Int. J. of Applied Mechanics and Engineering, 2006, vol.11, No.3, pp.609-621

HYDRODYNAMIC FLOWS THROUGH VERTICAL WAVY CHANNEL WITH TRAVELLING THERMAL WAVES EMBEDDED IN POROUS MEDIUM

M. GURIA* and R.N. JANA
Department of Applied Mathematics,
Vidyasagar University, Midnapore 721 102
West Bengal, INDIA
e-mail: mrinmoy9832@yahoo.com

This paper analyses the two dimensional free and forced convection flow and heat transfer in a vertical wavy channel with travelling thermal waves embedded in a porous medium. The set of non-linear ordinary differential equations is solved analytically. The velocity and temperature field have been obtained using perturbation technique. The effects of non-dimensional parameter on the velocity and temperature profile are shown graphically. It is observed that the main flow velocity increases with an increase in either permeability parameter or the Grashof number. It is remarkable that the flow is reversed at the middle of the channel. It is found that for small values of frequency parameter it increases while it decreases for large values of frequency parameter. On the other hand, the cross velocity decreases with an increase in the permeable parameter while it increases with an increase in the Grashof number. It is found that for small values of frequency parameter the cross velocity first increases and then decreases with increase in frequency parameter. The flow is also reversed for the cross flow. It is seen that the shear stresses at the plates rises with an increase in the permeable parameter. The temperature profile decreases with an increase in the Prandtl number while it increases with increase in the permeable parameter. Also it is seen that the heat transfer coefficient increases with an increase in the permeable parameter but decreases with an increase in the Grashof number.

Key words: porous medium, permeability, wavy, free and forced convection.

1. Introduction

Viscous flow and heat transfer through a porous medium is the subject of intensive studies due to its numerous and wide ranging applications. The study of natural convection in a vertical channel is an important subject due to increasing practical applications in industries. The study of viscous flows bounded by wavy wall is of special interest due to its application to transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. In view of these applications, Vajravelu (1989) studied the combined free and forced convection in hydrodynamic flows in vertical wavy channel with travelling thermal waves embedded in a porous medium. Recently, Chaudhury (2004) investigated the effect of injection on the three dimensional flow and heat transfer through a vertical parallel plate channel, which is embedded in a porous medium. Takhar (1990) studied the combined free and forced convection of an incompressible viscous fluid in a porous medium past a hot vertical plate. But they have not studied the flows through wavy channel in a porous medium.

The aim of this paper is to study the combined free and forced convection in hydrodynamic flows in vertical wavy channel with travelling thermal waves embedded in a porous medium (Fig.1). The problem is solved using perturbation technique. The solution is made on two parts, the mean part and the perturbed part. These two parts are obtained separately. The effects of the non-dimensional parameter on the velocity and temperature profile are shown graphically. It is observed that the main flow velocity increases with an increase in either permeability parameter or the Grashof number. It is remarkable that the flow is reversed at the middle of the channel. The temperature profile decreases with an increase in the Prandtl number while it

* To whom correspondence should be addressed

increases with an increase in the permeable parameter. Also it is seen that the heat transfer coefficient increases with an increase in the permeable parameter but decreases with an increase in the Grashof number.

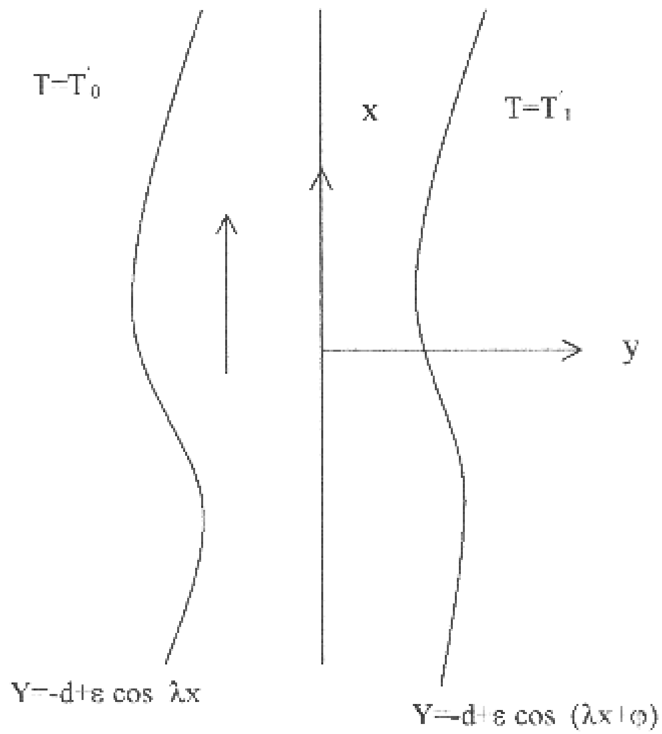

Fig.1. Geometry of the problem.

2. Basic equations

Consider the unsteady flow of a viscous incompressible fluid through the vertical channel bounded by two wavy walls embedded in a porous medium. We choose the x^*-axis along the direction of the flow, the y^*-axis is perpendicular to it. Let us consider the wavy walls $y^* = d + a\cos\lambda^* x^*$ and $y^* = -d - a\cos\lambda^* x^*$. For convenience we take $y^* = -d - a\cos\left(\lambda^* x^* + \varphi\right)$. Due to the density variation and temperature difference a force is created along the direction of the flow. This force is called buoyancy force. The fluid flowing with velocity V through the porous medium experiences a resistance gV/K per unit mass, where g is the acceleration due to gravity and K is constant called the permeability of the medium. The viscous dissipation term is neglected in the energy equation. We assume that the wavelength of the wavy walls, which is proportional to $1/\lambda$ is large. We study the combined convective heat transfer and fluid flow of viscous incompressible fluid through a vertical wavy channel for $\varphi = 0$.

