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# VIBRATION RESPONSE OF A DISC BRAKE: EVALUATION AND DESIGN

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Noise and vibration is an increasingly important consideration in the design and study of disc brakes. Certain vibrations may only result in minor annoying squeals, while others may be severe enough to result in structural damage or failure. In either case, it is desirable to predict the conditions under which these vibrations arise, so that they may be controlled, or eliminated.

This paper examines the contributions and interactions of four parameters to vibration response of a brake pad during braking: the applied load, the speed of rotation of the disc, the roughness of the disc and pad, and the Young's modulus of the disc and pad. The experiments are performed by simulating braking on a micro-tribometer. A statistical procedure, $2^k$ factorial design, is used to examine the effects and interactions of these four parameters on the vibration response of the pad in terms of the torque on the disc. Results suggested that the main effects. Disc tangential velocity, followed by Young's modulus, and applied load have the most significant influence. The model also suggested that the roughness is the least significant of the main effects, less significant than some interactions between other main effects, which indicate that the low frequency vibration at low speed is not necessarily associated with friction due to the low significance of the surface roughness.

**Key words:** braking simulation, vibration response of disc brakes.

## 1. Introduction

It has been recognized that friction and vibration have a mutual influence. A number of researchers have used the term "feedback" in studies involving friction-vibration relations. If one views the effect of frictional contact on the structural behavior of a mechanical system as the first effect then it has been shown that the vibration behavior of the mechanical system will in turn affect the frictional contact; the system feedback on friction.

The mechanical engineering literature treats mechanical vibration extensively, but there are relatively few studies of friction-vibration interaction. One of the first notable studies of friction-induced vibration was made by Thomas and Hoersch (1930) who demonstrated the possibility of stick-slip sliding where the force of static friction exceeds the kinetic friction. The same phenomenon was also observed by Bowden and Leben (1939) and by Bowden and Tabor (1964) in their experiments involving the sliding of metals. The assumption that the static friction force is larger than the kinetic friction force, has led researchers, since the early days of the discovery of the stick-slip sliding, to the belief that stick-slip is caused by the difference in their values.

Main theorems related to friction-induced vibration are summarized in Ibrahim and Rivin (1994a) and Ibrahim (1994b). Different forms of friction-induced vibration including stick-slip, chatter, squeal and

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chaos are also discussed. Ibrahim states that stick-slip may occur at low speeds where the motion of one surface sliding along another becomes discontinuous. So one has to assume slipping between contact surfaces and also situations in which the two surfaces in contact may have zero relative velocity. Ibrahim (1994b) concludes that friction-induced vibration is a complicated problem and still requires significant research effort. He advocates research focus on nonlinear and stochastic aspects of friction and its relation to structural behavior.

Brockley and Ko (1970a) discuss the friction-vibration interactions by describing the design and application of a pin-on-disk tribometer. In a second study, Brockley and Ko (1970b) provide an analysis of friction-velocity curves describing conditions that give rise to quasi-harmonic vibration. They found that the conditions for quasi-harmonic vibration are promoted when the friction rises to maximum, then falls off and slowly rises again, as a function of velocity.

A study of the friction-induced vibration and a review of relevant work on the subject are presented in Crolla and Lang (1990). The authors suggest that the friction-induced vibration study mostly comprise works that emphasize friction/velocity characteristics within the two areas of tribology and dynamics of friction systems. Aviles *et al*. (1995a; 1995b) discuss the low-frequency vibrations in disk brakes. They conclude that the vibration occurs when braking is initiated at high speed and under medium pressure. In a companion paper, Aviles *et al*. (1995a), the authors developed a mathematical model for judder in disk brake at highway speed. Ibrahim (1992b) states that in a multi-degree-of-freedom system, the phase differences between the coupled modes can redirect energy to induce vibrations. Earles and Lee (1976) and Krauter (1981) explain the importance of coupling between the system's degrees of freedom in their linear analysis of the generation of squeal in dry friction. The experimental clarification of Aronov *et al*. (1983) is found to support the importance of coupling in friction-induced vibration in their model of a pin sliding on a rotating disk.

