Brief note

HEAT TRANSFER IN A TURBULENT CHANNEL FLOW WITH A PERMEABLE WALL

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A problem of turbulent flow in a channel with a permeable wall recently considered in Hahn and Choi (2002) is extended to include heat transfer. A $k-\varepsilon$ model is utilized to investigate this problem numerically. The modeling is based on the assumption that the flow in the channel is turbulent while in the porous block the flow remains laminar. The wall functions approach is utilized to determine the boundary conditions for the k and ε equations. The dependence of the Nusselt number on the Darcy and Reynolds numbers is investigated.

1. Introduction

Fluid flow in a channel bounded on one side by a solid wall and on the other side by a porous block has been recently investigated by Hahn and Choi (2002) who were interested in extending the classical problem of Beavers and Joseph (1967) to the case when the flow in the channel is turbulent. The purpose of this paper is to extend this problem to include heat transfer due to turbulent forced convection flow in the channel.

Figure 1 displays a schematic diagram of the problem investigated in this paper. A parallel-plate channel of thickness H is bounded on the top by a solid wall and on the bottom by a porous block whose thickness is the same as that of the channel. The flow is driven by a constant pressure gradient. Two cases of the thermal boundary condition at the upper wall are considered: the upper wall is assumed to be either subjected to a constant heat flux or isothermal (with a temperature T_w). The lower wall (the one that bounds the porous block) is assumed adiabatic.

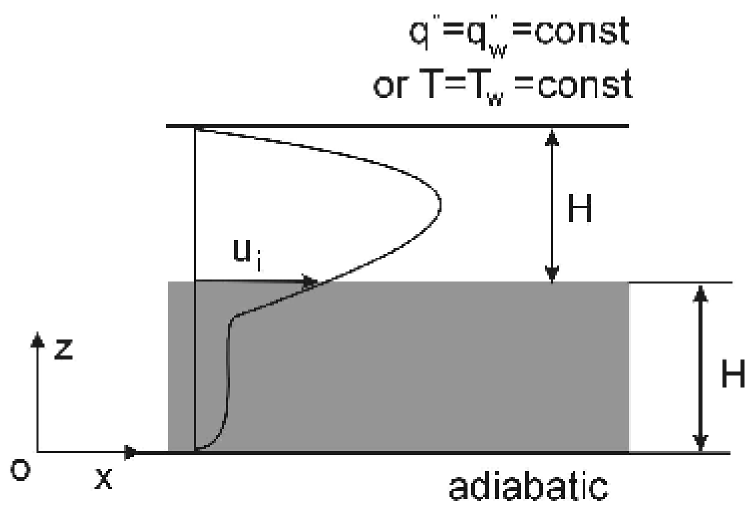

Fig.1. Schematic diagram of the problem.

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Although many recent papers considered the modeling of turbulence in porous media (see recent review chapter by Lage *et al*. (2002)), herein the assumption utilized in Hahn and Choi (2002), that the flow in the porous block remains laminar, was adopted. As discussed in Zhu and Kuznetsov (2005), this assumption will probably fail if the medium is hyperporous, but should hold for most practical porous media.

2. Mathematical model

As Fig.1 shows, the flow domain can be divided into two regions, the clear fluid region (the channel) where the flow is assumed turbulent, and the porous block, where the flow is assumed laminar. In the turbulent (clear fluid) region, a $k-\varepsilon$ model is utilized.

3. Momentum equation for the clear fluid region and the $k-\varepsilon$ model formulation

For a hydrodynamically fully developed flow, the velocity distribution in the clear fluid region can be computed from the following equation

$$0=\frac{1}{2H^+}+\frac{d}{dz^+}\left[\frac{du^+}{dz^+}+\mu_T^+\frac{du^+}{dz^+}\right] \tag{3.1}$$

where u^+ is the dimensionless velocity, u/u_τ; u is the longitudinal velocity; u_τ is the wall friction velocity, $\sqrt{|\tau_w|/\rho}$; τ_w is the shear stress at the upper wall (at $z=2H$); H^+ is the dimensionless thickness of the gap between the wall and the porous interface, $u_\tau H/\nu$; H is the thickness of the gap between the wall and the porous interface; ν is the fluid kinematic viscosity; z^+ is the dimensionless vertical coordinate, $u_\tau z/\nu$; x^+ is the dimensionless horizontal coordinate, $u_\tau x/\nu$; μ_T^+ is the dimensionless eddy viscosity, μ_T/μ; μ is the fluid dynamic viscosity; and μ_T is the eddy viscosity. It should be noted that H^+ can also be interpreted as a Reynolds number based on the channel width and the friction velocity at the upper wall.