Let u^*, v^* be the velocity component along the x^*- and y^*-axis respectively, the unsteady flow of viscous incompressible fluid is governed by the following equations

$$\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial z^*} = 0\,, \tag{2.1}$$

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}+v\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)-\frac{v}{K}u^*+\rho g, \tag{2.2}$$

$$\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+v\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)-\frac{\nu}{K'}v^*, \tag{2.3}$$

$$\frac{\partial T}{\partial t^*}+u^*\frac{\partial T}{\partial x^*}+v^*\frac{\partial T}{\partial y^*}=\alpha\left(\frac{\partial^2 T}{\partial x^{*2}}+\frac{\partial^2 T}{\partial y^{*2}}\right) \tag{2.4}$$

where ρ is the density, p^* is the pressure, v is the kinematic viscosity, T is the temperature, α is the thermal diffusivity, K' is the permeability of the medium.

The boundary conditions of the problem are

$$\begin{aligned}&u^*=0, \quad v^*=0, \quad T=T_0\left[1+\varepsilon\cos\left(\lambda^* x^*+\omega t\right)\right]=T_0', \quad \text{at} \quad y^*=d+a\cos\lambda^* x^*,\\ &u^*=0, \quad v^*=0, \quad T=T_1\left[1+\varepsilon\cos\left(\lambda^* x^*+\omega t\right)\right]=T_1', \quad \text{at} \quad y^*=-d+a\cos\left(\lambda^* x^*+\varphi\right).\end{aligned} \tag{2.5}$$

We introduce the non-dimensional variables

$$\begin{aligned}&(x,y)=\left(x^*,y^*\right)/d, \quad t=t^*\nu/d^2, \quad p=p^*d^2/\rho\nu^2, \quad \theta=\left(T-T_0'\right)/\left(T_1'-T_0'\right),\\ &(u,v)=\left(u^*,v^*\right)d/\nu, \quad \lambda=\lambda^* d, \quad \varepsilon=a/d.\end{aligned} \tag{2.6}$$

In terms of non-dimensional variables Eqs (2.1) to (2.4) become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.7}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{u}{K}+\mathrm{Gr}\,\theta, \tag{2.8}$$

$$\frac{\partial \nu}{\partial t}+u\frac{\partial \nu}{\partial x}+v\frac{\partial \nu}{\partial y}=-\frac{\partial p}{\partial y}+\left(\frac{\partial^2 \nu}{\partial x^2}+\frac{\partial^2 \nu}{\partial y^2}\right)-\frac{\nu}{K}, \tag{2.9}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right) \tag{2.10}$$

where $K=K'/d^2$, the permeable parameter; $\mathrm{Gr}=g\beta\left(T_1'-T_0'\right)d^3/\nu^2$, the Grashof number; $\mathrm{Pr}=\nu/\alpha$, the Prandtl number; $\lambda=\lambda^* d$, the non dimensional wave number; $\varepsilon=a/d$, the amplitude parameter.

Let us introduce the stream function ψ defined by

$$u=-\frac{\partial\psi}{\partial y} \quad \text{and} \quad v = \frac{\partial\psi}{\partial x} . \tag{2.11}$$

Using (2.11), Eqs (2.8)-(2.10) become

$$\begin{aligned}&\psi_{xxt}+\psi_{yyt}-\psi_y\left(\psi_{xxx}+\psi_{xyy}\right)+\psi_x\left(\psi_{xxy}+\psi_{yyy}\right)=\psi_{xxxx}+2\psi_{xxyy}+\\&+\psi_{yyyy}-\frac{1}{K}(\psi_{xx}+\psi_{yy})-\mathrm{Gr}\,\theta_y,\end{aligned} \tag{2.12}$$

$$\theta_t-\psi_y\theta_x+\psi_x\theta_y=\frac{1}{\mathrm{Pr}}\left(\theta_{xx}+\theta_{yy}\right). \tag{2.13}$$

The boundary conditions (2.5) become

$$\begin{aligned}&\psi_y=0, \quad \psi_x=0, \quad \theta=0 \quad \text{at} \quad y=1+\varepsilon\cos\lambda x,\\&\psi_y=0, \quad \psi_x=0, \quad \theta=1 \quad \text{at} \quad y=-1+\varepsilon\cos(\lambda x+\varphi).\end{aligned} \tag{2.14}$$

3. Solution of the problem

In order to solve Eqs (2.12) and (2.13) we assume that the solution in the form

$$\begin{aligned}&\psi(x,y,t)=\psi_0(y)+\psi_1(x,y,t),\\&\theta(x,y,t)=\theta_0(y)+\theta_1(x,y,t)\end{aligned} \tag{3.1}$$

where ψ_0, θ_0 are the mean parts and ψ_1, θ_1 are the perturbed parts also, we introduce

$$\psi_1(x,y,t)=\varepsilon e^{i(\lambda x+\omega t)}\overline{\psi_1}(y), \qquad \theta_1(x,y,t)=\varepsilon e^{i(\lambda x+\omega t)}\overline{\theta_1}(y).$$

