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RADIATIVE HEAT AND MASS TRANSFER EFFECTS ON MOVING ISOTHERMAL VERTICAL PLATE IN THE PRESENCE OF CHEMICAL REACTION

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The effect of thermal radiation on the unsteady free convective flow over a moving vertical plate with mass transfer in the presence of a homogeneous first order chemical reaction is considered. The fluid considered is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised to T_w and the concentration level near the plate is also raised to C'_w. The dimensionless governing equations are solved using the Laplace transform technique. The velocity and skin-friction are studied for different parameters like the radiation parameter, chemical reaction parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. It is observed that the velocity increases with decreasing radiation parameter or chemical reaction parameter.

Key words: chemical reaction, radiation, vertical plate.

1. Introduction

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion.

England and Emery (1969) have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar (1993) have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effects on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar (1996). In all above studies, the stationary vertical plate is considered. Raptis (1988) studied radiation effects on flow of a micropolar fluid past a continuously moving plate. Raptis and Perdikis (1999) studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das, *et al.* (1996) have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique.

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Diffusion rates can be altered tremendously by chemical reactions. The effect of a chemical reaction depends on whether the reaction is homogeneous or heterogeneous. It also depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate is proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

Chambre and Young (1958) have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das, *et al.* (1994) have studied the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction have been studied by Das, *et al.* (1999). The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

However, the effect of a chemical reaction on moving infinite isothermal vertical plate in the presence of thermal radiation is not studied in the literature. Therefore, it is proposed to study thermal radiation effects on flow past an impulsively started infinite isothermal vertical plate in the presence of first order chemical reaction. The dimensionless governing equations are solved using the Laplace transform technique.

2. Basic equations and analysis

The unsteady flow of a viscous incompressible fluid past an impulsively started infinite isothermal vertical plate with uniform mass diffusion is studied. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The x' – axis is taken along the plate in the vertically upward direction and the y – axis is taken normal to the plate. Initially the plate and fluid are at the same temperature and concentration. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity u_0 in a fluid, in the presence of thermal radiation. At the same time the plate temperature is raised to T_w and also the level of concentration is raised to C'_w. The fluid considered is a gray, absorbing-emitting radiation but a non-scattering medium. It is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\left(\frac{\partial^2 u}{\partial y^2}\right), \tag{2.1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_l C', \tag{2.3}$$

with the following initial and boundary conditions

$$\begin{aligned}
t' \le 0: &\quad u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y, \\
t' > 0: &\quad u = u_0, \quad T = T_w, \quad C' = C'_w \quad \text{at} \quad y = 0, \\
&\quad u = 0, \quad T \to T_\infty, \quad C' = C'_\infty \quad \text{as} \quad y \to \infty.
\end{aligned} \tag{2.4}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma \left(T_\infty^4 - T^4\right). \tag{2.5}$$

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 . \tag{2.6}$$

By using Eqs (2.5) and (2.6), Eq.(2.2) reduces to

$$\rho C_p = \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 \left(T_\infty - T\right). \tag{2.7}$$

Introducing the following non-dimensional quantities

$$U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\mathrm{Gr} = \frac{g\beta\nu\left(T_w - T_\infty\right)}{u_0^3}, \quad C = \frac{C' - C_\infty}{C'_w - C'_\infty}, \quad \mathrm{Gc} = \frac{\nu g \beta^* \left(C'_w - C'_\infty\right)}{u_0^3}, \tag{2.8}$$

$$\mathrm{Pr} = \frac{\mu C_p}{k}, \quad \mathrm{Sc} = \frac{\nu}{D}, \quad R = \frac{16 a^* \nu^2 \sigma T_\infty^3}{k u_0^2}, \quad K = \frac{\nu K_l}{u_0^2},$$

in Eqs (2.1) to (2.4), leads to

$$\frac{\partial U}{\partial t} = \mathrm{Gr}\theta + \mathrm{Gc}\, C + \frac{\partial^2 U}{\partial Y^2}, \tag{2.9}$$

$$\mathrm{Pr} \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial Y^2} - R\,\theta, \tag{2.10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{\mathrm{Sc}} \frac{\partial^2 C}{\partial Y^2} - KC . \tag{2.11}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{aligned}
& U = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all} \quad Y, \quad t \le 0, \\
t > 0: \quad & U = 1, \quad \theta = 1, \quad C = 1, \quad \text{at} \quad Y = 0, \\
& U = 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as} \quad Y \to \infty .
\end{aligned} \tag{2.12}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydromagnetic flow field in the presence of first order chemical reaction.

