Int. J. of Applied Mechanics and Engineering, 2006, vol.11, No.3, pp.547-556

A STUDY OF ROTATIONAL GAS FLOWS

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We decompose the compatibility equations of steady rotational gas flows and obtain solutions in three dimensional flows and also Mach numbers on the surfaces (i) small spherical balls (ii) in circular tunnels. The cavity ratio for the critical Mach number is discussed using hodograph transformation.

Key words: rotational flows, Mach number, hodograph transformation.

1. Introduction

The study of plane incompressible potential flow has been developed to a consistent and fairly complete mathematical theory. The determination of path profiles is an extremal problem in fluid dynamics. Among all profiles with a given ratio of width to length, we determine the shape of the profile for which the maximum critical Mach number is achieved. The critical Mach number is the value of the incident Mach number for which the sonic velocity is first attained on the profile. Fisher (1962) obtained the Mach number of subsonic cavities with sonic free streamlines. Bagewadi and Siddabasappa (1993) studied the plane rotating viscous MHD flows by using differential geometry techniques. Prim (1953) studied steady rotational flow of ideal gases. Further, Chandna and Smith (1971) studied some steady plane rotational flows of gases with an arbitrary equation of state. In this paper, we obtain the Mach number for rotational flows.

This paper is organized as follows: in section 2, the compatibility equations are stated and the decomposition of the equations for three dimensional flow is carried out to get the value of the Mach number. In section 3, the Mach number is obtained for the flow emanating from spherical ball and inside a circular tunnel. In section 4, the cavity ratio is calculated to obtain the values of the Mach number using a hodograph transformation.

2. Compatibility equations

In the absence of external forces and heat conduction, the system of equations governing steady motion of a compressible fluid is as follows (Berker, 1956)

$$\rho \boldsymbol{a} = -\text{grad}\, p \qquad \text{(Euler's dynamical equation)}, \tag{2.1}$$

$$\text{div}(\rho \boldsymbol{q}) = 0 \qquad \text{(continuity)}, \tag{2.2}$$

$$\boldsymbol{q} \cdot \text{grad}\, s = 0 \qquad \text{(energy)}, \tag{2.3}$$

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$$\rho = \rho(p, s) \qquad \text{(equation of state)} \tag{2.4}$$

where ρ is the density of fluid, p is the pressure, s is the specific entropy, $\boldsymbol{q}$ **is** the velocity vector, $\boldsymbol{a}$ is the acceleration vector.

From Eqs (2.1) and (2.2), we have

$$\boldsymbol{a} \cdot \operatorname{curl} \boldsymbol{a} = 0\,, \tag{i}$$

$$\operatorname{curl} \boldsymbol{u} = 0\,, \tag{ii}$$

where
$$\boldsymbol{u} = \operatorname{grad} \operatorname{In} \rho \frac{\operatorname{curl} \boldsymbol{a} \times \boldsymbol{q} - \boldsymbol{a} \operatorname{div} \boldsymbol{q}}{\boldsymbol{q} \cdot \boldsymbol{a}}\,, \qquad \boldsymbol{q} \cdot \boldsymbol{a} \neq 0\,. \tag{2.5}$$

The velocity of the fluid can be determined from Eqs (i) and (ii). The density and the pressure are then given by

$$\rho = \exp \int \boldsymbol{u} \cdot dr\,. \tag{2.6}$$

Using Eqs (2.3) and (2.4) , we obtain

$$\operatorname{grad} \frac{\mu}{\theta} \cdot \operatorname{curl} \boldsymbol{a} = 0\,, \tag{iii}$$

$$p = -\int \rho \mathrm{a} \cdot dr \tag{2.7}$$

where $\theta = \operatorname{div} \boldsymbol{q}$ and $\mu = \boldsymbol{q} \cdot \boldsymbol{a}$. Further, it can be shown that

$$\boldsymbol{a} \cdot \operatorname{curl} \boldsymbol{u} = 0\,. \tag{2.8}$$

Due to the compatibility equations, the ratio (μ/θ) is a function H of p and ρ only

$$\frac{\mu}{\theta} = H(p, \rho)\,. \tag{2.9}$$

The equation of state is found by integrating

$$\frac{\partial s}{\partial \rho} + H(p,\rho) \frac{\partial s}{\partial p} = 0\,. \tag{2.10}$$

For a Prim gas, we obtain

$$\operatorname{curl}\left(\frac{\theta}{\mu} \boldsymbol{a}\right) = 0\,, \tag{2.11}$$

instead of Eq.(iii).