Using (3.15), the Eqs (2.12) and (2.13) and the boundary conditions (2.14) become

$$\psi_0^{\mathrm{iv}}-\frac{1}{K}\psi_0''-\mathrm{Gr}\,\theta_0', \tag{3.2}$$

$$\theta_0''=0, \tag{3.3}$$

$$\begin{aligned}&\psi_0'=0, \quad \psi_0=0, \quad \theta_0=0 \quad \text{at} \quad y=1.0,\\&\psi_0'=0, \quad \psi_0=0, \quad \theta_0=1 \quad \text{at} \quad y=-1.0,\end{aligned} \tag{3.4}$$

to the zeroth order, and

$$\psi^{iv} - i\omega\left(\overline{\psi_1''} - \lambda^2\overline{\psi_1}\right) + i\lambda\psi_0'\left(\overline{\psi_1''} - \lambda^2\overline{\psi_1}\right) - i\lambda\overline{\psi_1}\psi_0''' - 2\lambda^2\psi_1'' + \\ + \lambda^4\overline{\psi_1} - \frac{1}{K}\left(\overline{\psi_1''} - \lambda^2\overline{\psi_1}\right) - \mathrm{Gr}\,\overline{\theta_1'} = 0, \tag{3.5}$$

$$\overline{\theta_{11}''} - i\,\mathrm{Pr}\,\omega\overline{\theta_1} - \lambda^2\overline{\theta_1} + i\,\mathrm{Pr}\,\lambda\left(\overline{\theta_1}\psi_0' - \overline{\psi_1}\theta_0'\right) = 0\,, \tag{3.6}$$

$$\overline{\psi_1'} = -\psi_0''e^{-i\omega t}\,, \quad \overline{\psi_1} = 0\,, \quad \overline{\theta_1} = -e^{-i\omega t}\theta_0' \quad \text{at} \quad y = 1.0\,, \\ \overline{\psi_1'} = -\psi_0''e^{-i(\varphi - \omega t)}, \quad \overline{\psi_1} = 0\,, \quad \overline{\theta_1} = -e^{-i(\varphi-\omega t)}\theta_0' \quad \text{at} \quad y = -1.0 \tag{3.7}$$

to the 1st order and prime denotes differentiation with respect to y. Now we assume

$$\overline{\psi_1}(\lambda, y) = \sum_{r=0}^{\infty}\lambda^r\overline{\psi_{1r}} \quad \text{and} \quad \overline{\theta_1}(\lambda, y) = \sum_{r=0}^{\infty}\lambda^r\bar{\theta}_{1r}\,. \tag{3.8}$$

Substituting (3.22) in Eqs (3.5)-(3.7), we get the following differential equations, neglecting the terms to the order of λ^2 and higher.

$$\overline{\theta_{10}''} - i\,\mathrm{Pr}\,\omega\overline{\theta_{10}} = 0\,, \tag{3.9}$$

$$\overline{\psi_{10}^{\mathrm{iv}}} - i\,\mathrm{Pr}\,\omega\overline{\psi_{10}} - \frac{1}{K}\overline{\psi_{10}''} - \mathrm{Gr}\overline{\theta_{10}'} = 0\,, \tag{3.10}$$

$$\overline{\theta_{11}''} - i\,\mathrm{Pr}\,\omega\overline{\theta_{11}} + i\,\mathrm{Pr}\left(\overline{\theta_{10}}\psi_0' - \overline{\psi_{10}}\theta_0'\right) = 0\,, \tag{3.11}$$

$$\overline{\psi_{11}^{\mathrm{iv}}} - i\omega\overline{\psi_{11}''} + i\psi_0'\overline{\psi_{10}''} - i\psi_0'''\overline{\psi_{10}} - \frac{1}{K}\overline{\psi_{11}''} - \mathrm{Gr}\,\overline{\theta_{11}'} = 0 \tag{3.12}$$

and

$$\overline{\psi_{10}'} = -\psi_0''e^{-i\omega t}, \quad \overline{\psi_{10}} = 0\,, \quad \overline{\theta_{10}} = -e^{-i\omega t}\theta_0' \quad \text{at} \quad y = 1.0\,, \\ \overline{\psi_{10}'} = -\psi_0''e^{i(\varphi-\omega t)}, \quad \overline{\psi_{10}} = 0\,, \quad \overline{\theta_{10}} = -e^{i(\varphi-\omega t)}\theta_0' \quad \text{at} \quad y = -1.0\,, \\ \overline{\psi_{11}} = 0\,, \quad \overline{\theta_{11}} = 0 \quad \text{at} \quad y = 1.0\,, \\ \overline{\psi_{11}} = 0\,, \quad \overline{\theta_{11}} = 0 \quad \text{at} \quad y = -1.0\,. \tag{3.13}$$

3.1. Zeroth order solution

The solutions of Eqs (3.2) and (3.3) subject to the boundary conditions (3.4) are

$$\psi_0(y) = A\cosh my + B + \frac{K\mathrm{Gr}y^2}{4}\,, \tag{3.14}$$

$$\theta_0(y) = \frac{1}{2}(1.0 - y) \tag{3.15}$$

where

$$A = \frac{-K\mathrm{Gr}}{2m\sinh m}, \qquad B = \frac{K\mathrm{Gr}(2\cosh m - m\sinh m)}{4m\sinh m}, \qquad m = \frac{1}{\sqrt{K}}.$$