Equations (2.9) to (2.11), subject to the boundary conditions (2.12), are solved by the usual Laplace-transform technique and the solutions are derived as follows

$$\theta = \frac{1}{2}\left[\exp\left(2\eta\sqrt{\Pr ct}\right)\mathrm{erfc}\left(\eta\sqrt{\Pr}+\sqrt{ct}\right)+\exp\left(-2\eta\sqrt{\Pr ct}\right)\mathrm{erfc}\left(\eta\sqrt{\Pr}-\sqrt{ct}\right)\right], \tag{2.13}$$

$$\begin{aligned}
U = &\left(1+\frac{\mathrm{Gr}}{a(1-\Pr)}+\frac{\mathrm{Gc}}{b(1-\mathrm{Sc})}\right)\mathrm{erfc}(\eta)+\\
&-\frac{\mathrm{Gr}\exp(at)}{2a(1-\Pr)}\left[\exp\left(2\eta\sqrt{at}\right)\mathrm{erfc}\left(\eta+\sqrt{at}\right)+\exp\left(-2\eta\sqrt{at}\right)\mathrm{erfc}\left(\eta-\sqrt{at}\right)\right]+\\
&-\frac{\mathrm{Gr}\exp(bt)}{2a(1-\mathrm{Sc})}\left[\exp\left(2\eta\sqrt{bt}\right)\mathrm{erfc}\left(\eta+\sqrt{bt}\right)+\exp\left(-2\eta\sqrt{bt}\right)\mathrm{erfc}\left(\eta-\sqrt{bt}\right)\right]+\\
&-\frac{\mathrm{Gr}}{2a(1-\Pr)}\left[\exp\left(2\eta\sqrt{Rt}\right)\mathrm{erfc}\left(\eta+\sqrt{ct}\right)+\exp\left(-2\eta\sqrt{Rt}\right)\mathrm{erfc}\left(\eta-\sqrt{ct}\right)\right]+\\
&+\frac{\mathrm{Gr}\exp(at)}{2a(1-\Pr)}\left[\exp\left(2\eta\sqrt{\Pr(a+c)t}\right)\mathrm{erfc}\left(\eta\sqrt{\Pr}+\sqrt{(a+c)t}\right)+\right.\\
&\left.+\exp\left(-2\eta\sqrt{\Pr(a+c)t}\right)\mathrm{erfc}\left(\eta\sqrt{\Pr}-\sqrt{(a+c)t}\right)\right]+\\
&-\frac{\mathrm{Gc}}{2b(1-\mathrm{Sc})}\left[\exp\left(2\eta\sqrt{Kt\,\mathrm{Sc}}\right)\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}+\sqrt{Kt}\right)+\right.\\
&\left.+\exp\left(-2\eta\sqrt{Kt\,\mathrm{Sc}}\right)\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}-\sqrt{Kt}\right)\right]+\\
&+\frac{\mathrm{Gc}\exp(bt)}{2b(1-\mathrm{Sc})}\left[\exp\left(2\eta\sqrt{\mathrm{Sc}(K+b)t}\right)\mathrm{erfc}\left(\eta+\sqrt{(K+b)t}\right)+\right.\\
&\left.+\exp\left(-2\eta\sqrt{\mathrm{Sc}(K+b)t}\right)\mathrm{erfc}\left(\eta-\sqrt{(K+b)t}\right)\right],
\end{aligned} \tag{2.14}$$

$$C = \frac{1}{2}\left[\exp\left(2\eta\sqrt{Kt\,\mathrm{Sc}}\right)\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}+\sqrt{Kt}\right)+\exp\left(-2\eta\sqrt{Kt\,\mathrm{Sc}}\right)\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}-\sqrt{Kt}\right)\right] \tag{2.15}$$

where $\eta=\frac{Y}{2\sqrt{t}}$, $a=\frac{R}{1-\Pr}$, $b=\frac{K\,\mathrm{Sc}}{1-\mathrm{Sc}}$, and $c=\frac{R}{\Pr}$.

