PLANE SH-WAVE RESPONSE FROM ELASTIC SLAB INTERPOSED BETWEEN TWO DIFFERENT SELF-REINFORCED ELASTIC SOLIDS

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Closed form expressions of reflection and transmission coefficients are obtained when a plane SH-wave becomes incident at a uniform elastic layer interposed between two different self-reinforced elastic solid half-spaces. It is found that the reflection and transmission coefficients are strongly influenced by the reinforcement parameters of the half-spaces. Numerical computations are performed for a specific model to study the effect of reinforcement parameters and angle of incidence of the incident wave on these coefficients. Numerical study reveals that both reflection and transmission coefficients are significantly influenced by the reinforcement parameters in the entire range of angle of incidence, except at normal and grazing incidence where the effect of reinforcement parameters is found minimum.

Key words: SH-wave, reflection and transmission coefficients, interposed layer, reinforcement, elastic half-spaces.

1. Introduction

Seismic waves, like lighting waves, undergo regular reflection and transmission across the boundary between layers of the Earth having different physical properties. The problems of reflection and transmission of seismic waves from these boundaries present in the earth medium are of great practical importance in seismology. They are not only helpful in investigating the internal structure of the Earth but are also very helpful in exploration of natural resources buried inside the Earth surface, e.g., oils, gases, deposits and other useful hydrocarbons and minerals.

The characteristic property of a self-reinforced material is that its components act together as a single anisotropic unit as long as they remain in elastic condition, i.e., the two components are bound together so that there is no relative displacement between them. There is sufficient evidence in the literature that the Earth crust may contain some hard/soft rocks or materials that may exhibit self-reinforcement properties. These rocks when they come in the way of seismic waves do affect their propagation and such seismic signals are always influenced by the elastic properties of the media through which they travel. Keeping this in mind, the problem of reflection and transmission of plane SH-wave through an elastic layer embedded between two different self-reinforced elastic solid half-spaces is studied.

Cerveny (1974) and Gupta (1962; 1966) investigated reflection and transmission coefficients for a transition layer. The problems of reflection and transmission of SH-waves through the transition layer have also been discussed by Sinha (1964), Thapliyal (1974), Singh (1977), Singh *et al*. (1978), Malhotra *et al*. (1983), Kaushik and Rana (1997) among others. Caviglia and Morro (2002) investigated reflection and transmission matrices, associated with obliquely incident plane harmonic waves for a planarly-stratified inhomogeneous layer. Recently, Chaudhary *et al*. (2004; 2005) studied the problems of transmission of a plane SH-wave through a self-reinforced elastic slab sandwiched between two elastic solid half-spaces.

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The idea of introducing a continuous reinforcement at every point of an elastic solid was given by Belfield *et al.* (1983). Later, Verma and Rana (1983) applied this model to the rotation of a tube, illustrating its utility in strengthening the lateral surface of the tube. Verma (1986) also discussed the magnetoelastic shear waves in self-reinforced bodies. The problem of magneto-elastic transverse surface waves in self-reinforced elastic solids was studied by Verma *et al.* (1988). Chattopadhyay and Choudhury (1990) studied the propagation; reflection and transmission of magnetoelastic shear waves in a self-reinforced elastic medium. Chattopadhyay and Choudhury (1995) studied the magnetoelastic shear waves in an infinite self-reinforced plate. Chattopadhyay and Venkateswarul (1998) investigated a two-dimensional problem of stress produced by a pulse of shearing force moving over the boundary of a fiber-reinforced medium. Sengupta and Nath (2001) studied the surface wave propagation in a fiber-reinforced anisotropic elastic media.

In the present work, we have attempted a two-dimensional problem of reflection and transmission of a cplane SH-wave through an isotropic homogeneous elastic layer interposed between two different self-reinforced elastic solid half-spaces. The expressions for reflection and transmission coefficients are obtained in closed form and the effect of various parameters such as reinforced parameters, normalized wave number and the angle at which the wave crosses the magnetic field, on these coefficients have been studied in detail. Results of some earlier workers have been reduced as a particular case of the present formulation.