A model of a pin-on-disk apparatus with elastic connections is used by Tworzydlo (1994) to study the friction-induced vibration in the form of self-excited oscillations and stick-slip motion. The first clear statement of the importance of the dynamic characteristics and vibrations is due to Tolstoi (1967) and Tolstoi *et al*. (1971). In these efforts kinetic friction was investigated experimentally. They concluded that the interface coefficient of friction does not explicitly depend on sliding velocity and that the difference between the apparent static and kinetic coefficient of friction is the consequence of frictional sliding. The observations were confirmed by experiments of other researchers including Polman and Lehffeldt (1965), Godfrey (1967), Broniec and Lenkiewicz (1980), Aronov *et al*. (1983).

The work presented is compared to the work done by Abdo *et al*. (2000). Due to their relevance to vibration response, the contributions and interactions of the applied load, speed of rotation of the disc, roughness of the disc and pad, and Young's modulus of the disc and pad of a brake pad during braking are considered. Mathematical modeling that relate those factors and experimental verifications are done by Abdo and Farhang (2005) and Abdo (2004; 2005).

## 2. Experimental design

A design of experiment with $k$ factors at two levels was considered for experimentation. The mathematical model on the basis of factorial design was formulated. The factorial design is a statistical method that is used to evaluate the effects and interactions of different independent variables on a response variable. This work utilizes a form of factorial design called $2^k$ factorial design. In the $2^k$ factorial design, $k$ different independent variables are analyzed, each with two different levels. Experiments are run for every combination of factors at each of the two levels. Hence, $2^k$ experiments are needed to observe each of the different conditions for $k$ factors at two levels. The benefit of this type of statistical design is that each of the different factors, or independent variables, may be varied during each experiment, instead of varying only one parameter at a time. Also, this design allows the experimenter to discover interactions between the independent variables, as well as direct effects. The functional relationship between the response parameter and the integrated variables for the postulation model was obtained in the form of a linear equation to interpolate the response by changing the values of controlling parameters.

## 3. Experimental procedure

The experiments to simulate braking were conducted using a Universal Micro Tribometer (UMT). The apparatus is equipped with control features to regulate the normal applied load and spindle speed. Appropriate sensors along with an integrated data acquisition system provide measurement of normal and tangential load on the specimen.

The samples were prepared and examined to study the coefficient of friction based on four independent variables. The UMT consists of a rotating carriage, and a spindle that can be forced against the carriage. A typical arrangement of this machine will find a disc sample fastened to the carriage, with a pin/stylus within the spindle pressing a sample against the disc. The carriage is rotated, and the resulting torque at the spindle is measured. A highly machinable type 304 Stainless Steel disk and specimen with radii *49.1 mm* and *2.7 mm*, respectively, and an Alloy 6061 Aluminum disk and specimen with the same dimensions as Stainless Steel are also used. Testing is done in a pin-on-surface mode where the pin is located at any radius up to *49.1 mm* from the centerline of the spindle. A coaxial suspension with the high lateral stiffness which works effectively in both clockwise and counter-clockwise directions is used. Data is displayed in real time on the monitor and is stored for subsequent review.

## 4. Vibration and the four factors design

In this work, there are four factors to be considered in the prediction of the pad vibration response: the load applied through the pin normal to the disc, the tangential velocity of the disc, the Young's modulus of the pin and disc, and the surface roughness of the disc. They are considered at each of two levels, a high level, and a low level. These levels are coded in such a way that within the design the high level for each factor has a pseudo-value of '*+1*', and the low level a pseudo-value '*–1*'. The factors and their corresponding low and high values are given in Tab.1.