At the upper wall, Eq.(3.1) must satisfy the following no-slip boundary condition

$$u^+=0 \quad \text{at} \quad z^+=2H^+. \tag{3.2}$$

In the clear fluid region, the following $k-\varepsilon$ model is utilized. For the fully developed flow, the turbulence kinetic energy equation can be presented as

$$\mu_T^+\left(\frac{\partial u^+}{\partial z^+}\right)^2-\varepsilon^+ +\frac{\partial}{\partial z^+}\left[\left(1+\mu_T^+\right)\frac{\partial k^+}{\partial z^+}\right]=0 \tag{3.3a}$$

where

$$k^+=k/u_\tau^2 \quad \text{and} \quad \varepsilon^+=\varepsilon\nu_f/u_\tau^4. \qquad (3.3b)\backslash$$

The dissipation rate equation for the fully developed flow is

$$C_{\varepsilon 1}\frac{\varepsilon^+}{k^+}\mu_T^+\left(\frac{\partial u^+}{\partial z^+}\right)^2-C_{\varepsilon 2}\frac{\left(\varepsilon^+\right)^2}{k^+}+\frac{\partial}{\partial z^+}\left[\left(1+\frac{\mu_T^+}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon^+}{\partial z^+}\right]=0\,. \tag{3.4}$$

The dimensionless eddy viscosity can be found from the following equation

$$\mu_T^+=C_\mu\frac{\left(k^+\right)^2}{\varepsilon^+}\,. \tag{3.5}$$

The closure coefficients for the $k-\varepsilon$ model are

$$C_{\varepsilon 1}=1.44\,,\qquad C_{\varepsilon 2}=1.92\,,\qquad \sigma_\varepsilon=1.3\qquad\text{and}\qquad C_\mu=0.09\,. \tag{3.6}$$

Boundary conditions for the k and ε equations are found from the wall functions (Henkse and Hoogendoorn, 1989). It should be noted that wall functions are traditionally used to determine boundary conditions at the solid wall, and the utilization of wall functions to determine boundary conditions at the interface is an additional approximation. For k^+, the following boundary condition is imposed at the interface and at the wall, respectively

$$k^+=\frac{1}{\sqrt{C_\mu}}\qquad\text{at}\qquad z^+=H^+\qquad\text{and}\qquad z^+=2H^+\,. \tag{3.7}$$

For ε^+, the following boundary condition is imposed at the first inner computational grid points from the interface and the wall (these grid points are located inside the clear fluid region)

$$\varepsilon^+=\frac{1}{0.41\Delta z^+}\qquad\text{at}\qquad z^+=H^++\Delta z^+\qquad\text{and}\qquad z^+=2H^+-\Delta z^+ \tag{3.8}$$

where Δz^+ is the mesh size in the *z*-direction (the grid points where this condition is imposed must be positioned outside the laminar sublayer, meaning that their distance from the interface and wall, respectively, must be larger than *11.5* in the wall coordinates).

In order to check whether the flow in the clear fluid gap is indeed turbulent, the Reynolds number based on the width of the clear fluid domain, *H*, and the mean velocity in this region, $\left(U_m\right)_{clear\,fl}$, is calculated as

$$\mathrm{Re}_{clear\,fl}=\left(U_m\right)_{clear\,fl}H/\nu_f=\left(U_m^+\right)_{clear\,fl}H^+ \tag{3.9}$$

where $\left(U_m^+\right)_{clear\,fl}$ is the dimensionless mean velocity in the clear fluid region, $\left(U_m\right)_{clear\,fl}/u_\tau$

$$\left(U_m^+\right)_{clear\,fl}=\frac{1}{H^+}\int_{H^+}^{2H^+}u^+dz^+\,. \tag{3.10}$$

For the flow in a channel, the critical Reynolds number is 4×10^3.