3. Discussion of results

The numerical values of the velocity and skin-friction are computed for different parameters such as the radiation parameter, chemical reaction parameter. Schmidt number, thermal Grashof number and mass Grashof number and time. The purpose of the calculations given here is to assess the effects of the parameters *R*, *K*, Gr, Gc and Sc upon the nature of the flow and transport. The solutions are in terms of exponential and complementary error function.

The velocity profiles for different values of the radiation parameter $(R=2, 5, 10)$, Gr=*2*, Gc=*2*, Sc=*0.6*, *K*=*0.2*, *M*=*2*, Pr=*0.71* and *t*=*0.2* are shown in Fig.1. It is observed that the velocity increases with decreasing the radiation parameter. This shows that velocity decrease in the presence of high thermal radiation.

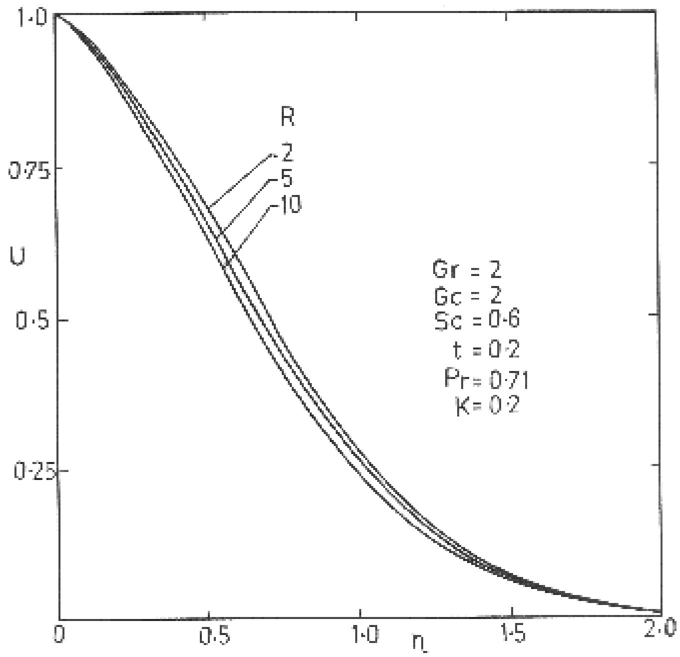

Fig.1. Velocity profiles for different *R*.

In Fig.2, the velocity profiles are studied for different Schmidt numbers $(\mathrm{Sc}=0.16, 0.6, 0.78)$ and chemical reaction parameter $(K=0.2, 2)$ Gr=*5*, Gc=*5*, *R*=*2*, Pr=*0.71* and *t*=*0.2*. It is observed that the velocity decreases with increasing *K* or Sc. This shows that the increase in the chemical reaction parameter or Schmidt number leads to a fall in the velocity.

