DUFOUR AND SORET EFFECTS ON STEADY FREE CONVECTION AND MASS TRANSFER FLOW PAST A SEMI-INFINITE VERTICAL POROUS PLATE IN A POROUS MEDIUM

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A steady two-dimensional free convection and mass transfer flow past a continuously moving semi-infinite vertical porous plate in a porous medium is studied theoretically, by taking into account the Dufour and Soret effects. The similarity equations of the problem considered are obtained by using the usual similarity technique. The resulting equations are then solved numerically by a shooting method using Runge-Kutta sixth-order integration scheme. The non-dimensional velocity, temperature and concentration profiles are displayed graphically for different values of the parameters entering into the problem. In addition, the skin-friction coefficient, the Nusselt number and Sherwood number are shown in a tabular form.

Key words: free convection, vertical plate, porous medium, steady flow, Dufour and Soret effects.

1. Introduction

Coupled heat and mass transfer by free convection in a porous medium has attracted considerable attention in the last several decades, due to its many important engineering and geophysical applications. Comprehensive reviews on this area have been made by many researchers such as Nield and Bejan (1999), Ingham and Pop (1998; 2002), Bejan and Khair (1985) and Trevisan and Bejan (1990).

In all the above studies, Dufour and Soret effects were neglected, since they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. There are, however, exceptions. The Soret effect, for instance, has been utilized for isotope separation and in a mixture between gases and with very light molecular weight (H_2, He) and of medium molecular weight (H_2, air) the Dufour effect was found to be of considerable magnitude such that it cannot be neglected (Eckert and Drake, 1972). Considering these aspects, model studies were carried out by many investigators of whom the names of Dursunkaya and Worek (1992), Kafoussias and Williams (1995), Anghel *et al*. (2000) and Postelnicu (2004) are worth mentioning.

In view of the above studies, our aim is to study the steady two-dimensional problem of free convection and mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium taking into account the Dufour and Soret effects.

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2. Mathematical analysis

A two-dimensional steady free convection and mass transfer flow of a viscous incompressible fluid past a continuously moving semi-infinite vertical porous flat plate in a porous medium is considered. The flow is assumed to be in the *x*-direction which is taken along the plate in the upward direction and the *y*-axis is taken normal to it. The coordinate system and the flow configuration are shown in the following Fig.1.

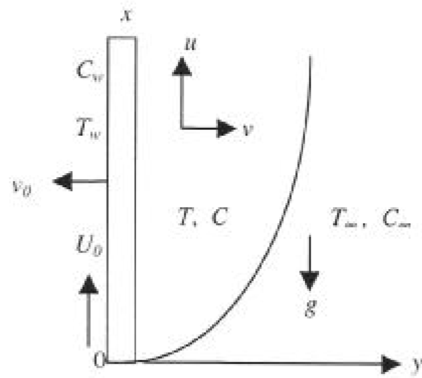

Fig.1. Flow configuration.

Then under the usual Boussinesq's approximation, the governing equations relevant to the problem are:
Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right)-\frac{\upsilon}{K'}u, \tag{2.2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \tag{2.3}$$

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{2.4}$$

where *u*, *v* are the velocity components in the *x* and *y* directions respectively, ν is the kinematic viscosity, *g* is the acceleration due to gravity, ρ is the density, β is the coefficient of volume expansion, β^* is the volumetric coefficient of expansion with concentration, T, T_w and T_∞ are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, while C, C_w and C_∞ are the corresponding concentrations. Also, K' is the permeability of a porous medium, α is the thermal diffusivity, D_m is the coefficient of mass diffusivity, c_p is the specific heat at constant pressure, T_m is the mean fluid temperature, k_T is the thermal diffusion ratio and c_s is the concentration susceptibility.

The boundary conditions for the model are

$$\left.\begin{array}{llllll} u = U_0, & \nu = \nu_0(x), & T = T_w, & C = C_w & \text{at} & y = 0, \\ u = 0, & \nu = 0, & T = T_\infty, & C = C_\infty & \text{as} & y \to \infty \end{array}\right\} \tag{2.5}$$

where U_0 is the uniform velocity and $\nu_0(x)$ is the velocity of suction at the plate.