3.2. First order solution

The solutions of Eqs (3.9)-(3.12) subject to the boundary conditions (3.13) are

$$\overline{\theta_{10}}(y) = B_1 \cosh ny + B_2 \sinh ny, \tag{3.16}$$

$$\overline{\psi_{10}}(y) = A_1 + A_2 y + A_3 \cosh py + A_4 \sinh py + A_5 \cosh ny + A_6 \sinh ny, \tag{3.17}$$

$$\begin{aligned}\overline{\theta_{11}}(y) &= B_3 \cos ny + B_4 \sinh ny - B_5 \sinh(m+n)y - B_6 \sinh(m-n)y + \\ &- B_7 \cosh(m+n)y - B_8 \cosh(m-n)y - B_9 y^2 \sinh ny - B_{10} y \cosh ny + \\ &- B_{11} y^2 \cosh ny - B_{12} y \sinh ny - B_{13} - B_{14} y - B_{15} \cosh py - B_{16} \sinh py,\end{aligned} \tag{3.18}$$

$$\begin{aligned}\overline{\psi_{11}}(y) &= C_1 \cosh py + C_2 \sinh py - (C_3 + C_{29})\sinh ny - (C_4 + C_{31})\cosh ny + \\ &- (C_5 + C_{22})\cosh(m+n)y - (C_6 + C_{23})\cosh(m-n)y - (C_7 + C_{20})\sinh(m+n)y + \\ &- (C_8 + C_{21})\sinh(m-n)y - C_9 y^2 \cosh ny - (C_{10} + C_{30}) y \sinh ny + \\ &- C_{11} y^2 \sinh ny - (C_{12} + C_{28}) y \cosh ny - C_{13} - (C_{14} + C_{25}) y \cosh py + \\ &- (C_{15} + C_{27}) y \sinh py - C_{16} \sinh(m+p)y - C_{17} \sinh(m-p)y + \\ &- C_{18} \cosh(m+p)y - C_{19} \cosh(m-p)y - C_{24} y^2 \sinh py - C_{26} y^2 \cosh py + \\ &- C_{32} \sinh my - C_{33} y \sinh my - C_{34} \cosh my\end{aligned} \tag{3.19}$$

where

$$\begin{gathered} B_1 = \frac{\cos(-\omega t) + \cos(\varphi - \omega t)}{4\cosh n}, \qquad B_2 = \frac{\cos(-\omega t) - \cos(\varphi - \omega t)}{4\sinh n}, \\ n = \sqrt{\frac{\omega \mathrm{Pr}}{2}}(1+i), \qquad p = \sqrt{\sqrt{\frac{1}{K^2} + \omega^2}}\left[\cos\left(\frac{1}{2}\tan^{-1}\omega K\right) + i\sin\left(\frac{1}{2}\tan^{-1}\omega K\right)\right]. \end{gathered} \tag{3.20}$$

The other constants are not given here to save space.

Obtaining the expressions of ψ_0 and ψ_1 we evaluate the main flow velocity u and the cross velocity v from Eq.(2.11) as

$$u = -[\psi_0'(y) + \varepsilon\{S_{1r}\cos(\lambda x + \omega t) - S_{1i}\sin(\lambda x + \omega t)\}] \tag{3.21}$$

and

$$v=\varepsilon\left[\overline{\psi}_{10}+\lambda\overline{\psi}_{11}(y)\right]i\lambda e^{i(\lambda x+\omega t)}, \qquad v=-\varepsilon\lambda\left[v_{1i}\cos(\lambda x+\omega t)+v_{1r}\sin(\lambda x+\omega t)\right]. \tag{3.22}$$

4. Discussion and results

Zeroth order solutions are naturally applicable to the case of a channel whose walls are flat and of constant temperature. We have obtained the zeroth order solutions ψ_0, θ_0 and perturbed solutions ψ_1, θ_1. From these we obtain the main flow velocity u, the cross flow velocity v and temperature distribution θ. In Figs 2-3 we have plotted the main flow velocity u for different values of the permeability parameter K, Grashof number Gr and frequency parameter ω for Pr=0.044, $\lambda=0.02$, $\lambda x=\pi/2.0$, $\omega t=\pi/4$, $\varphi=0$. It is observed that u increases with an increase in either K or Gr. It is remarkable that the flow is reversed in the region $0\le y\le 1$. From Fig.3 it is found that u increases for small values of ω and decreases for large values of ω. The cross velocity v is plotted in Figs 4-5 for different values of K, Gr and ω for Pr=0.044, $\lambda=0.02$, $\lambda x=\pi/2.0$, $\omega t=\pi/4$, $\varphi=0$. It is seen that v decreases with an increase in K but increases with an increase in Gr. From Fig.5 it is seen that v oscillates in nature with an increase in ω. As in primary flow, there is also a flow reversal for the cross flow in the region $-1\le y\le 0$. It is observed from Figs 2-5 that the flow is symmetrical about $y=0$.

Next we have drawn the temperature profile θ for different values of Pr for $\mathrm{Gr}=5.0$, $\omega=5.0$, $K=1.0$, $\lambda=0.02$, $\lambda x=\pi/2.0$, $\omega t=\pi/4$, $\varphi=0$ in Fig.6. From the figure it is seen that θ decreases with an increase in the Prandtl number.