Table 1. Factor levels.

| Factor | Description | Low value $(-1)$ | High value $(+1)$ |
|---|---|---|---|
| A | Applied Load, *N* | *8* | *18* |
| B | Disc Tangential Velocity, *cm/min* | *50* | *500* |
| C | Young's Modulus, *GPa* | *38.71* | *108.24* |
| D | Roughness, $\mu m$ | *10* | *100* |

Figure 1 shows an illustration of the pin-on-disc arrangement. The applied load (factor A), shown by the straight arrow, is a measure of the normal force applied to the pin, and subsequently to the disc. This force is controlled by the tribometer via the software accompanying the machine. The high and low values are *18* and *8* Newtons, respectively. Additionally, the radial distance of the pin from the center of the disc can be varied to allow a virgin surface for experimentation.

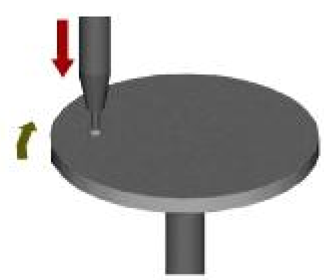


Fig.1. Pin on disc arrangement.

By controlling the rotation of the disc, the tangential velocity of the disc at the friction interface (factor B) can be held constant. The high and low values for the velocity are *500* and *50* centimeters per minute. The high and low values for the Young's modulus for each pin and disc (factor C) are *38.71* and *108.24 GPa* corresponding to an aluminum and a steel pin and disc. The surface roughness for each disc (factor D) is controlled by sanding each pin and disc with a two different grits of sandpaper and measuring the roughness using surface profilometry. The high and low values for roughness are *100* and *10* micro-millimeters (micron).

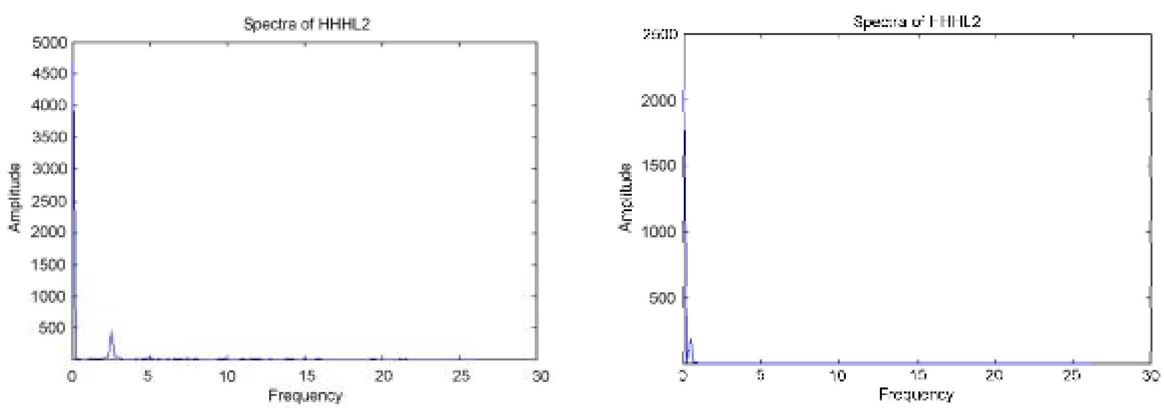
A prediction equation for the response variable can be generated from the design matrix in Tab.2. This equation will be of the form

$$\begin{aligned} y &= f\left(A, B, C, D, AB, AC, \mathbf{K}, ABCD\right) = \\ &= b_0 + b_1 A + b_2 B + b_3 C + b_4 D + b_{12} AB + b_{13} AC + \mathbf{L} + b_{1234} ABCD \end{aligned} \tag{4.1}$$

where *A*, *B*, etc. correspond to the direct effects of each factor on the response variable, and the combinations such as *AB*, *BC*, and *ABC* correspond to interactions between each of the factors. The coefficients, $b_1$, $b_{12}$, $b_{123}$, etc. define the contribution of a particular effect or interaction to the magnitude of the response variable. These coefficients are defined by

$$b_i = \sum_{j=1}^{2^k} \frac{\left(effect_i\right)_j}{2} \left(x_i\right)_j \tag{4.2}$$

where *i* is *0*, *1*, *2*, *3*, *4*, *12*, *13*, etc. corresponding to each combination of factors *A*, *B*, *C*, and *D*. The coefficient $b_0$ is the intercept, or median predicted value of the response variable. The degree of significance of the main effects are determined from the linear terms, $b_i$, and the interaction terms from the bi-linear terms, $b_{ij}$, and higher order products. The coefficients are calculated by adding the product of the effect, or the coded factor value $\left(\pm 1\right)$, for a particular run and the corresponding value of the response variable. Table 2 summarizes the interactions and corresponding coefficients.




