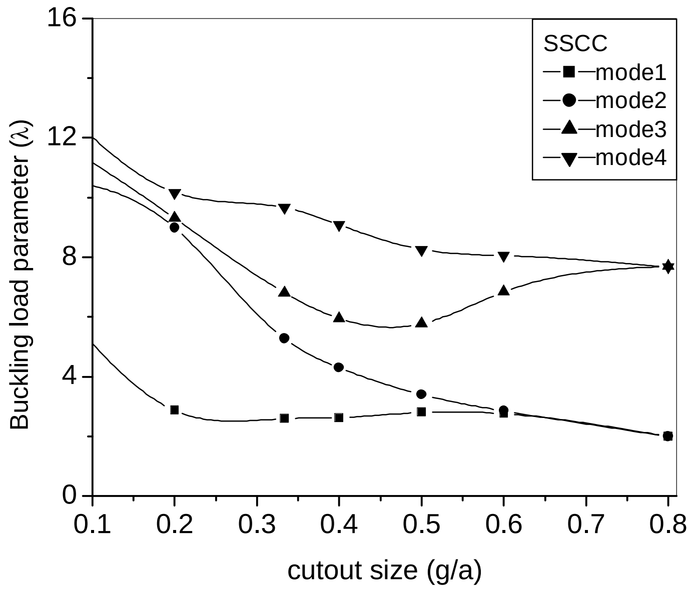

Fig.13. Buckling load parameter (λ) vs. hole/plate ratio (g/a) for biaxially loaded stiffened plate with one central stiffener ($\delta = 0.1$ and $\gamma = 10$) in various modes. The edges are SSCC.

4. Conclusion

The cutouts have considerable influence on the buckling loads, vibration frequencies and mode shapes. The effect is larger in higher modes than in the fundamental mode. The vibration frequencies increase for higher modes due to increased complexity in the mode shapes. The variation of the fundamental frequencies with increased in-plane forces is the same as that of uniaxial force in various modes. The curves for the uniaxial and biaxial loadings are identical for normalized compressive forces.

The buckling load decreases with the increase of cutout sizes for SSSS, SSCC and CCSS, but for edge condition CCCC, it increases for the cutout size g/a greater than 0.4.

The effect of shear deformation is more pronounced in the case of clamped plates than in simply supported plates. Vibration frequencies increase with the increase of restraint at the edges.

Nomenclature

a – plate dimension in longitudinal direction
A_S – cross sectional area of the stiffener
b – plate dimension in the transverse direction
b_s, d_s – web thickness and depth of a x-stiffener
d – cutout width
$[D_P]$ – rigidity matrix of plate
$[D_S]$ – rigidity matrix of stiffener
E, G – Young's and shear moduli for the plate material
g – cutout length
g/d – cutout width ratio
I_S – moment of inertia of the stiffener cross-section about reference axis
$[K_e]$ – elastic stiffness matrix of plate

$[K_G]$ – geometric stiffness matrix
$[K_S]$ – elastic stiffness matrix of stiffener
$[M_p], [M_S]$ – consistent mass matrix of plate, stiffener
$[N]_r$ – matrix of a shape function of a node r
P_{cr} – critical buckling load
P_S – polar moment of inertia of the stiffener element
t – plate thickness
T_S – torsional constant
$\{q\}_r$ – vector of nodal displacement a r^{th} node
υ – Poisson's ratio
ξ, η – non-dimensional element coordinate

References

Ali R. and Atwal S.J. (1987): *Prediction of natural frequencies of vibration of rectangular plates with rectangular cutouts*. – Computer and Structures, vol.12, pp.819-823.

Chang C.D. and Chiang F.K. (1988): *Vibration analysis of a thick plate with an interior cutout by a finite element method*. – Journal of sound and Vibration, vol.125, No.3, pp.477-486.

Corr R.B. and Jenning A. (1976): *A simultaneous iteration algorithms for symmetric eigenvalue problem*. – International Journal of Numerical Method Eng., vol.10, pp.647-663.

Lam K.Y. and Hung K.C. (1990): *Vibration study on plates with stiffened openings using orthogonal polynomials and partitioning method*. – Computer and Structures, vol.33, No.3, pp.295-301.

Laura P.A.A., Utjes J.C. and Palluzzi V.H. (1986): *On the effect of free, rectangular cutouts along the edge on the transverse vibrations of rectangular plates*. – Journal of Applied Mechanics, vol.19, pp.139-151.

Lee H.P., Lim S.P. and Chow S.T. (1990): *Prediction of natural frequencies of rectangular plates with rectangular cutouts*. – Computer and Structures, vol.36, No.5, pp.861-869.

Monahan I.J., Nemergut P.J. and Maddux G.E. (1970): *Natural frequencies and mode shapes of plates with interior cutouts*. – The Shock and Vibration Bulletin, vol.41, pp.37-49.

Mukherjee A. and Mukhopadhyay M. (1988): *Finite element free vibration of eccentrically stiffened plates*. – Computer and Structures, vol.30, pp.1303-1317.

Mundkur G., Bhat R.B. and Neria S. (1994): *Vibration of plates with cutouts using boundary characteristics orthogonal polynomial functions in the Raleigh-Ritz method*. – Journal of Sound and Vibration, vol.176, No.1, pp.136-144.

Olson,M.D and Hazell C.R. (1977): *Vibration studies of some integral rib stiffened plates*. – Journal of Sound and Vibration, vol.50, pp.43-61.

Paramasivan P. (1973a): *Free vibration of square plate with square openings*. – Journal of Sound and Vibration, vol.30, No.2, pp.173-178.

Paramshivam P and Sridhar J.K. (1973b): *Free vibration of square plate with stiffened square openings*. – International Journal of Mechanical Science, vol.15, pp.117-122.

Reddy J.N. (1982): *Large amplitude flexural vibration of layered composite plates with cutouts*. – Journal of Sound and Vibration, vol.83, No.1, pp.1-10.

Shastry B.P. and Rao G.V (1977): *Vibration of thin rectangular plates with arbitrary oriented stiffeners*. – Computer and Structures, vol.7, pp.627-629.

Sivasubramonium B., Rao G.V. and Krishnan A. (1999): *Free vibration of longitudinally stiffened curved panels with cutout*. – Journal of Sound and Vibration, vol.226, No.1, pp.41-55.

Srivastava A.K.L., Datta P.K. and Sheikh A.H. (2002): *Vibration and dynamic stability of stiffened plates subjected to in-plane harmonic edge loading*. – International J of Structural Stability and Dynamics, vol.2, No.2, pp.185-206.

Srivastava A.K.L., Datta P.K. and Sheikh A.H. (2003): *Buckling and vibration of stiffened plates subjected to partial edge loading*. – International Journal of Mechanical Sciences, vol.45, No.1, pp.73-93.

Srivastava A.K.L., Datta P.K. and Sheikh A.H. (2003): *Dynamic stability of stiffened plates subjected to non-uniform harmonic in-plane edge loading*. – Journal of Sound and Vibration, vol.262, No.5, pp.1171-1189.

Received: September 27, 2004

Revised: January 3, 2005