We now introduce the following dimensionless variables

$$\left.\begin{array}{l} \eta = y\sqrt{\dfrac{U_0}{2\upsilon x}} \\ \psi = \sqrt{\upsilon x U_0}\, f(\eta) \\ \theta(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty} \\ \phi(\eta) = \dfrac{C - C_\infty}{C_w - C_\infty} \end{array}\right\} \tag{2.6}$$

where $f(\eta)$ is the dimensionless stream function and ψ is the dimensional stream function defined by $u = \dfrac{\partial\psi}{\partial x}$ and $v = -\dfrac{\partial\psi}{\partial y}$, just to satisfy the equation of continuity (2.1).

Then introducing the relation (2.6), into Eq.(2.1) we obtain

$$u = U_0 f'(\eta) \qquad \text{and} \qquad \nu = \sqrt{\frac{\nu U_0}{2x}}\left(\eta f' - f\right). \tag{2.7}$$

Further introducing Eqs (2.6) and (2.7) into Eqs (2.2)-(2.4) we respectively have

$$f''' + ff'' + \mathrm{Gr}\theta + \mathrm{Gm}\phi - Kf' = 0, \tag{2.8}$$

$$\theta'' + \mathrm{Pr} f\,\theta' + \mathrm{PrDf}\,\phi'' = 0, \tag{2.9}$$

$$\phi'' + \mathrm{Sc} f \phi' + \mathrm{ScSr}\theta'' = 0 \tag{2.10}$$

where $\mathrm{Gr} = \frac{g\beta(T_w - T_\infty)2x}{U_0^2}$ is the Grashof number, $\mathrm{Gm} = \frac{g\beta^*(C_w - C_\infty)2x^2}{\nu U_0}$ is the modified Grashof number, $K = \frac{2\nu x}{K' U_0}$ is the permeability parameter, $\mathrm{Pr} = \frac{\nu}{\alpha}$ is the Prandtl number, $\mathrm{Sc} = \frac{\nu}{D_m}$ is the Schmidt number, $\mathrm{Sr} = \frac{D_m k_T (T_w - T_\infty)}{\nu T_m (C_w - C_\infty)}$ is the Soret number, and $\mathrm{Df} = \frac{D_m k_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)}$ is the Dufour number.

The corresponding boundary conditions are

$$\left.\begin{aligned} & f = f_w, \quad f' = 1, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0, \\ & f' = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{as} \quad \eta \to \infty \end{aligned}\right\} \tag{2.11}$$

where $f_w = -\nu_0 \sqrt{\frac{2x}{\nu U_0}}$ is the dimensionless suction velocity and primes denote partial differentiation with respect to the variable η.

The parameters of engineering interest for the present problem are the local skin-friction coefficient, the local Nusselt number and the local Sherwood number, which are given respectively by the following expressions

$$\frac{1}{2}\mathrm{Re}^{\frac{1}{2}} C_f = f''(0), \tag{2.12}$$

$$\mathrm{Nu}(\mathrm{Re})^{-\frac{1}{2}} = -\theta'(0), \tag{2.13}$$

$$\mathrm{Sh}(\mathrm{Re})^{-\frac{1}{2}} = -\phi'(0) \tag{2.14}$$

where $\mathrm{Re} = \frac{U_0 x}{\nu}$ is the Reynold's number.

The set of Eqs (2.8)-(2.10) together with the boundary conditions (2.11) have been solved numerically by applying Nachtsheim-Swigert (1965) shooting iteration technique along with Runge-Kutta sixth-order integration method. From the process of numerical computation, the skin-friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$, are also sorted out and their numerical values are presented in a tabular form.

The parameters for the present problem are the local Nusselt number and the local Sherwood number, which indicate physically the rate of heat transfer and the rate of mass transfer respectively.

Now the heat flux (q_w) and the mass flux (M_w) at the wall are given by

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k\Delta T \sqrt{\frac{U_\infty}{\nu x}}\theta'(0),$$

and $$M_w = -D_m \left(\frac{\partial C}{\partial y} \right)_{y=0} = -D_m \, \Delta C \sqrt{\frac{U_\infty}{\nu x}} \phi'(0)$$

where $\Delta T = T_w - T_\infty$ and $\Delta C = C_w - C_\infty$.