a) b)

Fig.2. Two examples of spectra of pin-on-disc friction torque: (a) Spectrum of run *37*, experimental condition 6 (b) Spectrum of run *41*, experimental condition *13*.

Table 2. Interaction coefficients.

| **Order of interaction** | **Coefficient** | **description** |
|---|---|---|
| Zero | $b_0$ | Intercept or median value |
| First | $b_i$ | Main effects: *A*,*B*,*C*, or *D* |
| Second | $b_{ij}$ | 2-factor interactions: *AB*, *BC*, or *BD* etc. |
| Third | $b_{ijk}$ | 3-factor Interactions: *ABC*, *BCD*, or *ACD* etc. |
| . | . | . |
| . | . | . |
| $n^{th}$ | $b_{ijk...}$ | *n* – factor interactions: *ABCD*, etc. |

By coding the independent variables, a design matrix can be generated which is comprised of the different experimental conditions. Each row corresponds to a different combination of factors and levels. Thus, there are $2^k\left(2^4 = 16\right)$ rows. Each column corresponds to the effect of each factor on the design. To eliminate any bias in the preparation of the experiments, they are performed in a random order. All interactions between independent variables are written as linear multiples of the variables. The first two columns of interactions illustrate these multiplications.

It is desirable to perform more than one experiment corresponding to each combination of factor levels. This replication increases the confidence level of the prediction model. For this design there are four replicates for each combination of the factor levels and two center-point measurements used to test for quadratic terms in the low-to-high range. Thus, the total number of rows are

$$N = r\,2k + C \tag{4.3}$$

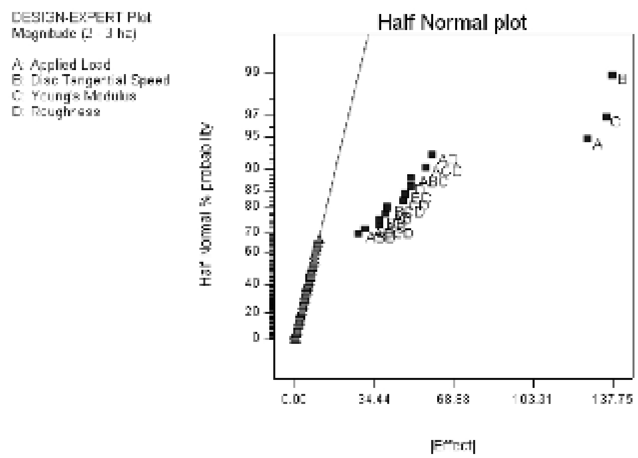
where *k* represents the number of factors, *r* referred to as the number of replicates, and *C* represents the number of center-points. Thus, $N = 4\left(2^4\right) + 2 = 64$.

The experiments revealed a recurring vibration between *2* and *3 Hz*. Figure 2 shows examples of the transformed (FFT) data signal revealing dominant frequencies within the friction torque data. The frequency range of $2-3\,Hz$ is examined for all experiments, and the corresponding magnitude of vibration in this range is recorded as the response value for that experiment.