2. Formulation of the problem and basic equations

We consider a homogeneous isotropic elastic layer H_2 having uniform width '*d*' and interposed (sandwiched) between two perfectly conducting self-reinforced linearly elastic solid half-spaces H_1 and H_3. The *Z*-axis of the rectangular Cartesian coordinate system is pointing into the lower half-space H_1 such that $z=0$ and $z=-d$ are the interfaces between H_1-H_2 and H_2-H_3 respectively. Let a train of a plane SH-wave propagating through the lower medium H_1 become incident at the interface $z=0$ making an angle 'θ_0' with the *Z*-axis. The geometry of the problem is shown in Fig.1.

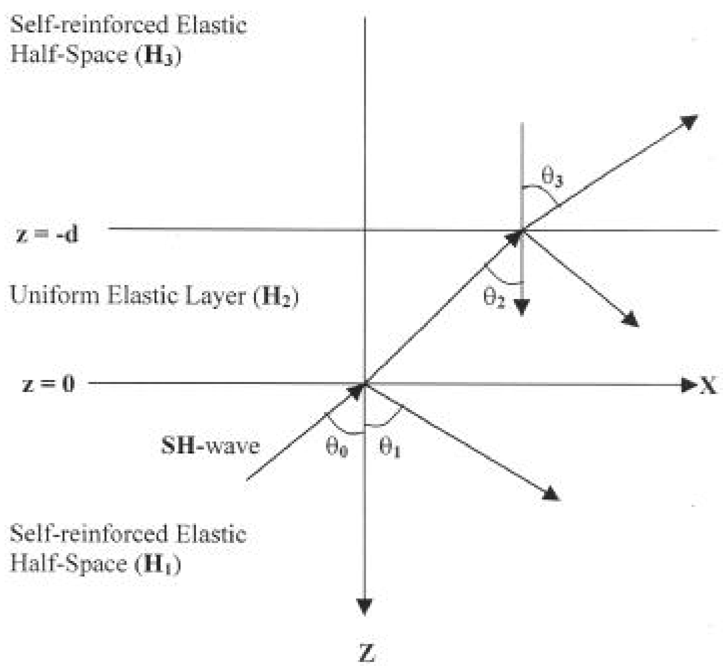

Fig.1. Geometry of the problem.

The governing equations of propagation of small disturbances in a self -reinforced elastic medium in the absence of body forces and displacement current are given by (Verma, 1986)

$$\tau_{ij,j}+(\boldsymbol{J}\times\boldsymbol{B})_i=\rho\frac{\partial^2 u_i}{\partial t^2}, \qquad (i,\ j=1,2,3) \tag{2.1}$$

where τ_{ij} are the components of the stress tensor, $\boldsymbol{J}$ is the electric current, $\boldsymbol{B}$ is the magnetic induction, u_i are the components of the displacement vector $\boldsymbol{u}$ and ρ is the density of the medium. The vectors $\boldsymbol{J}$ and $\boldsymbol{B}$ are given by the following well known Maxwell's fundamental equations

$$\nabla\times\boldsymbol{H}=\boldsymbol{J}, \qquad \nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t}, \qquad \nabla\cdot\boldsymbol{B}=0, \qquad \boldsymbol{B}=\mu_e\boldsymbol{H}, \qquad \boldsymbol{J}=\sigma\left(\boldsymbol{E}+\frac{\partial\boldsymbol{u}}{\partial t}\times\boldsymbol{B}\right) \tag{2.2}$$

where $\boldsymbol{E}$ is the induced electric field, $\boldsymbol{H}=(H_x,H_y,H_z)$ is the total applied and induced magnetic field, μ_e is the magnetic permeability and σ is the electrical conductivity.

The stress – strain relation in a self – reinforced elastic solid medium, in which reinforcement is to make the material locally transversely isotropic whose preferred direction is that of unit vector $\boldsymbol{a}$, is (Belfield *et al.*, 1983)

$$\begin{gathered}\tau_{ij}=\lambda e_{kk}\delta_{ij}+2\mu_T e_{ij}+\alpha\left(a_k a_m e_{km}\delta_{ij}+e_{kk}a_i a_j\right)+\\ +2(\mu_L-\mu_T)\left(a_i a_k e_{kj}+a_j a_k e_{ki}\right)+\beta a_k a_m e_{km}a_i a_j\end{gathered} \tag{2.3}$$

where $\lambda,\mu_T,\mu_L,\alpha$ and β are the material constants, τ_{ij} are the components of stress tensor, e_{ij} are the components of infinitesimal strain tensor and a_i are the components of $\boldsymbol{a}$.