Hence the Nusselt number (Nu) and Sherwood number (Sh) are obtained as

$$\mathrm{Nu} = \frac{x q_w}{k \Delta T} = -(\mathrm{Re})^{\frac{1}{2}} \theta'(0),$$

i.e., $$\mathrm{Nu}(\mathrm{Re})^{-\frac{1}{2}} = -\theta'(0),$$

and $$\mathrm{Sh} = \frac{x M_w}{D_m \Delta C} = -(\mathrm{Re})^{\frac{1}{2}} \phi'(0),$$

i.e., $$\mathrm{Sh}(\mathrm{Re})^{-\frac{1}{2}} = -\phi'(0).$$

3. Results and discussion

During the course of discussion about the effects of various parameters on the flow field the following considerations are made:

(i) The value of Prandtl number Pr is taken equal to *0.71*, which corresponds, physically to air.

(ii) The value of Schmidt number Sc is chosen *0.22*, which represents hydrogen at approx. $T_m = 25^{\mathbf{0}} C$ and *1 atm*.

(iii) The values of Dufour number Df and Soret number Sr are chosen in such a way that their product is constant provided that the mean temperature T_m is kept constant as well.

(iv) Finally, the values of Grashof number Gr, modified Grashof number Gm, suction parameter f_w and permeability parameter *K* are chosen arbitrarily.

Under the above assumptions our results are shown in Figs 2-10 and in Tab.1. The effects of Grashof number and permeability parameter on the velocity field are shown in Fig.2. It is seen from this figure that the velocity decreases with the increase of permeability parameter while the increase of Grashof number (or increase of free convection current) increases the velocity, which is usually expected. In Fig.3, the effects of modified Grashof number and suction parameter on the velocity field are shown. Figure 3 thus shows that the velocity increases when the concentration difference between the mean and free stream values increases whereas the velocity decreases with the increase of suction parameter.

Table 1. Skin-friction coefficient, Nusselt number and Sherwood number for $\mathrm{Gr} = 10$, $\mathrm{Gm} = 4$, $f_w = 0.5$, $K = 0.3$, $\mathrm{Pr} = 0.71$ and $\mathrm{Sc} = 0.22$.

Df	Sr	C_f	Nu	Sh
0.030	*2.0*	*6.2285*	*1.1565*	*0.1531*
0.037	*1.6*	*6.1491*	*1.1501*	*0.2283*
0.050	*1.2*	*6.0720*	*1.1428*	*0.3033*
0.075	*0.8*	*6.0006*	*1.1333*	*0.3781*
0.150	*0.4*	*5.9553*	*1.1157*	*0.4540*

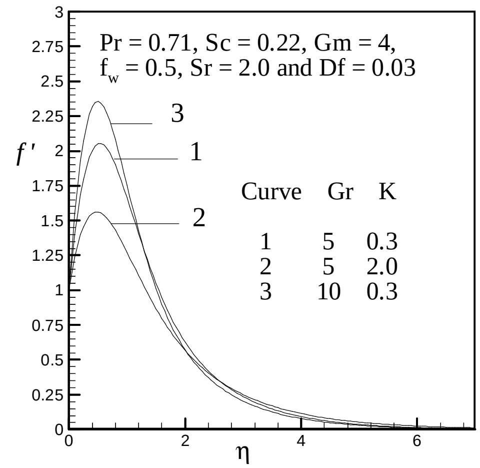

Fig.2. Velocity profiles for different values of Gr and *K*.

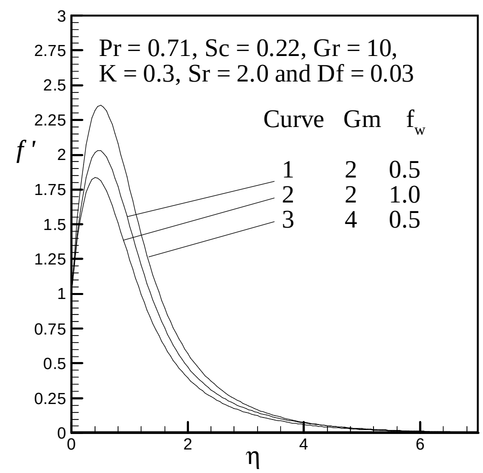

Fig.3. Velocity profiles for different values of Gm and f_w.

The influence of Soret number Sr and Dufour number Df on the velocity field is shown in Fig.4. Quantitatively, when $\eta = 0.90$ and Sr decreases from *2* to *0.4* (or Df increases from *0.03* to *0.15*), there is *6.2%* decrease in the velocity value, whereas the corresponding decrease is *10.95%* when Sr decreases from *0.4* to *0.1*.