2.1. Decomposition of compatibility equations

Let (ξ, η, ψ) be the orthogonal curvilinear co-ordinates in which the arc length in this system is of the form

$$ds^2 = g_1^2(\xi,\eta,\psi)d\xi^2 + g_2^2(\xi,\eta,\psi)d\eta^2 + g_3^2(\xi,\eta,\psi)d\psi^2 \tag{2.12}$$

where g_1, g_2, g_3 are metric coefficients.

Taking the curve through any point, along which ξ increases as our streamline, we get the orthogonal curvilinear net in natural co-ordinates. Let e_1, e_2, and e_3 be the unit tangential vectors to the three orthogonal curves at a point in the increasing directions ξ, η and ψ respectively, then the velocity $\boldsymbol{q}$ is given by

$$\boldsymbol{q} = u(\xi,\eta,\psi)e_1 . \tag{2.13}$$

We can express the compatibility conditions (i) (ii) and (iii) in terms of the curvilinear co-ordinates introduced above as follows

$$\boldsymbol{a} = \frac{u}{g_1}\frac{\partial u}{\partial \xi}e_1 + \frac{u^2}{g_1 g_2}\frac{\partial g_1}{\partial \eta}e_2 + \frac{u^2}{g_1 g_3}\frac{\partial g_1}{\partial \psi}e_3, \tag{2.14}$$

$$\theta = div\boldsymbol{q} = \frac{1}{g_1 g_2 g_3}\frac{\partial}{\partial \xi}(g_2 g_3 u), \tag{2.15}$$

$$\mu = \boldsymbol{q}\cdot\boldsymbol{a} = \frac{u^2}{g_1}\frac{\partial u}{\partial \xi}, \tag{2.16}$$

$$\begin{aligned}\mathrm{curl}\,\boldsymbol{a} = {} & \frac{1}{g_2 g_3}\left\{\frac{\partial}{\partial \psi}\left(\frac{u^2}{g_1}\frac{\partial g_1}{\partial \eta}\right) - \frac{\partial}{\partial \eta}\left(\frac{u^2}{g_1}\frac{\partial g_1}{\partial \psi}\right)\right\}e_1 + \\ & + \frac{1}{g_1 g_3}\left\{\frac{\partial}{\partial \xi}\left(\frac{u^2}{g_1}\frac{\partial g_1}{\partial \psi}\right) + \frac{\partial}{\partial \psi}\left(u\frac{\partial u}{\partial \xi}\right)\right\}e_2 + \\ & - \frac{1}{g_1 g_2}\left\{\frac{\partial}{\partial \xi}\left(\frac{u^2}{g_1}\frac{\partial g_1}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(u\frac{\partial u}{\partial \xi}\right)\right\}e_3 .\end{aligned} \tag{2.17}$$

Using Eqs (2.13) and (2.17) in Eq.(2.5) and consequently using this value of u and Eq.(2.14) in Eq.(2.8), we have

$$\frac{\partial}{\partial \xi}\left[\frac{\dfrac{\partial^2 u}{\partial \xi \partial \eta} + \dfrac{\partial}{\partial \eta}\left\{\log\left(\dfrac{g_1}{g_2 g_3}\right)\right\}\dfrac{\partial u}{\partial \xi} + \left\{\dfrac{\partial^2}{\partial \xi \partial \eta}(\log g_1) - \dfrac{\partial}{\partial \eta}(\log g)\dfrac{\partial}{\partial \xi}(\log g_2 g_3)\right\}u}{\dfrac{\partial u}{\partial \xi}}\right] = 0, \tag{2.18}$$

$$\frac{\partial}{\partial\xi}\left[\frac{\frac{\partial^2 u}{\partial\xi\partial\psi}+\frac{\partial}{\partial\psi}\left\{\log\left(\frac{g_1}{g_2g_3}\right)\right\}\frac{\partial u}{\partial\xi}+\left\{\frac{\partial^2}{\partial\xi\partial\psi}(\log g_1)-\frac{\partial}{\partial\psi}(\log g)\frac{\partial}{\partial\xi}(\log g_2g_3)\right\}u}{\frac{\partial u}{\partial\xi}}\right]=0\,, \quad (2.19)$$