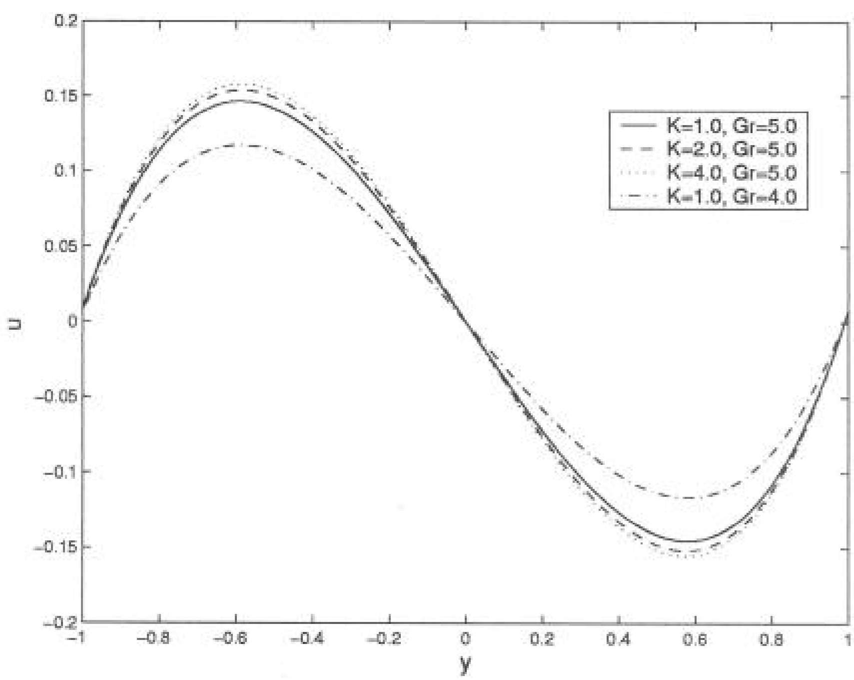

Fig.2. Main velocity u for $\omega=5.0$, Pr=0.044, $\lambda=0.02$, $\lambda x=\pi/2$, $\omega t=\pi/4$, $\varphi=0$.

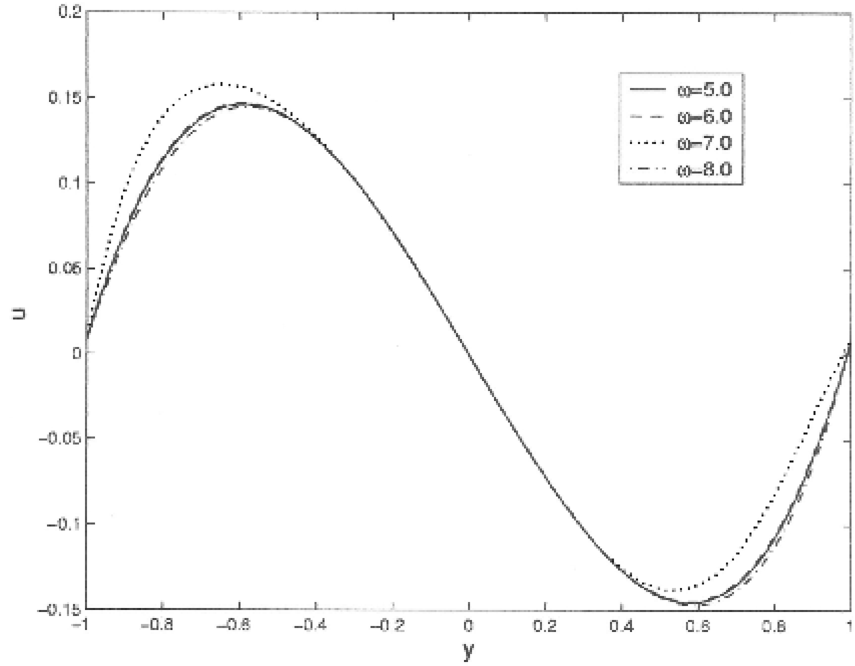

Fig.3. Main velocity *u* for *K=1.0*, Gr=*5.0*, Pr=*0.044*, $\lambda = 0.02$, $\lambda x = \pi/2$, $\omega t = \pi/4$, $\varphi = 0$.

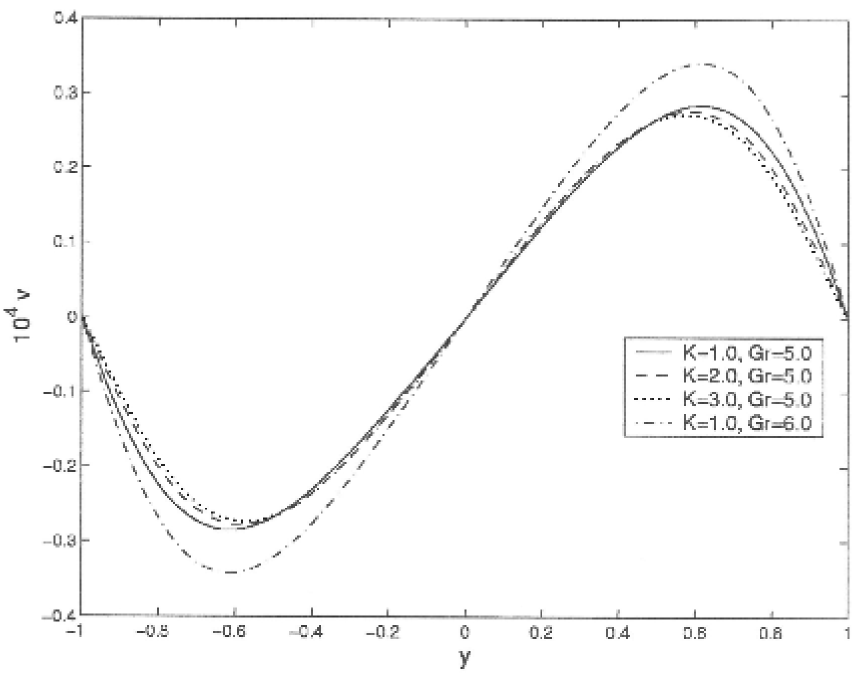

Fig.4. Cross velocity *v* for $\omega = 5.0$, Pr=*0.044*, $\lambda = 0.02$, $\lambda x = \pi/2$, $\omega t = \pi/4$, $\varphi = 0$.