The last Eq.(2.2) can be written as

$$\nabla^2\boldsymbol{H}=\sigma\mu_e\left[\frac{\partial\boldsymbol{H}}{\partial t}-\nabla\times\left(\frac{\partial\boldsymbol{u}}{\partial t}\times\boldsymbol{H}\right)\right]. \tag{2.4}$$

For the motion of SH-wave in the $x-z$ plane, we take $\boldsymbol{u}=(0,\ V_m,\ 0)$, $m=1,3$ in medium H_m and $\frac{\partial}{\partial y}\equiv 0$. From Eq.(2.4), we have

$$\frac{\partial H_X}{\partial t}=\frac{1}{\sigma\mu_e}\nabla^2 H_X, \qquad \frac{\partial H_z}{\partial t}=\frac{1}{\sigma\mu_e}\nabla^2 H_z, \tag{2.5}$$

$$\frac{\partial H_y}{\partial t}=\frac{1}{\sigma\mu_e}\nabla^2 H_y+\frac{\partial}{\partial x}\left(H_x\frac{\partial V_m}{\partial t}\right)+\frac{\partial}{\partial z}\left(H_z\frac{\partial V_m}{\partial t}\right). \tag{2.6}$$

For perfectly conducting medium, that is, when $\sigma\to\infty$, it can be seen from Eqs (2.5) and (2.6) that there is no perturbation in H_x and H_z. Thus in this case, Eq.(2.6) becomes

$$\frac{\partial H_y}{\partial t} = \frac{\partial}{\partial x}\left(H_x \frac{\partial V_m}{\partial t} \right) + \frac{\partial}{\partial z}\left(H_z \frac{\partial V_m}{\partial t} \right). \tag{2.7}$$

We may take small perturbation ' h_2 ' in H_y only as follows

$$H_x = H_{01}, \qquad H_y = H_{02} + h_2, \qquad H_z = H_{03},$$

such that

$$\boldsymbol{H} = \left(H_0 \cos\phi,\ h_2,\ H_0 \sin\phi \right) \tag{2.8}$$

where $H_0 = |\boldsymbol{H}_0|$ and ϕ is the angle at which the wave crosses the magnetic field.

Plugging Eq.(2.8) into Eq.(2.7) and integrating with respect to 't' we get

$$h_2 = H_0 \left(\cos\phi \frac{\partial V_m}{\partial x} + \sin\phi \frac{\partial V_m}{\partial z} \right). \tag{2.9}$$

Since there is no perturbation in the magnetic field initially, i.e., $h_2 = 0$, therefore, from Eq.(2.2), we have

$$\boldsymbol{J} \times \boldsymbol{B} = \mu_e \left[(\boldsymbol{H} \cdot \nabla)\boldsymbol{H} - \frac{1}{2} \nabla \boldsymbol{H}^2 \right]. \tag{2.10}$$

Using Eq.(2.8) into Eq.(2.10), it can be shown that

$$(\boldsymbol{J} \times \boldsymbol{B})_2 = \mu_e H_0 \left(\cos\phi \frac{\partial h_2}{\partial x} + \sin\phi \frac{\partial h_2}{\partial z} \right), \tag{2.11}$$

is the only non-zero component. Equation (2.11) with the aid of Eq.(2.9) yields

$$(\boldsymbol{J} \times \boldsymbol{B})_2 = \mu_e H_0^2 \left(\cos^2\phi \frac{\partial^2 V_m}{\partial x^2} + \sin 2\phi \frac{\partial^2 V_m}{\partial x \partial z} + \sin^2\phi \frac{\partial^2 V_m}{\partial z^2} \right). \tag{2.12}$$

The non-zero stress components from Eq.(2.3) with $\boldsymbol{a} = (a_1, 0, a_3)$ are

$$\tau_{23} = \mu_T \frac{\partial V_m}{\partial z} + (\mu_L - \mu_T) \left[a_3^2 \frac{\partial V_m}{\partial z} + a_1 a_3 \frac{\partial V_m}{\partial x} \right], \tag{2.13}$$

$$\tau_{21} = \mu_T \frac{\partial V_m}{\partial x} + (\mu_L - \mu_T) \left[a_1^2 \frac{\partial V_m}{\partial x} + a_1 a_3 \frac{\partial V_m}{\partial z} \right]. \tag{2.14}$$