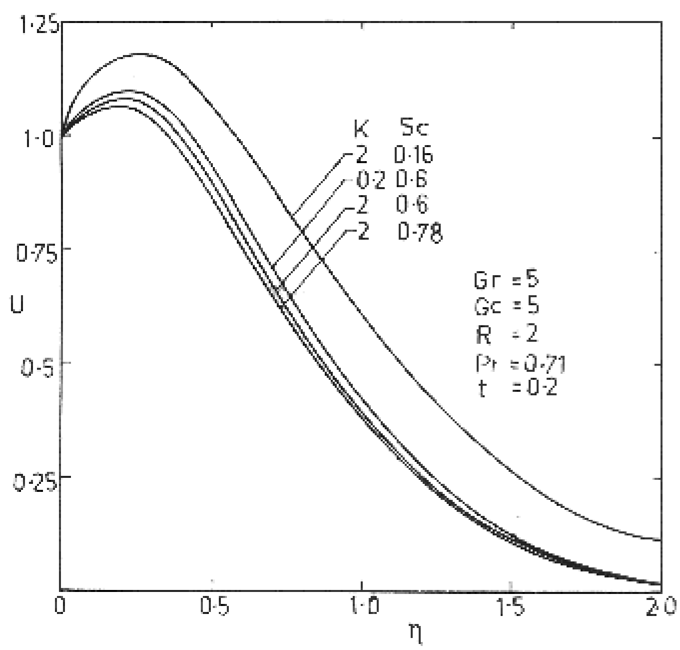

Fig.2. Velocity profiles for different *K* and Sc.

The velocity profiles for different thermal Grashof numbers $(\mathrm{Gr}=2,5)$, mass Grashof number $(\mathrm{Gc}=2,5)$, K=*0.2*, R=*2*, Sc=*0.6*, Pr=*0.71* and time (t=*0.2*, *0*, *4*) are shown in Fig.3. It is clear that the velocity increases with increasing the thermal Grashof number of mass Grashof number. The trend is also same with respect to time.

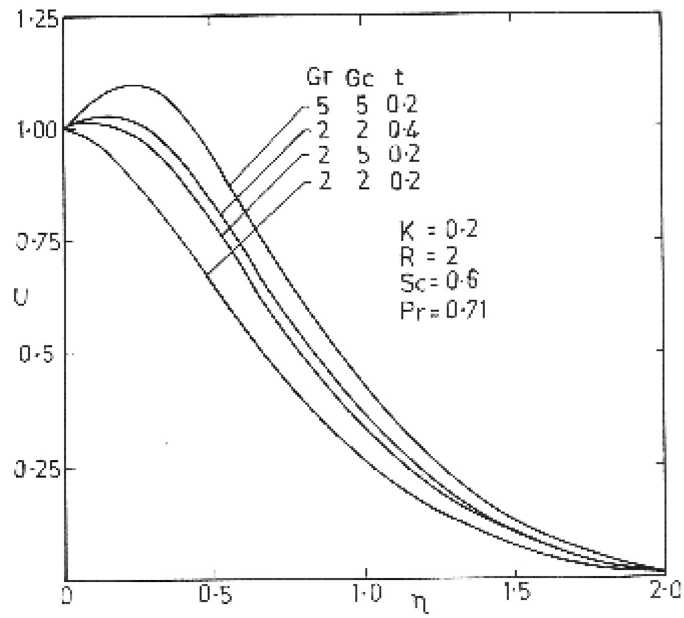

Fig.3. Velocity profiles for different Gr, Gc and *t*.

From the velocity field, the effect of mass transfer on the skin-friction is studied and given in dimensionless form as

$$\tau=-\left(\frac{dU}{dY}\right)_{Y=0}=-\frac{1}{2\sqrt{t}}\left(\frac{dU}{d\eta}\right)_{\eta=0}. \tag{3.1}$$

Hence, from Eqs (2.14) and (3.1)

$$\begin{aligned}\tau=&\frac{1}{\pi\sqrt{t}}\left[\left(1+\frac{\mathrm{Gr}}{a(1-\mathrm{Pr})}+\frac{\mathrm{Gc}}{b(1-\mathrm{Sc})}\right)+\right.\\&-\frac{\mathrm{Gr}\exp(at)}{a(1-\mathrm{Pr})}\left(1+\sqrt{a\pi t}\ \mathrm{erf}\left(\sqrt{at}\right)\right)-\frac{\mathrm{Gc}\exp(bt)}{b(1-\mathrm{Sc})}\left(1+\sqrt{b\pi t}\ \mathrm{erf}\left(\sqrt{bt}\right)\right)+\\&-\frac{\mathrm{Gr}}{a(1-\mathrm{Pr})}\left(\sqrt{\mathrm{Pr}}+\sqrt{\pi Rt}\ \mathrm{erf}\ \sqrt{ct}\right)+\\&+\frac{\mathrm{Gr}\exp(at)}{a(1-\mathrm{Pr})}\left(\sqrt{\mathrm{Pr}}+\sqrt{\mathrm{Pr}(a+c)\pi t}\ \ \mathrm{erf}\left(\sqrt{(a+c)t}\right)\right)+\\&-\frac{\mathrm{Gc}}{b(1-\mathrm{Sc})}\left(\sqrt{\mathrm{Sc}}+\sqrt{K\mathrm{Sc}\pi t}\ \mathrm{erf}\left(\sqrt{Kt}\right)\right)+\\&\left.+\frac{\mathrm{Gc}\exp(bt)}{b(1-\mathrm{Sc})}\left(\sqrt{\mathrm{Sc}}+\sqrt{\mathrm{Sc}(K+b)\pi t}\ \ \mathrm{erf}\left(\sqrt{(K+b)t}\right)\right)\right].\end{aligned} \tag{3.2}$$

