Int. J. of Applied Mechanics and Engineering, 2006, vol.11, No.3, pp.715-722

Letter to the Editor

A NOTE ON THE EFFECT OF RADIATION ON FREE CONVECTION OVER A VERTICAL FLAT PLATE EMBEDDED IN A NON-NEWTONIAN FLUID SATURATED POROUS MEDIUM

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The effect of radiation on the free convection from a vertical plate embedded in a power-law fluid saturated porous media has been considered. Similarity equations have been obtained and solved numerically. It was found that there is an increase in the boundary layer thickness with an increase in the radiation parameter N and a decrease in the power-law index n was observed.

Key words: porous media, non-Darcy law, boundary layer, radiation.

1. Introduction

Heat transfer in porous media occurs in practical applications in geophysics, energy related problems, environment, etc. An excellent summary of the work on this subject is given in the monographs by Ingham and Pop (1998; 2002), Nield and Bejan (1999), Vafai (2000), Pop and Ingham (2001), Bejan and Kraus (2003), Ingham *et al.* (2004) and Bejan *et al.* (2004).

Many fluids involved in industrial applications have a non-Newtonian behaviour. On the other hand, if the processes take place at a high temperature, radiative effects cannot be neglected (Modest, 2003; Siegel and Howell, 1992). The effects of radiation on free convection past a horizontal plate with a variable wall temperature and embedded in a non-Newtonian fluid saturated porous medium has been studied by Mehta and Rao (1994). Mansour and Gorla (1998) have considered the case of mixed convection from a wedge and Mohammadein and El-Amin (2000) have studied the case of mixed convection from a horizontal plate in a porous medium.

The object of this Note is to study the radiation effects on free convection from a vertical flat plate embedded in a power-law fluid saturated porous medium using the Rosseland model.

2. Basic equations

Consider a semi-infinite vertical flat plate which is maintained at a constant temperature T_w and which is embedded in a non-Newtonian fluid saturated porous medium of ambient temperature T_∞, see Fig.1. Using the boundary layer and Boussinesq approximations the mathematical model is given by the continuity, modified Darcy law and energy equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{2.1}$$

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$$u^n = \frac{g\,K^*(n)\beta}{\nu^*}(T - T_\infty), \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_f}\frac{\partial q^r}{\partial y}, \tag{2.3}$$

respectively, where x and y are the Cartesian co-ordinates along and normal to the plate, respectively, u and v are the velocity components along x- and y- axes, respectively, T is the fluid temperature , n is the power-law index, β is the thermal expansion coefficient, ν^* is the modified kinematic viscosity, α_m is the thermal diffusivity of the medium, ρ is the density, c_p is the specific heat at constant pressure, q^r is the radiative heat flux, and the lower indices ∞ and f refer to the ambient conditions and fluid phase, respectively, and the upper index * refers to the modified quantities in the power-law fluid. The modified permeability $K^*(n)$ is given by

$$K^*(n) = \begin{cases} \dfrac{6}{25}\left(\dfrac{n\varphi}{3n+1}\right)^n \left(\dfrac{\varphi d}{3(1-\varphi)}\right)^{n+1} & \text{(Christopher and Middleman, 1965)} \\[2ex] \dfrac{2}{\varphi}\left(\dfrac{d\varphi^2}{8(1-\varphi)}\right)^{n+1} \dfrac{6n+1}{10n-3}\left(\dfrac{16}{75}\right)^{\frac{3(10n-3)}{10n+11}} & \text{(Darmadhikari and Kale, 1985)} \end{cases} \tag{2.4}$$

where φ is the porosity and d is the particle diameter. It is worth mentioning that $n < 1$ corresponds to a pseudoplastic fluid, $n = 1$ to a Newtonian fluid and $n > 1$ to a dilatant fluid. The radiative heat flux q^r under the Rosseland approximation has the form

$$q^r = -\left(\frac{4\sigma}{3\chi}\right)\frac{\partial T^4}{\partial y} \tag{2.5}$$

where σ is the Stefan-Boltzman's constant and χ is the mean absorption coefficient.