Using expressions given by Eqs (2.12) to (2.14) in Eq.(2.1), we have the following equation of motion

$$\left[\mu_T+\mu_e H_0^2\cos^2\phi+a_1^2(\mu_L-\mu_T)\right]\frac{\partial^2 V_m}{\partial x^2}+$$
$$+\left[2a_1a_3(\mu_L-\mu_T)+\mu_e H_0^2\sin 2\phi\right]\frac{\partial^2 V_m}{\partial x\partial z}+ \tag{2.15}$$
$$+\left[\mu_T+\mu_e H_0^2\sin^2\phi+a_3^2(\mu_L-\mu_T)\right]\frac{\partial^2 V_m}{\partial z^2}=\rho\frac{\partial^2 V_m}{\partial t^2}.$$

For time harmonic SH-wave propagating in the positive x- direction, the general solution to this equation in the half space H_1 may be taken as

$$V_1=A_1\exp\left[ik\left\{x+\left(\gamma_1+\frac{S}{2P}\right)z-ct\right\}\right]+B_1\exp\left[\imath k\left\{x-\left(\gamma_1+\frac{S}{2P}\right)z-ct\right\}\right] \tag{2.16}$$

where A_1 and B_1 are unknown, k is the wave number, c is the phase velocity and

$$\gamma_1=\sqrt{\frac{S^2}{4P^2}+\frac{1}{P}\left(\frac{\rho\omega^2}{k^2}-Q\right)},\qquad S=2a_1a_3(\mu_L-\mu_T)+\mu_e H_0^2\sin 2\phi,$$
$$P=\mu_T+\mu_e H_0^2\sin^2\phi+a_3^2(\mu_L-\mu_T),\qquad Q=\mu_T+\mu_e H_0^2\cos^2\phi+a_1^2(\mu_L-\mu_T), \tag{2.17}$$

ω being the angular frequency.

Using Eq.(2.16) in Eq.(2.13), the stress component in the self-reinforced elastic half-space H_1 is given by

$$(\tau_{23})_1=ikA_1\left\{\left(\gamma_1+\frac{S}{2P}\right)\left[\mu_T+a_3^2(\mu_L-\mu_T)\right]+a_1a_3(\mu_L-\mu_T)\right\}\exp\left[ik\left\{x+\left(\gamma_1+\frac{S}{2P}\right)z-ct\right\}\right]+$$
$$-ikB_1\left\{\left(\gamma_1+\frac{S}{2P}\right)\left[\mu_T+a_3^2(\mu_L-\mu_T)\right]-a_1a_3(\mu_L-\mu_T)\right\}\exp\left[ik\left\{x-\left(\gamma_1+\frac{S}{2P}\right)z-ct\right\}\right]. \tag{2.18}$$

In the self-reinforced medium H_3, denoting the quantities by superscript prime $(\ /\ )$, the relevant displacement and stress components respectively are given by

$$V_3=A_3\exp\left[ik\left\{x+\left(\gamma_1'+\frac{S'}{2P'}\right)z-ct\right\}\right], \tag{2.19}$$

$$(\tau_{23})_3=ikA_3\left\{\left(\gamma_1'+\frac{S'}{2P'}\right)\left[\mu_T'+a_3'^2(\mu_L'-\mu_T')\right]+a_1'a_3'(\mu_L'-\mu_T')\right\}\times$$
$$\times\exp\left[ik\left\{x+\left(\gamma_1'+\frac{S'}{2P'}\right)z-ct\right\}\right] \tag{2.20}$$

where γ_1', P', S' and Q' can be obtained from Eq.(2.17) by putting primes appropriately.

For the propagation of SH-waves in a homogeneous, isotropic elastic layer H_2, the displacement component V_2 satisfies the following equation of motion

$$\frac{\partial^2 V_2}{\partial x^2}+\frac{\partial^2 V_2}{\partial z^2}=\frac{1}{\beta_2^2}\frac{\partial^2 V_2}{\partial t^2}.$$

The time harmonic solution to this equation is given by

$$V_2 = A_2 \exp\{\iota k(x+\gamma_2 z - ct)\} + B_2 \exp\{\iota k(x-\gamma_2 z - ct)\} \tag{2.21}$$

where A_2 and B_2 are constants, $\gamma_2^2 = \dfrac{c^2}{\beta_2^2}-1$ and $\beta_2^2 = \dfrac{\mu_2}{\rho_2}$.