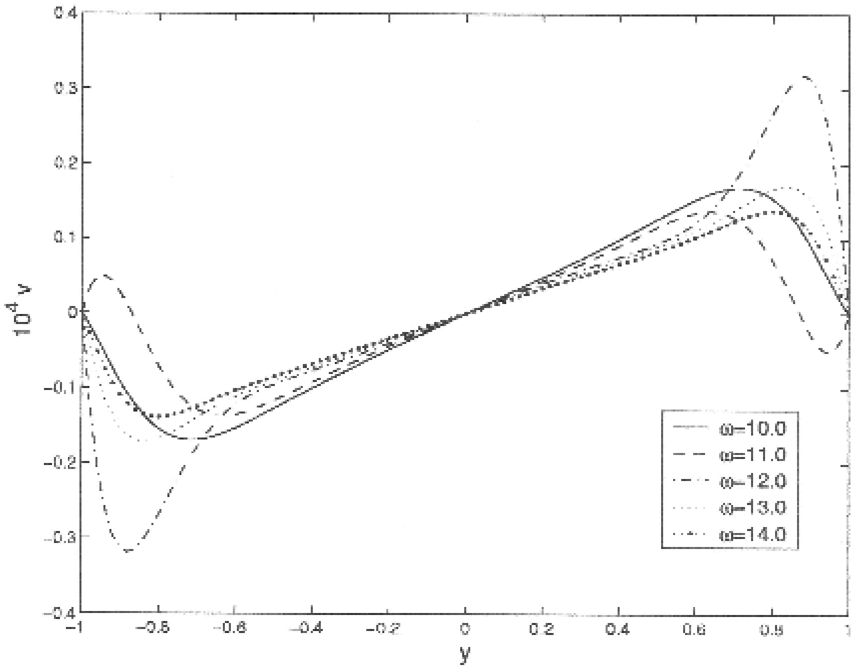

Fig.5. Cross velocity v for $K=1.0$, Gr=5.0, Pr=0.044, $\lambda = 0.02$, $\lambda x = \pi/2$, $\omega t = \pi/4$, $\varphi = 0$.

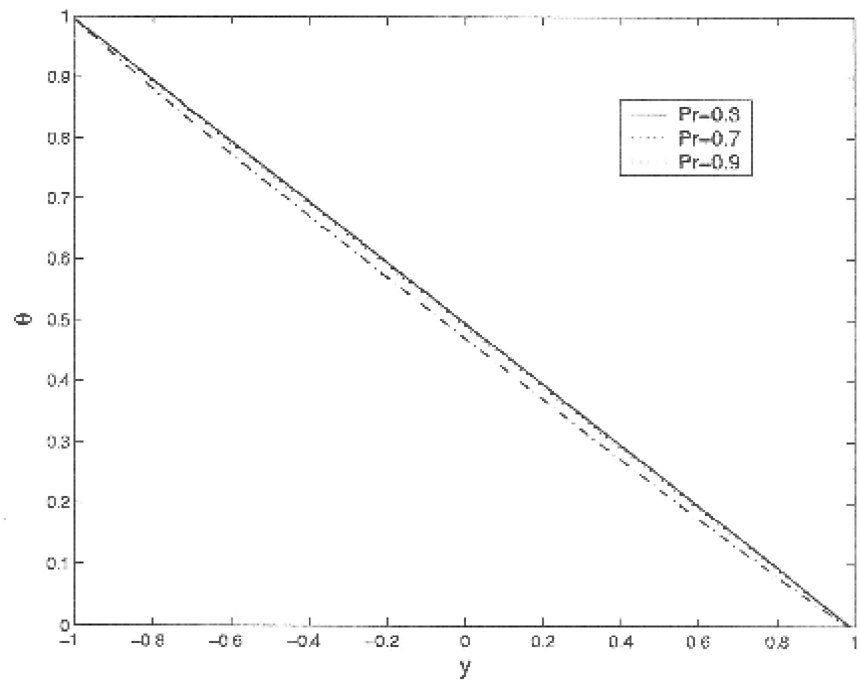

Fig.6. Temperature profile θ for $K=1.0$, Gr=5.0, $\omega = 5.0$, $\lambda = 0.02$, $\lambda x = \pi/2$, $\omega t = \pi/4$, $\varphi = 0$.

Also we have evaluated the numerical value of θ for different values of K, which is shown in Tab.1. From the table it is seen that the temperature θ increases with an increase in K but the effect of K on the temperature field θ is negligible.

Table 1. Temperature θ for $\mathrm{Gr}=5.0$, $\omega=5.0$, $\lambda=0.02$, $\lambda x=\pi/2.0$, $\omega t=\pi/4$, $\varphi=0$.