## 5. Analysis and discussion

A statistical software program is used for performing the statistical calculations and the generation of associated plots. In order to analyze the factorial design, the significant factor effects must be identified and separated from the insignificant effects. Figure 3a is a plot of the effect levels and the corresponding distributions. A line can be drawn through most of the points that represent insignificant effects. Effects not falling on this line correspond to combinations of factors which may have a significant effect on the prediction equation. Figure 3a suggests that the main effects: Disc tangential velocity (B), Young’s modulus (C) and Applied load (A) are the most significant, in that order. Several interactions and the main effect roughness (D) are suggested to be less significant, but not insignificant. It is interesting to compare these results to the result found by Abdo *et al.* (2000) for analysis of the coefficient of friction. In their case, Young’s modulus alone appeared to be the dominant factor (of the four considered) affecting the resulting coefficient of friction. The other main effects are shown to be less significant than one interaction, that between the disc tangential velocity, and the Young's modulus.

a)




b)

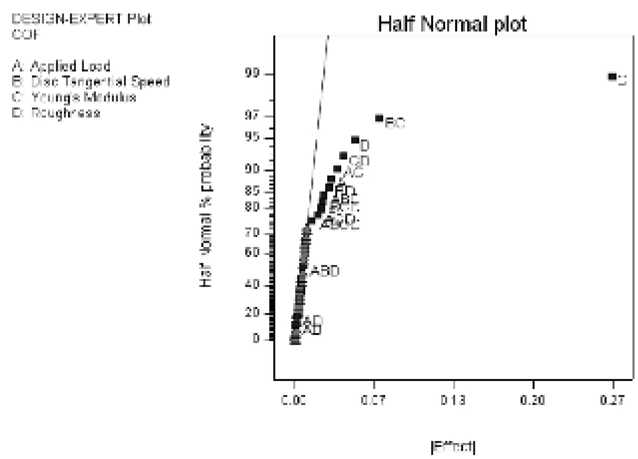



Fig.3. Plot of relative significance of effects: (a) Vibration response between *2* and *3Hz* (b) Coefficient of friction (Abdo *et al.*, 2000).

The software has suggested that the fifteen effects shown above (A through ABCD) may not be insignificant. An F-value for the model shows that the model is significant, and that there is only a ***0.01%*** chance that an F-value this large could be due to noise. Additionally, an 'Adequate Precision' value of *17.958* shows that an adequate signal to the noise ratio exists, since *17.958* is greater than the desired ration of *4*. The two R-squared values show a correlation of greater than *93%* between the prediction model and actual values.

The software provides a prediction equation for pseudo-values $(\pm 1)$. This is the prediction model corresponding to Eq.(4.1) above. One way to evaluate the significance of terms within the prediction equation is to compare the coefficients to the intercept value, and eliminate the terms whose magnitudes are less than a certain percentage of the intercept value, and are therefore not large contributors to the predicted value.

The intercept value is *112.52*, and the smallest coefficient in magnitude is *13.79*, that of the interaction between Young's modulus (C) and Roughness (D). However this coefficient is more than *10%* of the intercept, and should not be eliminated. Additionally, the software provides a prediction equation with the coefficients scaled so that actual values of the different factor levels may be used to predict the magnitude of vibration in the $2-3\,Hz$ range.

Figure 4 is a plot of the actual values versus the values estimated by the prediction equation. Had all values fallen on the straight line shown, there would be a one-to-one correspondence between the prediction model and experiment. Therefore, this plot shows the relative accuracy of the predictive model. Many of the predicted and actual response values are zero and appear as a single point at the lowewr right of the plot.

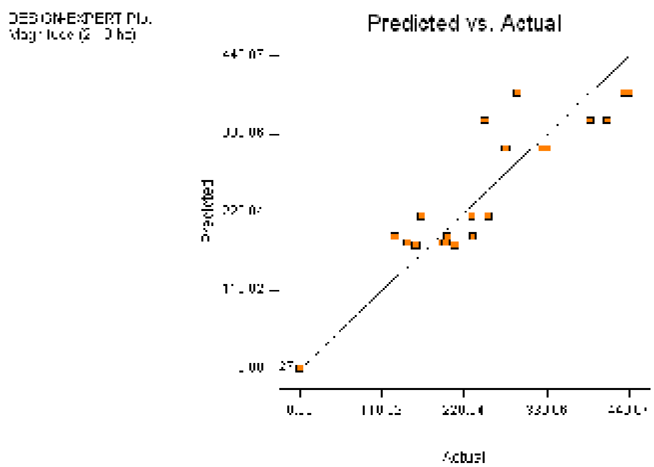



Fig.4. Predicted response vs. actual values.