$$\begin{aligned}&\frac{\partial}{\partial\eta}\left[\frac{1}{u\frac{\partial u}{\partial\xi}}\left\{\frac{\partial}{\partial\xi}\left(\frac{u^2}{g_1}\frac{\partial g_1}{\partial\psi}\right)\right\}-\frac{1}{g_1g_2g_3\frac{\partial u}{\partial\xi}}\frac{\partial g_1}{\partial\psi}\frac{\partial}{\partial\xi}(g_2g_3u)\right]+\\ &-\frac{\partial}{\partial\psi}\left[\frac{1}{u\frac{\partial u}{\partial\xi}}\left\{\frac{\partial}{\partial\xi}\left(\frac{u^2}{g_1}\frac{\partial g_1}{\partial\eta}\right)\right\}-\frac{1}{g_1g_2g_3\frac{\partial u}{\partial\xi}}\frac{\partial g_1}{\partial\eta}\frac{\partial}{\partial\xi}(g_2g_3u)\right]=0,\end{aligned} \quad (2.20)$$

Equation (2.11) can be decomposed into the following equations

$$\frac{\partial}{\partial\psi}\left[\frac{\frac{\partial}{\partial\xi}(g_2g_3u)\frac{\partial g_1}{\partial\eta}}{g_1g_2g_3\frac{\partial u}{\partial\xi}}\right]-\frac{\partial}{\partial\eta}\left[\frac{\frac{\partial}{\partial\xi}(g_2g_3u)\frac{\partial g_1}{\partial\psi}}{g_1g_2g_3\frac{\partial u}{\partial\xi}}\right]=0\,, \quad (2.21)$$

$$\frac{\partial}{\partial\psi}\left[\frac{\frac{\partial}{\partial\xi}(g_2g_3u)}{g_2g_3u}\right]+\frac{\partial}{\partial\xi}\left[\frac{\frac{\partial}{\partial\xi}(g_2g_3u)\frac{\partial g_1}{\partial\psi}}{g_1g_2g_3\frac{\partial u}{\partial\xi}}\right]=0\,, \quad (2.22)$$

$$\frac{\partial}{\partial\xi}\left[\frac{\frac{\partial}{\partial\xi}(g_2g_3u)\frac{\partial g_1}{\partial\eta}}{g_1g_2g_3\frac{\partial u}{\partial\xi}}\right]+\frac{\partial}{\partial\eta}\left[\frac{\frac{\partial}{\partial\xi}(g_2g_3u)}{g_2g_3u}\right]=0\,. \quad (2.23)$$

Hence velocity can be determined by using Eqs (2.21), (2.22), (2.23) and (2.5). Let

$$u=U(\xi\,\eta,\psi)e_1\,, \quad (2.24)$$

be the velocity solution for the problem. Using Eq.(2.24) in Eq.(2.2), we get

$$\rho=\frac{\phi(\eta,\psi)}{g_2g_3U} \quad (2.25)$$

where $\phi(\eta,\psi)$ is an arbitrary function of η and ψ. Employing components of ***a*** given by Eq.(2.14) in Eq.(2.1), we get

$$\frac{\partial p}{\partial \xi} = \frac{-\phi(\eta,\psi)}{g_2 g_3}\frac{\partial}{\partial \xi}U \,,$$

$$\frac{\partial p}{\partial \eta} = U\frac{\partial}{\partial \eta}\left(\log\left(g_1\right)\right)\frac{\phi(\eta,\psi)}{g_2 g_3}\,, \tag{2.26}$$

$$\frac{\partial p}{\partial \psi} = U\frac{\partial}{\partial \psi}\left(\log\left(g_1\right)\right)\frac{\phi(\eta,\psi)}{g_2 g_3}\,.$$

Solving Eq.(2.26), we obtain p. Using Eqs (2.15) and (2.16) in (2.9), we have

$$H(p,\rho) = \frac{g_2 g_3 U^2 \frac{\partial}{\partial \xi}U}{\frac{\partial}{\partial \xi}\left(g_2 g_3 U\right)}\,. \tag{2.27}$$

Finally, the Mach number $\mathrm{M} = (U/\mathbf{c})$, $\boldsymbol{c}$ velocity of sound $\left(c^2 = dp/d\rho\right)$ is given by

$$\mathrm{M} = \sqrt{1 + \frac{\frac{\partial}{\partial \xi}\left(\log\left(g_2 g_3\right)\right)}{\frac{\partial}{\partial \xi}\left(\log(U)\right)}}\,. \tag{2.28}$$