Y	θ			
	K=1.0	*K=1.5*	*K=2.0*	*K=2.5*
-1.00	0.9949970	0.9949970	0.9949970	0.9949970
-0.80	0.8949917	0.8949919	0.8949919	0.8949920
-0.60	0.7949890	0.7949893	0.7949894	0.7949895
-0.40	0.6949879	0.6949881	0.6949882	0.6949883
-0.20	0.5949874	0.5949875	0.5949876	0.5949876
0.00	0.4949869	0.4949869	0.4949869	0.4949869
0.20	0.3949865	0.3949864	0.3949863	0.3949863
0.40	0.2949864	0.2949862	0.2949861	0.2949860
0.60	0.1949873	0.1949871	0.1949870	0.1949869
0.80	0.0949904	0.0949902	0.0949901	0.0949901
1.00	-0.005003	-0.005003	-0.005003	-0.0050030

The shear stress at any point in the fluid is given by

$$\tau^*_{xy}=\mu\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right). \tag{4.1}$$

In non dimensional form

$$\tau_{xy}=\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right). \tag{4.2}$$

At the wavy walls $y=1+\varepsilon\cos\lambda x$

$$\tau_1=\tau_1^{(0)}-\varepsilon\left[\cos\lambda x\psi_0'''(1)+e^{i(\lambda x+\omega t)}\psi_1''(1)\right]. \tag{4.3}$$

At the wavy walls $y=-1+\varepsilon\cos(\lambda x+\varphi)$

$$\tau_2=\tau_2^{(0)}-\varepsilon\left[\cos(\lambda x+\varphi)\psi_0'''(-1)+e^{i(\lambda x+\omega t)}\psi_1''(-1)\right] \tag{4.4}$$

where $\tau_1^{(0)}=-\psi_0''(1)$ and $\tau_2^{(0)}=-\psi_0''(-1)$.

The heat transfer coefficient is given by

$$h=-K\frac{\partial\theta}{\partial y}, \tag{4.5}$$

or,

$$\mathrm{Nu}=\theta_0'(y)+\mathrm{Re}\,al\ Part\,of\left(\varepsilon e^{i(\lambda x+\omega t)}\theta_1'(y)\right) \tag{4.6}$$

where Nu is the non-dimensional heat transfer coefficient.

At the wavy walls $y = 1 + \varepsilon \cos \lambda x$

$$\begin{aligned} &\mathrm{Nu} = \theta_0'(1 + \varepsilon \cos \lambda x) + \mathrm{Re}\,al\ \ Part\,of\ \left(\varepsilon e^{i(\lambda x + \omega t)} \theta_1'(1 + \varepsilon \cos \lambda x)\right), \\ &(\mathrm{Nu})_{y=1} = \theta_0'(1) + \varepsilon\left[\cos \lambda x\, \theta_0''(1) + \mathrm{Re}\,al\ \ Part\ \ of\ \ \left(e^{i(\lambda x + \omega t)} \theta_1'\right)\right] = \mathrm{Nu}_1^{(0)} + \varepsilon\, \mathrm{Nu}_1^{(1)}. \end{aligned} \tag{4.7}$$

At the wavy walls $y = -1 + \varepsilon \cos(\lambda x + \varphi)$

$$\begin{aligned} &\mathrm{Nu} = \theta_0'(-1 + \varepsilon \cos(\lambda x + \varphi)) + \mathrm{Re}\,al\ \ Part\,of\ \left(\varepsilon e^{i(\lambda x + \omega t)} \theta_1'(-1 + \varepsilon \cos(\lambda x + \varphi))\right), \\ &(Nu)_{y=1} = \theta_0'(1) + \varepsilon\left[\cos(\lambda x + \varphi) \theta_0''(1) + \mathrm{Re}\,al\ \ Part\ \ of\ \ \left(e^{i(\lambda x + \omega t)} \theta_1'\right)\right] = \mathrm{Nu}_2^{(0)} + \varepsilon\, \mathrm{Nu}_2^{(1)}. \end{aligned} \tag{4.8}$$

Next we have drawn the shear stresses at the plates $y = -1$ and $y = 1$ in Fig.7 for different values of K against ω Pr=*0.044*, $\lambda = 0.02$, $\lambda x = \pi/2.0$, $\omega t = \pi/4$, $\varphi = 0$. It is observed that the total shear stress τ is same at the two plates and it is significantly affected by K. Here the shear stress at the plates increases with an increase in K. The heat transfer coefficient $\mathrm{Nu}_1^{(1)}$ at the plates $y = 1$ and $y = -1$ are also same and is shown in Fig.8 for different values of K and Gr for Pr=*0.044*, $\lambda = 0.02$, $\lambda x = \pi/2.0$, $\omega t = \pi/4$, $\varphi = 0$. However, the heat transfer coefficient $\mathrm{Nu}_1^{(1)}$ is significantly affected by K and Gr.