4. Momentum equation for the porous region

The Brinkman-Forchheimer-extended Darcy equation (1999) is utilized to model the laminar flow in the porous region. Utilizing the dimensionless variables defined above, this equation can be presented as

$$\frac{1}{2H^+}+\left(\frac{\mu_{eff}}{\mu}\right)\frac{d^2u^+}{\left(dz^+\right)^2}-\frac{u^+}{\mathrm{Da}\left(H^+\right)^2}-\frac{c_F}{\mathrm{Da}^{1/2}H^+}\left(u^+\right)^2=0 \tag{4.1}$$

where c_F is the Forchheimer coefficient; Da is the Darcy number, K/H^2 ; K is the permeability of the porous medium; and μ_{eff} is the effective viscosity in the porous region.

To match the laminar flow velocity in the porous region with the turbulent flow velocity in the clear fluid region the following matching conditions are utilized at the porous/fluid interface

$$u^+\Big|_{z^+=H^+-0}=u^+\Big|_{z^+=H^+ +0}=u_i^+ \quad \text{and} \quad \left(\frac{\mu_{eff}}{\mu_f}\right)\frac{\partial u^+}{\partial z^+}\bigg|_{z^+=H^+-0}=\frac{\partial u^+}{\partial z^+}\bigg|_{z^+=H^+ +0},$$
$$\text{at} \qquad z^+=H^+. \tag{4.2}$$

The second equation in (4.2) implies direct matching of the shear stress on the porous and clear fluid sides of the interface. This assumption could be relaxed to allow for a possible jump in the shear stress at the interface; however, for the purpose of this study the additional complexity is not necessary. The second equation in (4.2) also assumes that the wall and the interface are both hydraulically smooth; this assumption could be relaxed utilizing the approach developed in Zhu and Kuznetsov (2005).

Finally, at the lower wall the following no-slip boundary condition is imposed

$$u^+=0 \qquad \text{at} \qquad z^+=0. \tag{4.3}$$

5. Heat transfer in the channel

The dimensionless temperature is defined as

$$\phi=\frac{T-T_w}{T_m-T_w} \tag{5.1}$$

where the mean flow temperature is defined as follows

$$T_m=\frac{1}{u_m(2H)}\int_0^{2H}uTdz. \tag{5.2}$$

The Nusselt number is defined as

$$\mathrm{Nu}=\frac{2Hq''_w}{k\left(T_w-T_m\right)}.$$

For the case of a constant wall heat flux, the dimensionless energy equation in the porous region $\left(0 \le z^+ \le H^+\right)$ is

$$-\frac{1}{4\left(H^+\right)^2}\frac{u^+}{u_m^+}\mathrm{Nu} = \frac{d}{dz^+}\left[\left(\frac{k_{eff}}{k_f}\right)\frac{d\phi}{dz^+}\right] \tag{5.3}$$

where

$$\frac{k_{eff}}{k_f} = \frac{k_m}{k_f} + C\mathrm{Pr}\mathrm{Re}_p u^+ , \tag{5.4}$$

is the dimensionless effective thermal conductivity of the porous medium, which accounts for thermal dispersion (Nield and Bejan, 1999).

The dimensionless mean flow velocity in Eq.(5.3) is defined as follows

$$u_m^+ = \frac{1}{2H^+}\int_0^{2H^+} u^+ dz^+ . \tag{5.5}$$

For the case of a constant wall heat flux, the dimensionless energy equation in the clear fluid region $\left(H^+ \le z^+ \le 2H^+\right)$ is

$$-\frac{1}{4\left(H^+\right)^2}\frac{u^+}{u_m^+}\mathrm{Nu} = \frac{d}{dz^+}\left[\left(1 + \mu_T^+ \frac{\mathrm{Pr}}{\mathrm{Pr}_t}\right)\frac{d\phi}{dz^+}\right]. \tag{5.6}$$

Equations (5.3) and (5.6) must be solved subject to the following boundary conditions

$$\phi = 0 \quad \text{at} \quad z^+ = 2H^+ ,$$

$$\left.\phi\right|_{z^+=H^+-0} = \left.\phi\right|_{z^+=H^++0} \quad \text{and} \quad \left.\left(\frac{k_{eff}}{k_f}\right)\frac{\partial\phi^+}{\partial z^+}\right|_{z^+=H^+-0} = \left.\frac{\partial\phi^+}{\partial z^+}\right|_{z^+=H^++0} \quad \text{at} \quad z^+ = H^+ \tag{5.7}$$

$$\frac{d\phi}{dz^+} = 0 \quad \text{at} \quad z^+ = 0 .$$

Finally, the Nusselt number can be found from the following compatibility condition