The effects of Grashof number and permeability parameter on the temperature field are shown in Fig.5. From this figure we observe that the temperature increases with the increase of permeability parameter and decreases with the increases of free convection current. The effects of modified Grashof number and suction parameter on the temperature profiles are shown in Fig.6. This figure shows that the temperature decreases with the increase of both the suction parameter and modified Grashof number. From Fig.7 when $\eta = 1.5$ and Sr decreases from *2* to *0.4* (or Df increases from 0.03 to 0.15), there is *27.70%* increase in the temperature value, whereas the corresponding increase is *48.78%* when Sr decreases from *0.4* to *0.1*.

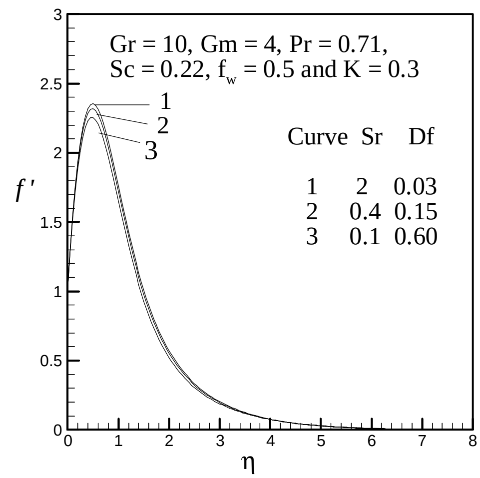

Fig.4. Velocity profiles for different values of Sr and Df.

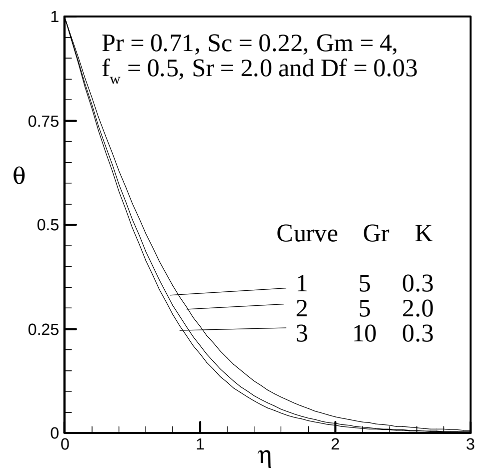

Fig.5. Temperature profiles for different values of Gr and *K*.

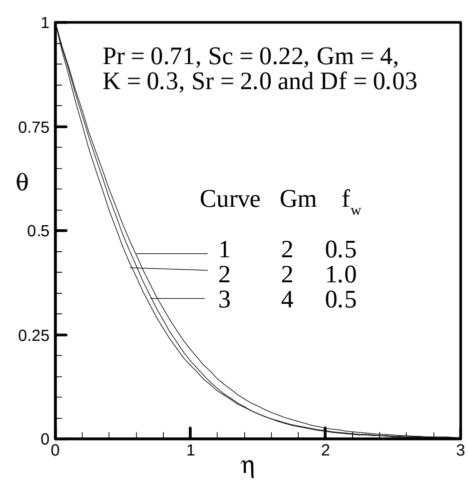

Fig.6. Temperature profiles for different values of Gm and f_w.

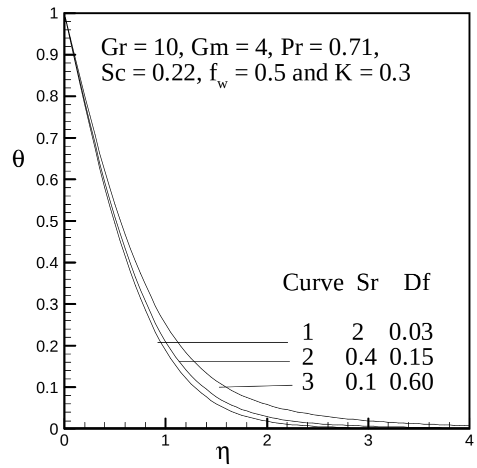

Fig.7. Temperature profiles for different values of Sr and Df.

The effects of Grashof number and permeability parameter on the concentration field are shown in Fig.8. From this figure we observe that the concentration increases with the increase of permeability parameter and decreases with the increases of free convection current. The effects of modified Grashof number and suction parameter on the concentration profiles are shown in Fig.9. This figure shows that the concentration decreases with the increase of both the suction parameter and modified Grashof number. From Fig.10 when $\eta = 1.0$ and Sr decreases from *2* to *0.4* (or Df increases from 0.03 to 0.15), there is *27.47%* decrease in the concentration value, whereas the corresponding decrease is *7.42%* when Sr decreases from *0.4* to *0.1*.