The numerical values of τ are presented in Tab.1.

K	*R*	*t*	Gr	Gc	Sc	τ
0.2	*0.2*	*0.2*	*5*	*5*	*0.62*	*- 23.0480*
2	*0.2*	*0.2*	*5*	*5*	*0.6*	*- 13.4888*
0.2	*2*	*0.2*	*5*	*5*	*0.6*	*- 16.2388*
2	*2*	*0.2*	*5*	*5*	*0.6*	*- 6.6796*
0.2	*2*	*0.2*	*2*	*5*	*0.6*	*- 13.9903*
0.2	*2*	*0.2*	*2*	*2*	*0.6*	*- 5.7386*
0.2	*2*	*0.4*	*5*	*5*	*0.6*	*- 19.6810*
0.2	*0.2*	*0.2*	*5*	*5*	*0.3*	*- 28.9072*
0.2	*0.2*	*0.2*	*5*	*5*	*0.16*	*- 36.3552*

It is observed that a decrease in the Schmidt number or the chemical reaction parameter leads to a fall in the value of the skin-friction but an increase in the chemical reaction parameter leads to a rise in the value of skin-friction. As time advances the value of the skin-friction decreases. Moreover, the value of the skin-friction decreases with increasing the thermal Grashof number or mass Grashof number.

4. Conclusions

An exact analysis is performed to study thermal radiation effects on flow past an impulsively started infinite isothermal vertical plate in the presence of a chemical reaction of first order. The dimensionless governing equations are solved by the usual Laplace -transform technique. The effect of different parameters such as the radiation parameter, thermal Grashof number, mass Grashof number Schmidt number, chemical reaction parameter and time are studied. The conclusions of the study are as follows.

(i) It is observed that the velocity increases with decreasing the chemical reaction parameter or radiation parameter.
(ii) The velocity increases with increasing the thermal Grashof number or mass Grashof number.
(iii) The skin-friction increases with decreasing *R* or *K*.

Nomenclature

a^* – absorption coefficient
C – dimensionless concentration
C_p – specific heat at constant pressure
C' – species concentration in the fluid
C'_w – concentration on the plate
C'_∞ – concentration in the fluid far away from the plate
D – mass diffusion coefficient
g – acceleration due to gravity
Gc – mass Grashof number
Gr – thermal Grashof number
k – thermal conductivity of the fluid
K – dimensionless chemical reaction parameter
K_l – chemical reaction parameter
Pr – Prandtl number
q_r – radiative heat flux in the *y*-direction
R – radiation parameter
Sc – Schmidt number
t – dimensionless time

t' – time
T – temperature of the fluid near the plate
T_w – temperature of the plate
T_∞ – temperature of the fluid far away from the plate
u – velocity of the fluid in the *x*-direction
u_0 – velocity of the plate
U – dimensionless velocity
y – coordinate axis normal to the plate
Y – dimensionless coordinate axis normal to the plate
β – volumetric coefficient of thermal expansion
β^* – volumetric coefficient of expansion with concentration
μ – coefficient of viscosity
ν – kinematic viscosity
ρ – density
σ – Stefan-Boltzmann constant
τ – dimensionless skin-friction
θ – dimensionless temperature
η – similarity parameter
erfc – complementary error function

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