$$\int_0^{2H^+} u^+ \phi dz^+ = 2H^+ u_m^+ . \tag{5.8}$$

For the case of a constant wall temperature, the dimensionless energy equation in the porous region $\left(0 \le z^+ \le H^+\right)$ is

$$-\frac{1}{4\left(H^{+}\right)^{2}}\frac{u^{+}}{u_{m}^{+}}\mathrm{Nu}\phi=\frac{d}{dz^{+}}\left[\left(\frac{k_{eff}}{k_{f}}\right)\frac{d\phi}{dz^{+}}\right], \tag{5.9}$$

and the dimensionless energy equation in the clear fluid region $\left(H^{+}\le z^{+}\le 2H^{+}\right)$ is

$$-\frac{1}{4\left(H^{+}\right)^{2}}\frac{u^{+}}{u_{m}^{+}}\mathrm{Nu}\phi=\frac{d}{dz^{+}}\left[\left(1+\mu_{T}^{+}\frac{\mathrm{Pr}}{\mathrm{Pr}_{t}}\right)\frac{d\phi}{dz^{+}}\right]. \tag{5.10}$$

The boundary conditions defined by Eq.(5.7) stand, and the compatibility condition for finding the Nusselt number becomes

$$\mathrm{Nu}=-2H^{+}\left.\frac{\partial\phi}{\partial z^{+}}\right|_{z+=2H^{+}}. \tag{5.11}$$

6. Results and discussion

Computations are performed for the following parameter values

$$\begin{gathered}H^{+}=10^{3};\quad \mathrm{Da}=10^{-2},\quad 10^{-3},\quad \text{and}\quad 10^{-4};\quad c_{F}=0.55;\quad \mu_{eff}/\mu_{f}=1;\\ \varphi=0.95;\quad k_{m}/k_{f}=1;\quad C=0.1;\quad \mathrm{Pr}=1;\quad \mathrm{Pr}_{t}=1;\quad \mathrm{Re}_{p}=H^{+}\frac{\sqrt{180}\,(1-\varphi)}{\varphi^{3/2}}\mathrm{Da}^{1/2}.\end{gathered} \tag{6.1}$$

Figure 2a displays velocity distributions in the channel for three different values of the Darcy number. As expected, the increase of the Darcy number leads to an increase of the filtration velocity in the porous block. It also leads to an increase of the dimensionless velocity at the porous/fluid interface, u_{i}^{+}, and, consequently, to a velocity increase in the clear fluid region. Figures 2b and 2c display distributions of the dimensionless temperature for the constant wall heat flux and the constant wall temperature at the upper wall, respectively. The kinks in the profiles of the dimensionless temperature at the interface (at $z^{+}=1000$), which are clearly visible on the curves that correspond to Darcy numbers equal to 10^{-3} and 10^{-2}, are explained by the influence of thermal dispersion on thermal conductivity in the porous layer. According to Eq.(5.4), mixing at a pore scale results in increasing effective thermal conductivity of the porous layer as velocity in the porous layer increases. The kinks are not visible on the temperature profiles that correspond to $\mathrm{Da}=10^{-4}$ because, according to Fig.2a, for this Darcy number the dimensionless velocity at the porous/fluid interface is too small to cause a significant increase of thermal conductivity in the porous layer.