Figure 5 illustrates the normal plot of the distribution of the difference between the actual and the predicted values (residuals). A plot of residuals that appears linear suggests a normal distribution of these residuals. The residuals corresponding to the predicted and actual response values appears as a vertical line at zero on the abcissa. Again, it is interesting to compare this plot to the same plot generated by Abdo *et al.* (2000) shown in Fig.6, whose plot does suggest a normal distribution of residuals.

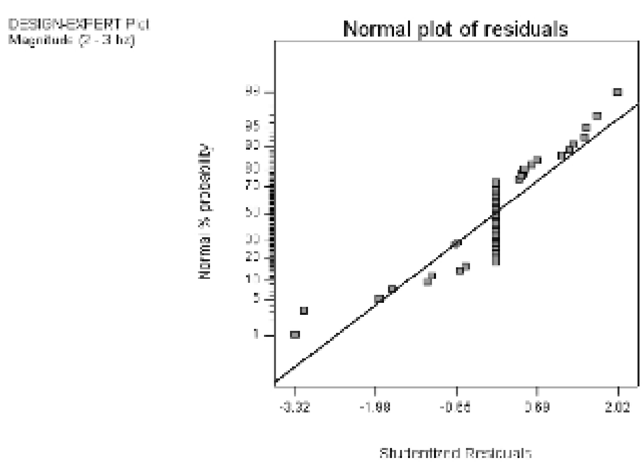



Fig.5. Normal plot of residuals for vibration response $2-3\,Hz$ .

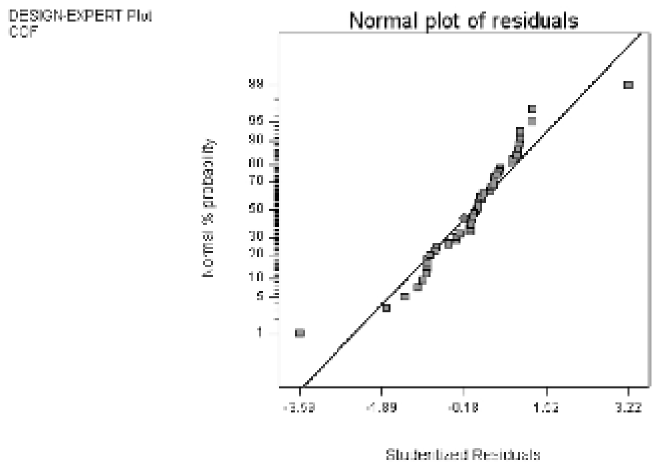



Fig.6. Normal plot of residuals for the COF (Abdo *et al.*, 2000).

The software allows for the visualization of the predicted values under varying levels of each factor by arranging these in the shape of a cube. In four factor factorial design, one factor is held to a specific level while the other three variables are represented by the different axes of the cube. Figure 7 shows an example of this plot with Roughness (Factor D) held to a constant value corresponding to the coded value of *0* which is equivalent to the median between the high value of *100* and the low value of *10*. The plot shows quickly what the predicted value of the response may be for different factor levels. For example, a high value of each factor, with the exception of Roughness which has the median value of *45* micron, the response is expected to be a vibration magnitude of *349.434* between *2* and *3Hz*.