3. Calculation of the Mach number

3.1. Radial flow from the surface of a small spherical ball

We study this problem in the spherical co-ordinate system whose metric coefficients are given by

$$g_1(\xi,\eta,\psi) = 1\,, \qquad g_2(\xi,\eta,\psi) = \xi\,, \qquad g_3(\xi,\eta,\psi) = \xi\sin\eta\,. \tag{3.1}$$

Using Eq.(3.1) in Eqs (2.18) and (2.19), we obtain the equations satisfied by the velocity $u(\xi,\eta,\psi)$ as

$$\frac{\partial}{\partial \xi}\left[\frac{\frac{\partial^2 u}{\partial \xi \partial \eta} - \cot\eta\frac{\partial u}{\partial \xi}}{\frac{\partial u}{\partial \xi}}\right] = 0\,, \tag{3.2}$$

$$\frac{\partial}{\partial \xi}\left[\frac{\frac{\partial^2 u}{\partial \xi \partial \eta}}{\frac{\partial u}{\partial \xi}}\right] = 0\,, \tag{3.3}$$

and Eq.(2.20) is automatically satisfied. Equation (iii) gives

$$\frac{\partial u}{\partial \xi}\frac{\partial^2 u}{\partial \xi \partial \psi}-\frac{\partial u}{\partial \psi}\frac{\partial^2 u}{\partial \xi \partial \eta}=0\,. \tag{3.4}$$

Also from Eqs (3.2), (3.3) and (3.4), we get

$$u(\xi,\eta,\psi)=a(\eta,\psi)\,b(\xi)+c(\eta,\psi) \tag{3.5}$$

where $a(\eta,\psi)\,b(\xi)$ and $c(\eta,\psi)$ are the arbitrary functions.

Substituting (3.5) and (2.17) in the Euler's dynamical equation in the direction of ξ - increasing and using p as a function of ξ alone, we get

$$\rho=\frac{\rho_0}{\xi^2 a(\eta,\psi)\{a(\eta,\psi)\,b(\xi)+c(\eta,\psi)\}} \tag{3.6}$$

where ρ_0 is an arbitrary constant. Employing (3.5) and (3.6), we get

$$p(\xi,\eta,\psi)=p_0-\int\frac{b'(\xi)}{\xi^2}d\xi\,. \tag{3.7}$$

Then we obtain the Mach number $\mathrm{M}=(U/\mathbf{c})$ as

$$\mathrm{M}=\sqrt{1+\frac{2\{a(\eta,\psi)b(\xi)+C(\eta,\psi)\}}{\xi a(\eta,\psi)b'(\xi)}}\,. \tag{3.8}$$

3.2. Flow of gases in a circular tunnel

We study the flow of gases, inside a circular tunnel when there are no external forces. We take the natural co-ordinate system to be a cylindrical coordinate system in which metric coefficients are given by

$$g_1=\eta\,,\qquad g_2=1\,,\qquad \text{and}\qquad g_3=1\,. \tag{3.9}$$

Compatibility Eqs (i) and (iii) give

$$\frac{\partial}{\partial \xi}\left(\log\frac{\partial u}{\partial \psi}\right)=\frac{\partial}{\partial \xi}(\log u)\,, \tag{3.10}$$

$$\frac{\partial u}{\partial \eta}\frac{\partial^2 u}{\partial \xi \partial \psi}-\frac{\partial u}{\partial \psi}\frac{\partial^2 u}{\partial \xi \partial \eta}=0\,. \tag{3.11}$$

Therefore,

$$u(\xi,\eta,\psi)=a(\eta,\psi)\,b(\xi)+c(\eta,\psi)\,. \tag{3.12}$$

Using Eq.(3.12) in Eqs (3.10) and (3.11), we get

$$\frac{\partial}{\partial\psi}\left(\log\frac{c(\eta,\psi)}{a(\eta,\psi)}\right)=0\,,\qquad \frac{\partial}{\partial\eta}\left(\log\frac{c(\eta,\psi)}{a(\eta,\psi)}\right)=0\,,\qquad c(\eta,\psi)=ka(\eta,\psi). \tag{3.13}$$