The requisite component of stress in medium H_2 is given by

$$(\tau_{23})_2 = \iota k\gamma_2\mu_2 A_2 \exp\{\iota k(x+\gamma_2 z - ct)\} - \iota k\gamma_2\mu_2 B_2 \exp\{\iota k(x-\gamma_2 z - ct)\}. \tag{2.22}$$

3. Boundary conditions

We assume that the interposed layer and the half spaces are in welded contact. Therefore, the boundary conditions are the continuity of displacement and stresses at the interfaces. Mathematically, these boundary conditions can be expressed as:

(i) $V_1 = V_2$, (ii) $[\tau_{23}]_1 = [\tau_{23}]_2$, at $z=0$,

(iii) $V_2 = V_3$, (iv) $[\tau_{23}]_2 = [\tau_{23}]_3$, at $z=-d$.

Snell's law is given by

$$c=\frac{\beta_1}{\sin\theta_1}=\frac{\beta_2}{\sin\theta_2}=\frac{\beta_1'}{\sin\theta_3} \tag{3.1}$$

where $\beta_1^2 = \mu_T/\rho$ and $\beta_1'^2 = \mu_T'/\rho'$.

Using Eqs (2.16) to (2.22) in these four boundary conditions and making use of Snell's law given by Eq.(3.1), we obtained the following equations written in a matrix form as

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}\begin{pmatrix} R \\ T' \\ R' \\ T \end{pmatrix}=\begin{pmatrix} -1 \\ S_1 \\ 0 \\ 0 \end{pmatrix} \tag{3.2}$$

where the elements of matrices are given by

$$a_{11}=1, \quad a_{12}=a_{13}=-1, \quad a_{14}=a_{24}=a_{31}=a_{41}=0, \quad a_{21}=S_2, \quad a_{22}=-a_{23}=\gamma_2\mu_2,$$

$$a_{32}=\exp\left(-\iota k\gamma_2 d\right),\qquad a_{33}=\exp\left(ik\gamma_2 d\right),\qquad a_{34}=-\exp\left\{-\iota k\left(\gamma_1'+\frac{S'}{2P'}\right)d\right\},$$

$$a_{42}=\gamma_2\mu_2\exp\left(-ik\gamma_2 d\right),\quad a_{43}=-\gamma_2\mu_2\exp\left(-ik\gamma_2 d\right),\quad a_{44}=-S_3\exp\left\{-ik\left(\gamma_1'+\frac{S'}{2P'}\right)d\right\},$$

$$S_1=\left(\gamma_1+\frac{S}{2P}\right)\left[\mu_T+a_3^2\left(\mu_L-\mu_T\right)\right]+a_1a_3\left(\mu_L-\mu_T\right),$$

$$S_2=\left(\gamma_1+\frac{S}{2P}\right)\left[\mu_T+a_3^2\left(\mu_L-\mu_T\right)\right]-a_1a_3\left(\mu_L-\mu_T\right),$$

$$S_3=\left(\gamma_1'+\frac{S'}{2P'}\right)\left[\mu_T'+a_3'^2\left(\mu_L'-\mu_T'\right)\right]-a_1'a_3'\left(\mu_L'-\mu_T'\right),$$

and
$$R=\frac{B_1}{A_1},\qquad T'=\frac{A_2}{A_1},\qquad R'=\frac{B_2}{A_1},\qquad T=\frac{A_3}{A'}.$$

On solving Eq.(3.2) for R and T, we obtain

$$R=\frac{\Delta_1}{\Delta},\qquad T=\frac{\Delta_2}{\Delta} \tag{3.3}$$

where

$$\Delta=\left[\left\{-\gamma_2\mu_2\left(S_2+S_3\right)-\left(\gamma_2^2\mu_2^2+S_2S_3\right)\right\}\exp\left(ik\gamma_2 d\right)+\right.$$
$$\left.+\left\{-\gamma_2\mu_2\left(S_2+S_3\right)+\left(\gamma_2^2\mu_2^2+S_2S_3\right)\right\}\exp\left(-ik\gamma_2 d\right)\right]\exp\left\{-ik\left(\gamma_1'+\frac{S'}{2P'}\right)d\right\},$$