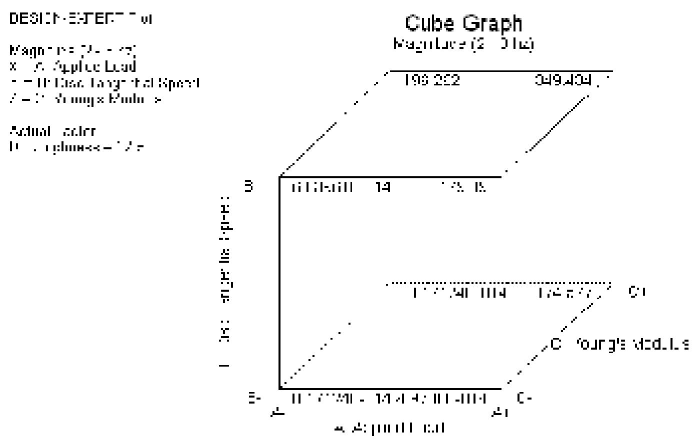
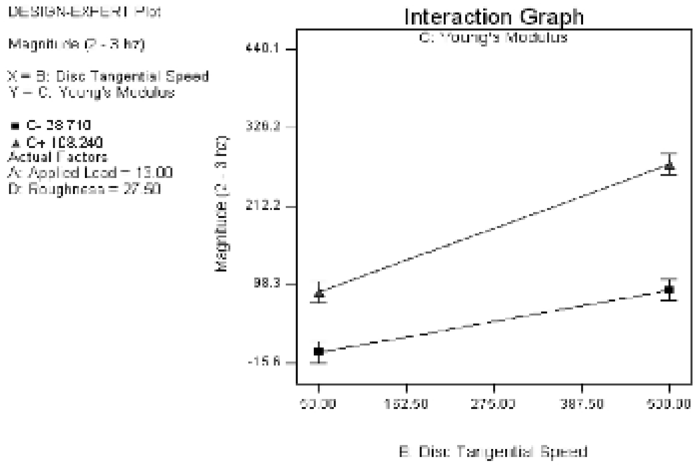
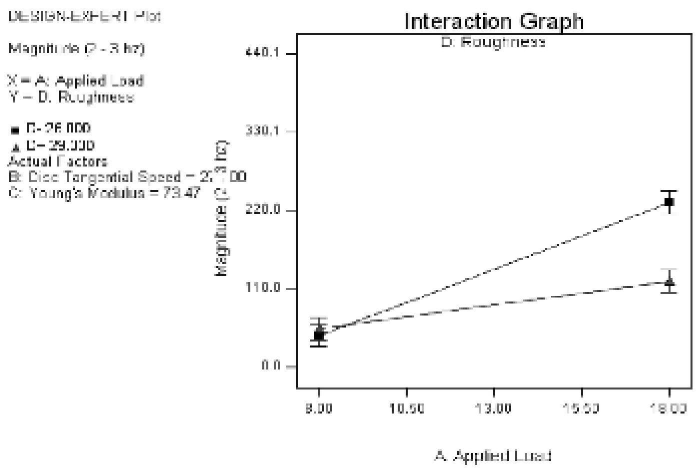


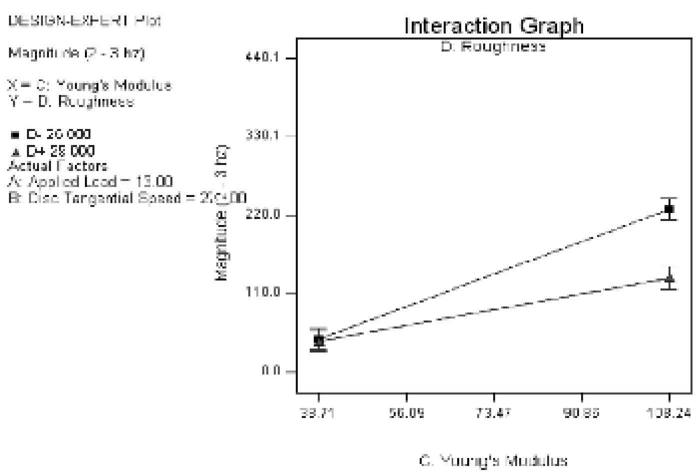

Fig.7. Cube graph with constant roughness.