Finally, we have $$u(\xi,\eta,\psi)=a(\eta,\psi)\,d(\xi). \tag{3.14}$$

From Eq.(2.14), we get

$$\boldsymbol{a}=\frac{u}{\eta}\frac{\partial u}{\partial\xi}e_1+\frac{u^2}{\eta}e_2. \tag{3.15}$$

Substituting Eqs (3.15) and (2.25) in Euler's dynamical equation in the directions of ξ and η increasing, we get

$$\rho(\xi,\eta,\psi)=\frac{\rho_0}{\eta a^2(\eta,\psi)d(\xi)} \tag{3.16}$$

where ρ_0 is an arbitrary constant. Using Eqs (3.12) and (3.16) in Eq.(2.26) and integrating, we get

$$p(\xi,\eta,\psi)=p_0-\frac{\rho_0 d(\xi)}{\eta}. \tag{3.17}$$

Finally, by using Eq.(2.28) the Mach number is $\mathrm{M}=1$.

4. Transformation to the Hodograph plane

The stream function $\psi(x,y)$ for a subsonic irrotational steady compressible fluid flow is given by a non linear elliptic partial differential equation

$$\left[(\rho c)^2-\psi_y^2\right]\,\psi_{xx}+2\psi_x\psi_y\psi_{xy}+\left[(\rho c)^2-\psi_y^2\right]\,\psi_{yy}=0\,,$$

i.e., $$\left[1-\frac{\psi_y^2}{(\rho c)^2}\right]\psi_{xx}+\frac{2\psi_x\psi_y}{(\rho c)^2}\psi_{xy}+\left[1-\frac{\psi_x^2}{(\rho c)^2}\right]\psi_{yy}=0 \tag{4.1}$$

where ρ is the fluid density, c is the speed of sound, and ρ and c are functions of ψ_x and ψ_y. The velocity components (u,v) of the fluid are related to ψ by

$$\rho u=\psi_y \qquad \text{and} \qquad \rho v=-\psi_x. \tag{4.2}$$

Taking the x-axis as the axis of symmetry parallel to the incident flow and the y-axis as the axis of symmetry orthogonal to the incident flow, the region of interest consists of the semi infinite domain D bounded by the x-axis exterior to the plates, the two parallel half plates and the free streamline surmounting the half plates as illustrated in Fig.1. The condition that Eq.(4.1) is elliptic

$$q^2 = u^2 + v^2, \qquad \theta = \tan^{-1}\frac{v}{u}. \tag{4.3}$$

Namely, $u^2 + v^2 < c^2$, is equivalent to the relation $\mathrm{M} < 1$ where $\mathrm{M} = q/c$ is the Mach number and the velocity of sound is given by $c^2 = dp/d\rho$ and the pressure density is taken to be $p = \text{constant } \rho^r$. Under the normalization $q = c = 1$ for $\mathrm{M} = 1$.

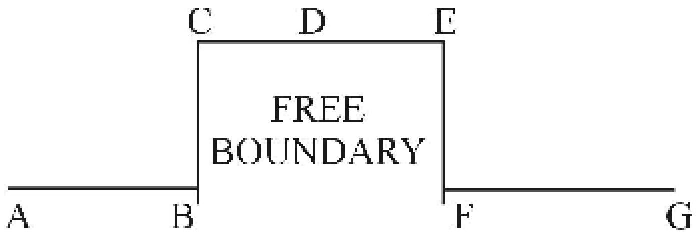

Fig.1. Region of interest in the physical plane.

Further, the transformation to the Hodograph (θ, λ)- plane given by

$$\frac{d\lambda}{dq} = \frac{\sqrt{1-\mathrm{M}^2}}{q}. \tag{4.4}$$

Under this transformation, Eq.(4.1) becomes

$$\psi_{\lambda\lambda} + \psi_{\theta\theta} + g(\lambda)\psi_\lambda = 0, \qquad g(\lambda) = -\frac{\gamma+1}{2}\frac{\mathrm{M}^4}{\left(1-\mathrm{M}^2\right)^{3/2}}, \tag{4.5}$$

and the corresponding domain is depicted in Fig.2. In the (θ, λ) –plane the differential equation is linear and is simplified to the form of Laplacian plus a lower order term. Further, the region of interest is a semi-infinite slit rectangular domain with the original free boundary transformed into the segment of the θ-axis between $-\pi/2$ and $\pi/2$.