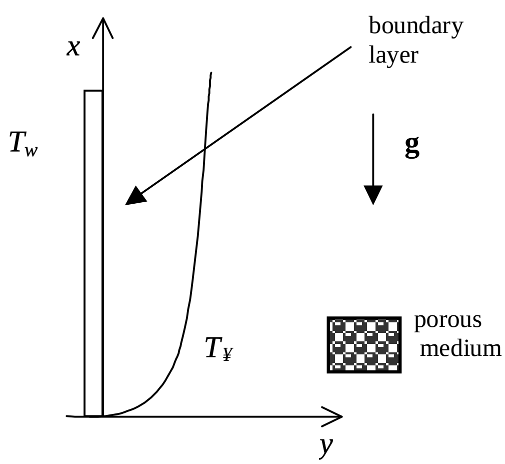

Fig.1. Physical model and co-ordinate system.

Equations (2.1)-(2.3) have to be solved subject to the boundary conditions

$$\begin{aligned} &v = 0\,, \quad T = T_w \quad \text{for} \quad y = 0\,, \quad x > 0\,, \\ &u \to 0\,, \quad T \to T_\infty \quad \text{for} \quad y \to \infty\,, \quad -\infty < x < \infty\,. \end{aligned} \tag{2.6}$$

In order to obtain similarity solutions, i.e., the governing partial differential equations reduce to ordinary differential equations, of Eqs (2.1)-(2.3) subject to the boundary conditions (2.6), we introduce the following variables

$$f(\eta) = \psi / \alpha_m \mathrm{Ra}_x^{*1/2}\,, \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\,, \qquad \eta = \mathrm{Ra}_x^{*1/2}(y/x) \tag{2.7}$$

where ψ is the stream function defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ and Ra_x^* is the modified local Rayleigh number which is defined as

$$\mathrm{Ra}_x^* = \left(\frac{g\beta K^*(n)(T_w - T_\infty)x^n}{v^* \alpha_m^n} \right)^{\frac{1}{n}}. \tag{2.8}$$

Substituting (2.7) into Eqs (2.1)-(2.3) and (2.6), we obtain the following ordinary differential equations

$$(f')'' = 0\,, \tag{2.9}$$

$$\left\{ \left[1 + \frac{4}{3} N [1 + (\theta_w - 1)\theta]^3 \right] \theta' \right\}' + \frac{1}{2} f\,\theta' = 0\,, \tag{2.10}$$

subject to the boundary conditions

$$f(0) = 0, \qquad \theta(0) = 1, \qquad f'(\infty) = 0 \tag{2.11}$$

where N is the radiation parameter

$$N = \frac{4\sigma T_\infty^3}{k\chi}\,, \tag{2.12}$$

and θ_w is the temperature parameter

$$\theta_w = \frac{T_w}{T_\infty}. \tag{2.13}$$

Using the energy balance on the surface of the plate it is possible to calculate the coefficient of the convective heat transfer, *h*, defined as

$$-k\left(\frac{\partial T}{\partial y}\right)_{y=0} + q^r = h\left(T_w - T_\infty\right), \tag{2.14}$$

and thus we can obtain the local Nusselt number

$$\mathrm{Nu}_x \Big/ \mathrm{Ra}_x^{*\,1/2} = -\left(1 + \frac{4}{3} N \theta_w^3\right)\theta'(0). \tag{2.15}$$

We mention that in the case when the radiation effect is absent $(N = 0)$, Eqs (2.9) and (2.10) reduce to those obtained by Chen and Chen (1988).

3. Results and discussion

Equations (2.9) and (2.10), subject to the boundary conditions (2.11) were solved numerically using the Runge-Kutta method in combination with a shooting technique for different values of the parameters N, θ_w and n $(N = 0,1,5,10;\ \theta_w = 1.1,1.5,2.0;\ n = 0.5,0.8,1,1.5,2.5)$. The obtained values of the local Nusselt number when the radiation effect is absent $(N = 0)$ are given in Tab.1 and the values of Chen and Chen (1988) are also included in this table for the sake of comparison. It is seen that the present results are in very good agreement with those reported by Chen and Chen (1988) and this confirms the accuracy of the present method. The values of the local Nusselt number for different values of the parameters $N\ (\neq 0)$, θ_w and n are given in Tab.2.

Figures 2-4 show the dimensionless temperature profiles for some values of the governing parameters of interest. We can see that the thickness of the boundary layer increases as the radiation parameter N and the temperature parameter θ_w increase. The radiation effect is more evident by increasing the boundary layer thickness for the pseudoplastic fluids $(n < 1)$, the thickness becoming lower for the Newtonian $(n = 1)$ and dilatant $n > 1$ fluids.