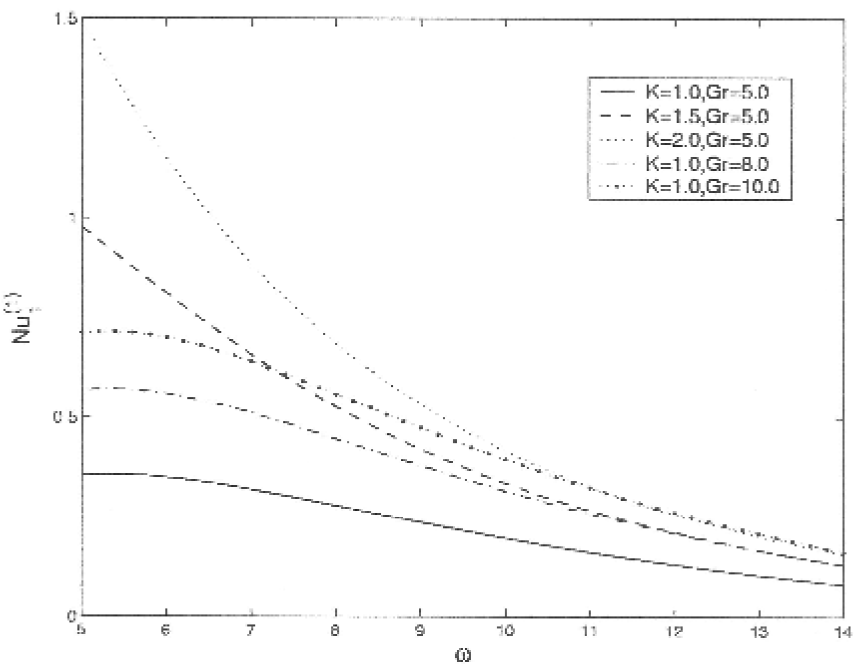

Fig.7. Shear stress at the plates for Pr=*0.044*, $\lambda = 0.02$, $\lambda x = \pi/2$, $\omega t = \pi/4$, $\varphi = 0$.

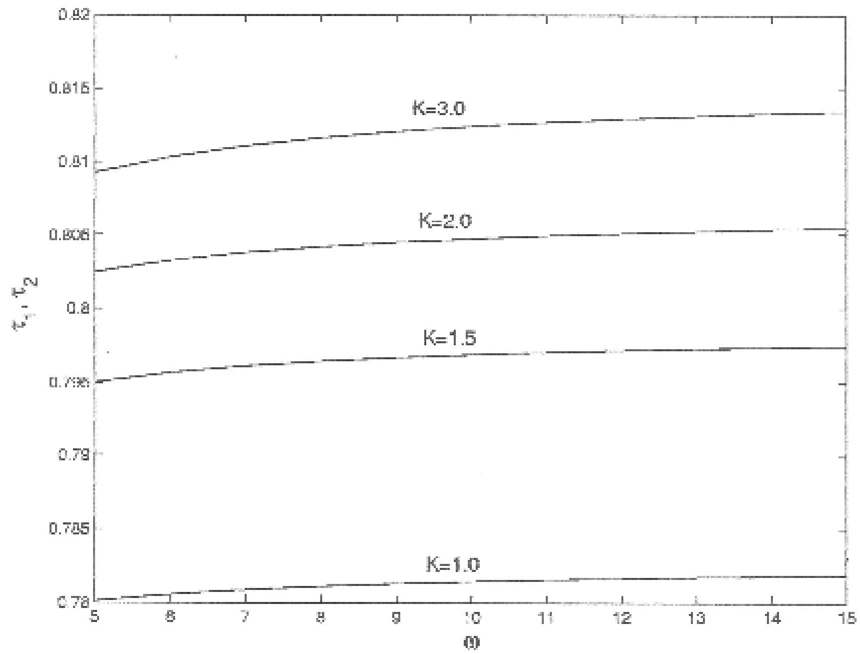

Fig.8. Heat transfer coefficient $Nu_1^{(1)}$ for Pr=0.044, $\lambda = 0.02$, $\lambda x = \pi/2$, $\omega t = \pi/4$, $\varphi = 0$.

Acknowledgment

One of the author M. Guria wishes to express his thanks to U.G.C for financial provision for this research.

Nomenclature

A, B – constants
$A_i\ (i = 1, ..., 6)$ – constants
$B_i\ (i = 1, ..., 16)$ – constants
$C_i\ (i = 1, ..., 34)$ – constants
d – half the distance between the wavy wall
g – gravitational acceleration
Gr – Grashof number
i – imaginary unity $\left(i = \sqrt{-1}\right)$
K – non-dimensional permeability parameter
K' – permeability of the porous medium
m – constant
n – constant
p^* – pressure
p – non-dimensional pressure
Pr – Prandtl number
S_{1r}, S_{1i} – constants
T – fluid temperature
T_0' – temperature of the left wall of the channel
T_1' – temperature of the right wall of the channel

ν – kinematic viscosity
x, y, z – non-dimensional Cartesian co-ordinates
α – thermal diffusivity
ε – non-dimensional amplitude parameter
λ – non-dimensional wave number
θ – non-dimensional temperature
θ_0, θ_1 – mean and perturbed parts of the temperature
ρ – density
ω – non-dimensional frequency parameter
ψ – stream function
ψ_0, ψ_1 – mean and perturbed parts of the stream function

Reference

Vajravelu K. (1989): ***Combined free and forced convection in hydro magnetic flows in vertical wavy channels with traveling thermal waves.*** – Int. J. Eng. Sci., vol.27, No.3, pp.289-300.

Chaudhary and Tara Chand (2004): ***Effect of injection through one side of a long vertical channel embedded in porous medium with transpiration cooling.*** – Bull. Cal. Math. Soc., vol.96, No.1, pp.65-70.

Takhar H.S., Soundalgekar V.M. and Gupta A.S (1990): ***Mixed convection of an incompressible viscous fluid in a porous medium past a hot vertical plate.*** – Int. J. Non Linear Mechanics, vol.25, No.6, pp.723-728.

Received: April 29, 2005
Revised: October 5, 2005