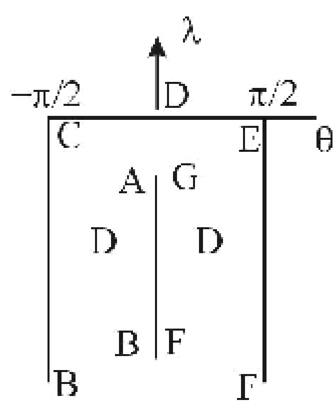

Fig.2. Region of interest in a pseudo logarithmic Hodograph plane.

The singularity $S(\theta, \lambda)$ for a compressible subsonic flow is

$$S(\theta, \lambda)\mathrm{Im}\{i[\theta + i(\lambda - \lambda_0)]\}^{1/2} \tag{4.6}$$

where λ_0 corresponds to point A (Fig.2) in our formulation.

After obtaining a solution for Eq.(4.3) in the (θ, λ) plane one obtains in the physical plane from the integrations

$$x(\theta,\lambda) = \int x_\theta d\theta + x_\lambda d\lambda = \int x_\theta d\theta + x_q q_\lambda d\lambda \quad \text{and} \quad y(\theta,\lambda) = \int y_\theta d\theta + y_q q_\lambda d\lambda \tag{4.7}$$

where $$x_q = -\frac{1}{\rho q}\left[\psi_\theta \frac{(1-\mathrm{M}^2)}{q}\cos\theta + \psi_q \sin\theta\right], \qquad x_\theta = \frac{1}{\rho q}\left[q\psi_q \cos\theta - \psi_\theta \sin\theta\right],$$

and $$y_q = \frac{1}{\rho q}\left[-\psi_\theta \frac{(1-\mathrm{M}^2)}{q}\sin\theta + \psi_q \cos\theta\right], \qquad y_\theta = \frac{1}{\rho q}\left[q\psi_q \sin\theta + \psi_\theta \cos\theta\right].$$

All integrations are carried out from a fixed point [taken as $\left(\frac{\pi}{2}, -\infty\right)$] to a variable point (θ, λ). The integrands in Eq.(4.7) are $\psi_\theta(\theta, 0) = 0$ and $\psi_\lambda(\pi/2, \lambda) = 0$.

Therefore, Eq.(4.7) can be written as

$$\begin{aligned} x_c &= \frac{1}{\rho^*}\int_{\substack{\theta=\pi/2\\ \lambda=0}}^{\theta=0} \sqrt{(1-\mathrm{M}^2)}\,\psi_\lambda \cos\theta\, d\theta, \\ y_c &= \frac{1}{\rho^*}\int_{\substack{\theta=\pi/2\\ \lambda=0}}^{\theta=0} \sqrt{(1-\mathrm{M}^2)}\,\psi_\lambda \sin\theta\, d\theta - \int_{\substack{\lambda=-\infty\\ \theta=\pi/2}}^{\lambda=0} \frac{1}{\rho q}\sqrt{(1-\mathrm{M}^2)}\,\psi_\theta d\lambda \end{aligned} \tag{4.8}$$

where $\rho^* = \rho(c)$. The integrands for θ in Eq.(4.8) are indeterminate in the present form since $(1-\mathrm{M}^2)^{1/2}$ tends to zero and ψ_λ becomes infinite as λ tends to zero. Therefore, the partial differential Eq.(4.4) approaches the Tricomi equation $\Delta\psi + \frac{\psi_\lambda}{3\lambda} = 0$ and one can show that $\lim_{\lambda\to 0} \psi_\lambda \sqrt{(1-\mathrm{M}^2)} = -\frac{2}{3}\frac{\psi}{\lambda^{2/3}}\left[\frac{3}{2}(\gamma+1)\right]^{1/3}$.

Acknowledgment

The authors are grateful to the referees for their valuable suggestions in revising the paper.

Nomenclature

- $\boldsymbol{a}$ – acceleration vector
- $\boldsymbol{c}$ – velocity of sound
- e_1, e_2, e_3 – unit tangential vectors

g_1, g_2, g_3 – metric coefficients and
p – pressure of the gas
s – specific entropy
$\boldsymbol{q}$ – velocity vector
ρ – density of gas
μ – viscosity of the gas

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Received: November 8, 2004
Revised: February 11, 2005