Another useful plot to consider that is a result of factorial design is the plot of interactions between the main effects. Relative significance of the interactions is determined by comparing the relative difference in magnitude of the response corresponding to the high values of each of two factors, and to the response value of the low values of the same two factors. Visually, this is shown in the plot by connecting the high and low values of each factor, and comparing the line of one factor to the other. If the lines appear to be parallel, the interaction between the two factors can be considered to be insignificant. Figure 8 shows the interaction plots for the response. Each interaction appears to be significant, with the interaction between the

Applied load and Roughness (AD), the Young's modulus and Roughness (CD), and the Disc Tangential Speed (B) and Roughness (D) appearing to be more significant than the other interactions.



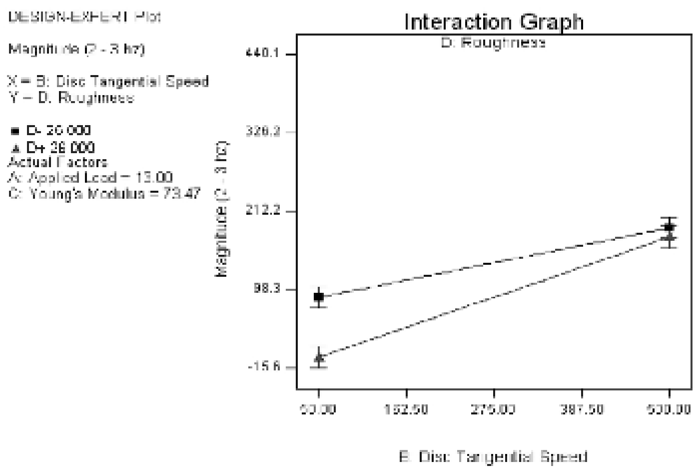



Fig.8. Two factor interactions.

For any given factor value, in between the high and low level, it may be desirable to know what corresponding interacting factor value will produce a certain response. This can be determined by examining iso-response curves. Figure 9 shows the iso-response curve in terms of the interacting effects of the applied load (A) and Roughness (D). For example, for a roughness of *45* microns, to ensure the response magnitude is less than *74.3* it is predicted that an applied load of approximately *9* Newtons or less should be used, when the other two factors, Young's modulus and Disc Tangential Speed, are at median values.

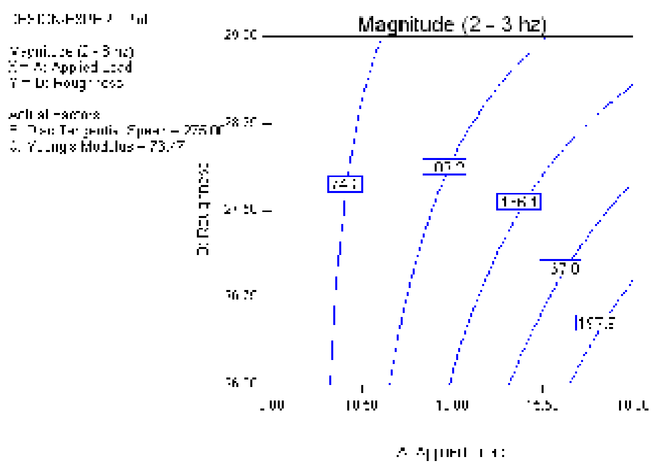



Fig.9. Iso-response curves for applied load and roughness.

## 6. Conclusions

A factorial design model has been presented considering the effect of variations in four factors, the applied load, disc tangential velocity, Young's modulus, and roughness on the resulting magnitude of vibration between *2* and *3Hz*. A prediction model resulted from the statistical calculations that showed the roughness was the least significant of the main effects, less significant than some interactions between other main effects. This is interesting in that, according to the model, low frequency vibration at low speed is not necessarily associated with friction due to the low significance of the surface roughness. However, variation in the friction torque is related to the applied load and Young's modulus, which may be affected by misalignment between the pin and the disc. The software ANOVA table showed that the model has a good correlation with the actual values. Generally, a correlation greater than *95%* is desired.

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