Table 1. Values of the local Nusselt number $-\theta'(0)$ for $N = 0$ and different values of the parameter n.

N	Chen and Chen (1988)	present results
0.5	*0.3768*	*0.377670*
0.8	*0.4238*	*0.423999*
1.0	*0.4437*	*0.443885*
1.5	*0.4752*	*0.475379*
2.0	*0.4938*	*0.493804*
2.5	*0.5059*	*0.505912*

Table 2. Values of the local Nusselt number $-\theta'(0)$ for different values of the parameters N, n and θ_w.

n	N	$-\theta'(0)$		
		$\theta_w = 1.1$	$\theta_w = 1.5$	$\theta_w = 2.0$
0.5	1	0.129473	0.109367	0.095490
	5	0.056356	0.049000	0.043692
	10	0.039518	0.034618	0.030997
0.8	1	0.140203	0.118992	0.104629
	5	0.062999	0.052978	0.047731
	10	0.042194	0.037386	0.033836
1.0	1	0.144489	0.122876	0.108364
	5	0.061836	0.054560	0.049370
	10	0.043234	0.038486	0.034990
1.5	1	0.150952	0.128776	0.114095
	5	0.064111	0.056939	0.051870
	10	0.044768	0.040138	0.036760
2.0	1	0.154573	0.132108	0.117363
	5	0.065364	0.058269	0.053288
	10	0.045611	0.041059	0.037747
2.5	1	0.156894	0.134251	0.119478
	5	0.066159	0.059119	0.054202
	10	0.046144	0.041649	0.038390

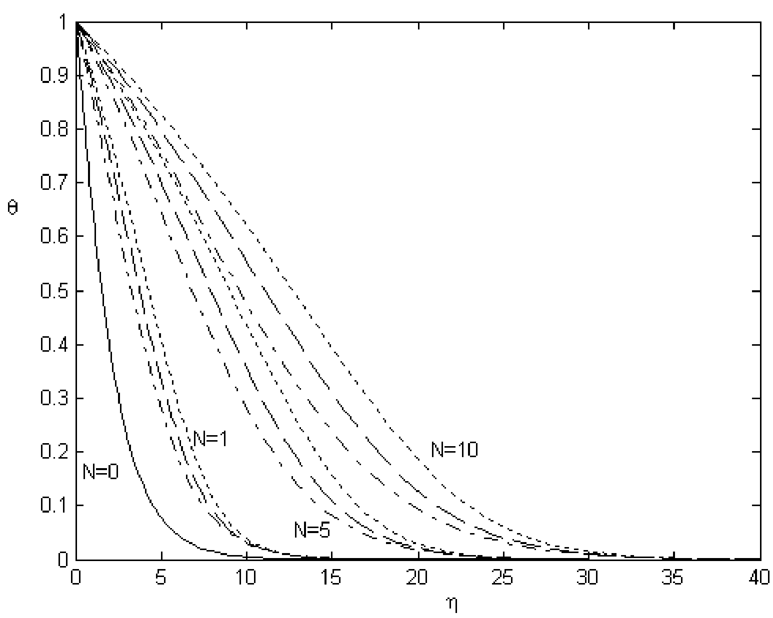

Fig.2. Dimensionless temperature profiles for $n = 0.5$, $N = 0, 1, 5, 10$ and $\theta_w = 1.1(\_\cdot\_\cdot)$, $1.5(\_\_\_)$ and $2(......)$.