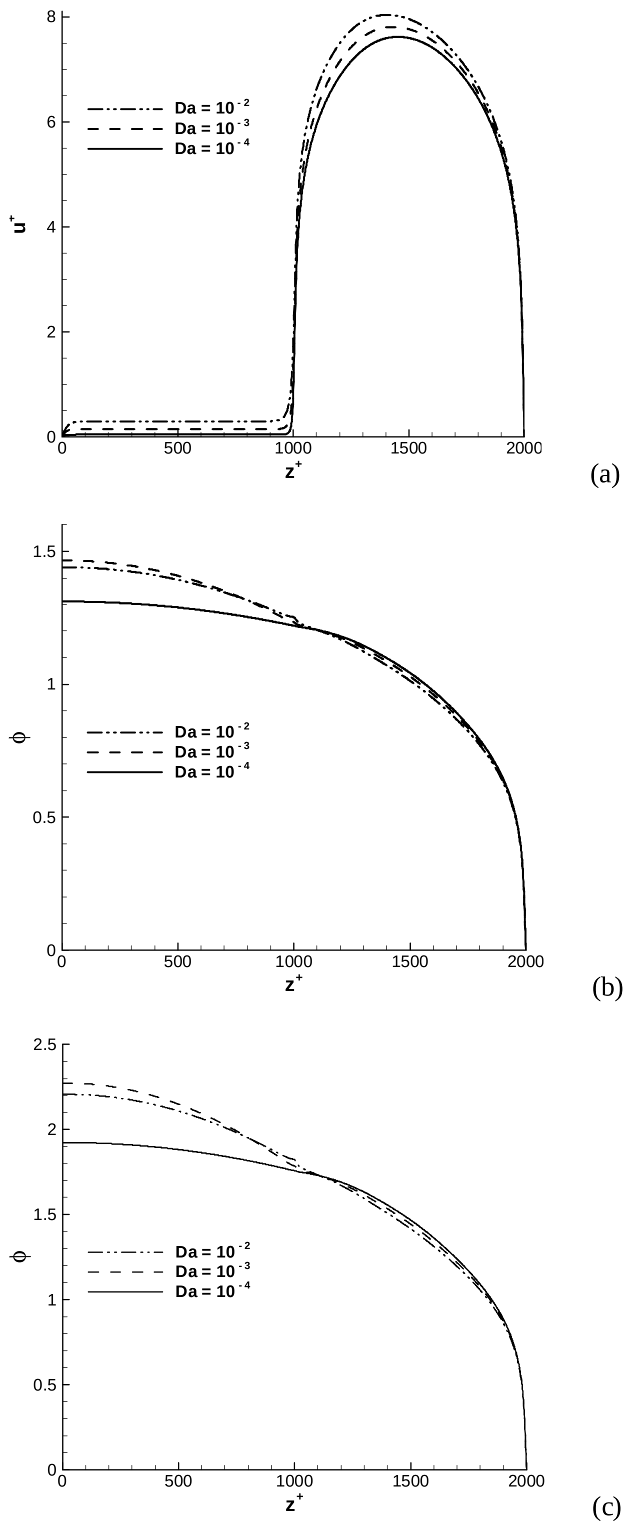
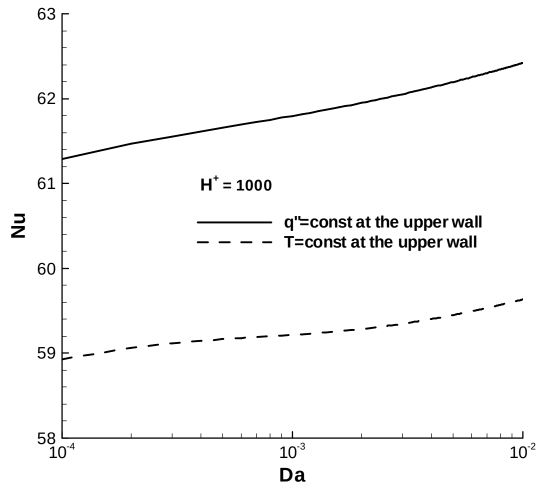
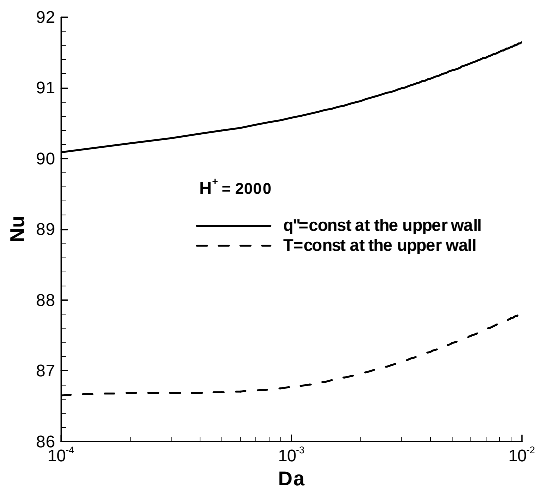


Fig.2. Profiles of the dimensionless velocity (a) and dimensionless temperature for the isoflux (b) and isothermal (c) upper wall.

Figures 3a and 3b display the dependence of the Nusselt number on the Darcy number for $H^+ = 1000$ and *2000*, respectively. As expected, the Nusselt number increases as the Darcy number increases. Nu also increases as the Reynolds number based on the channel width and the friction velocity at the upper wall, $H^+ = 1000$, increases.



(a)



(b)

Fig.3. Nusselt number versus Darcy number for $H^+ = 1000$ (a) and $H^+ = 2000$ (b).

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