Finally, the effects of Soret and Dufour number on skin-friction coefficient, Nusselt number and Sherwood number are shown in Tab.1. The behaviour of these numbers is self-evident from Tab.1 and hence any further discussion about them seems to be redundant.

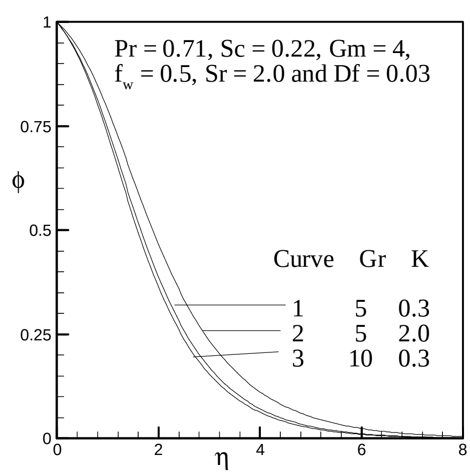

Fig.8. Concentration profiles for different values of Gr and *K*.

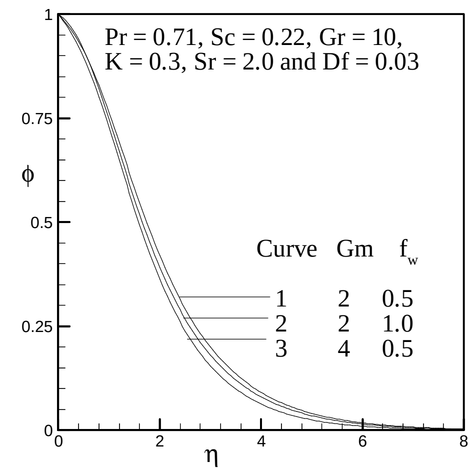

Fig.9. Concentration profiles for different values of Gm and f_w.

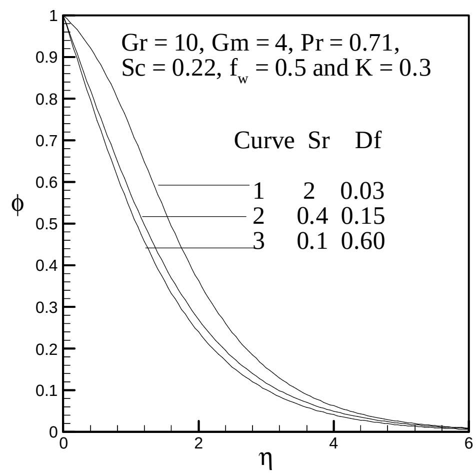

Fig.10. Concentration profiles for different values of Sr and Df.

4. Conclusion

In this paper, the Dufour and Soret effects on a steady free convection and mass transfer flow past a continuously moving semi-infinite vertical porous flat plate in a porous medium have been studied numerically for a hydrogen-air mixture as a non-chemical reacting fluid pair. The analysis has shown that the temperature and concentration fields is appreciably influenced by the Dufour and Soret effects. Therefore, we can conclude that for fluids with medium molecular weight (H_2, air), the Dufour and Soret effects should not be neglected.

Nomenclature

C – concentration
c_p – specific heat at constant pressure
c_s – concentration susceptibility
D_m – mass diffusivity

Df – Dufour number
f – dimensionless stream function
f_w – dimensionless suction velocity
g – acceleration due to gravity
Gr – Grashof number
Gm – modified Grashof number
K – permeability parameter
k_T – thermal diffusion ratio
Nu – Nusselt number
Pr – Prandtl number
Re – Reynod's number
Sc – Schmidt number
Sh – Sherwood number
Sr – Soret number
T – temperature
T_m – mean fluid temperature
U_0 – uniform velocity
u, v – velocities in the x and y-direction respectively
x, y – Cartesian coordinates along the plate and normal to it, respectively
α – thermal diffusivity
β – coefficient of thermal expansion
β^* – coefficient of concentration expansion
ρ – density of the fluid
ν – kinematic viscosity
θ – dimensionless temperature
ϕ – dimensionless concentration

Subscripts

w – condition at wall
∞ – condition at infinity

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