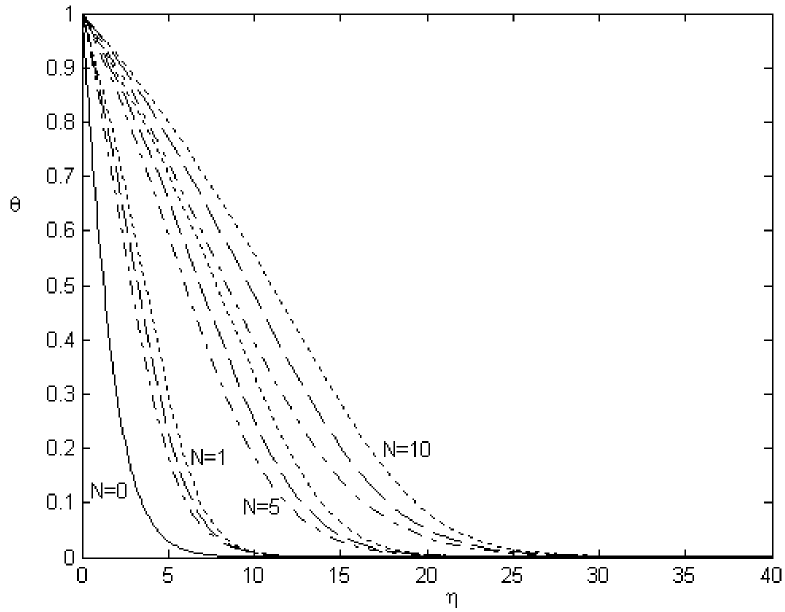

Fig.3. Dimensionless temperature profiles for $n = 1$, $N = 0, 1, 5, 10$ and $\theta_w = 1.1(\_\cdot\_\cdot)$, $1.5(\_\_\_)$ and $2(......)$.

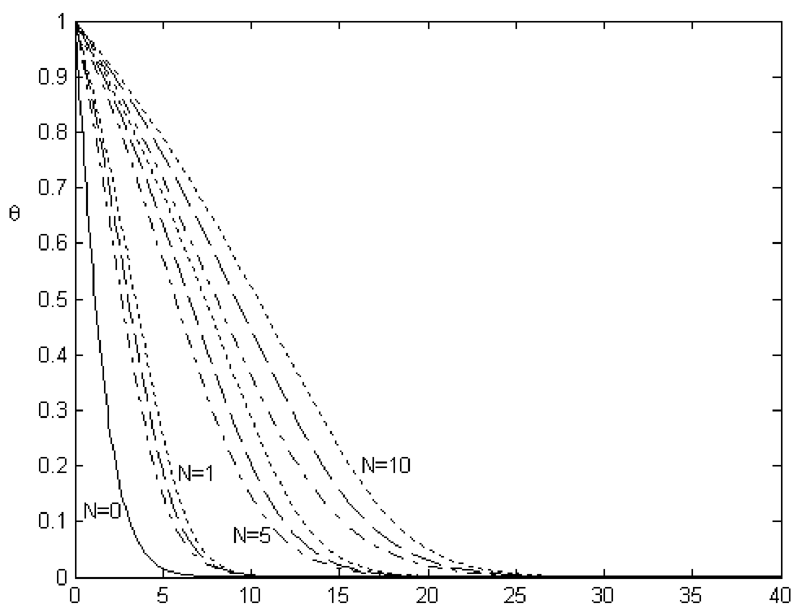

Fig.4. Dimensionless temperature profiles for $n = 1.5$, $N = 0, 1, 5, 10$ and $\theta_w = 1.1(\_\cdot\_\cdot)$, $1.5(\_\_\_)$ and $2(......)$.

4. Conclusions

Natural convection over a vertical flat plate embedded in a fluid-saturated porous medium exposed to thermal radiation has been investigated. Numerical solutions are given in terms of three parameters, namely the radiation parameter, N, the plate temperature, θ_w, and the power-law index, n. It is found that both the heat transfer, $-\theta'(0)$, and the non-dimensional temperature profiles, $\theta(\eta)$, are greatly affected by these parameters. The solution showed that the values of heat transfer from the plate without radiation $(N=0)$ are in very good agreement with those reported by Chen and Chen (1988).

Acknowledgment

The authors wish to express their thanks to Prof. Derek B. Ingham for his valuable comments.

Nomenclature

c_p – specific heat at constant pressure
d – particle diameter
f – reduced stream function
g – gravitational acceleration
h – heat transfer coefficient
k – thermal conductivity
K^* – modified permeability of the porous medium
n – power-law index
N – radiation parameter
Nu_x – local Nusselt number
q^r – radiative heat flux
Ra^* – modified Rayleigh number for porous medium
T – fluid temperature
T_w – wall temperature
T_∞ – ambient temperature
u,v – velocity components along the x- and y-axes, respectively
x,y – Cartesian coordinates
α_m – effective thermal diffusivity
β – coefficient of thermal expansion
χ – mean absorption
φ – porosity
η – similarity variable
ν^* – modified kinematic viscosity
θ – non-dimensionless temperature
θ_w – temperature parameter
ρ – fluid density
σ – Stefan-Boltzman constant
ψ – stream function

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Received: November 25, 2004