$$\Delta_1=\left[\left\{\gamma_2\mu_2\left(S_3-S_1\right)+\left(\gamma_2^2\mu_2^2-S_2S_3\right)\right\}\exp\left(ik\gamma_2 d\right)+\right.$$
$$\left.+\left\{\gamma_2\mu_2\left(S_3-S_1\right)-\left(\gamma_2^2\mu_2^2-S_2S_3\right)\right\}\exp\left(-ik\gamma_2 d\right)\right]\exp\left\{-ik\left(\gamma_1'+\frac{S'}{2P'}\right)d\right\},$$

$$\Delta_2=-2\gamma_2\mu_2\left(S_1+S_2\right).$$

The formulae for reflection and transmission coefficients given by Eq.(2.21) show that they depend on reinforcement parameters, angle of incidence and on the thickness of the sandwiched layer.

4. Special cases

(i) When the sandwiched layer is absent, we shall be left with two different self-reinforced elastic solid half-spaces in welded contact. In this case, putting $d=0$ in the formulae (3.3), we obtain following expressions of the reflection and transmission coefficients at the welded contact interface for the relevant problem

$$R = \frac{S_1 - S_3}{S_2 + S_3}, \qquad T = \frac{S_1 + S_2}{S_2 + S_3}. \tag{4.1}$$

(ii) When the reinforcement of both the half-spaces is neglected, we obtain two isotropic elastic half-spaces in welded contact. In this case, we put $\mu_L = \mu_T = \mu$, $\mu'_L = \mu'_T = \mu'$, $a_1 = a_3 = a'_1 = a'_3 = 0$ in the formulae (4.1), we obtain the reflection and transmission coefficients for the relevant problem as

$$R = \frac{\mu \tan\theta_0 - \mu' \tan\theta_3}{\mu \tan\theta_0 + \mu' \tan\theta_3}, \qquad T = \frac{2\mu \tan\theta_0}{\mu \tan\theta_0 + \mu' \tan\theta_3} \tag{4.2}$$

where $\tan\theta_0 = \sqrt{\dfrac{c^2}{\beta_1^2} - 1}$, $\tan\theta_3 = \sqrt{\dfrac{c^2}{\beta_1'^2} - 1}$.

Equation (4.2) gives the reflection and transmission coefficients of the incident SH-wave at the interface between two uniform elastic half-spaces in welded contact, already given in (Bullen and Bolt, 1985, pp.143) for the relevant problem.

5. Numerical results

In order to study the effects of the self-reinforced elastic parameters ε_H and ε'_H, the normalized wave number '*kd*' and the angle 'ϕ', on the reflection and transmission coefficients due to the incident SH-wave, we consider the following values of parameters for the layer and half-spaces:

In the half-space H_1 (Chattopadhaya and Venkateswarul, 1998)

$$\mu_L = 5.66\times10^9\ N/m^2, \quad \mu_T = 2.46\times10^9\ N/m^2, \quad \rho = 7800\ kg/m^3 \quad \text{and} \quad a_1 = 0.0316.$$

In the layer H_2, the values of the elastic constants taken are

$$\mu_2 = 42.98\times10^9\ N/m^2, \qquad \rho_2 = 2900\ kg/m^3.$$

In the half-space H_3

$$\mu'_L = 4.80\times10^9\ N/m^2, \quad \mu'_T = 1.20\times10^9\ N/m^2, \quad \rho' = 4200\ kg/m^3 \quad \text{and} \quad a'_1 = 0.0421.$$

Figure 2 shows the variation of reflection coefficient with the angle of incidence for different values of $\varepsilon_H \left(= \mu_e H_0^2/\mu_T\right)$ and $\varepsilon'_H \left(= \mu'_e H_0'^2/\mu'_T\right)$. When $(\varepsilon_H, \varepsilon'_H) = (0, 0), (0.02, 0.04), (0.1, 0.2)$, $kd = 0.0$, $(\phi, \phi') = (30^\circ, 45^\circ)$ and $c/\beta_1 = 1.20$, it is clear from the figure that the reflection coefficient has maximum value near the normal incidence for $(\varepsilon_H, \varepsilon'_H) = (0,0)$, after that it starts decreasing monotonically with angle of incidence and attains its minimum value at the grazing incidence. But as soon as the values of ε_H and ε'_H increase, the value of reflection coefficient decreases fast near the normal incidence, while it increases slowly at the grazing incidence.

Figure 3 shows that the transmission coefficient's behaviour is contrary to that of the reflection coefficient for the same values of parameters.

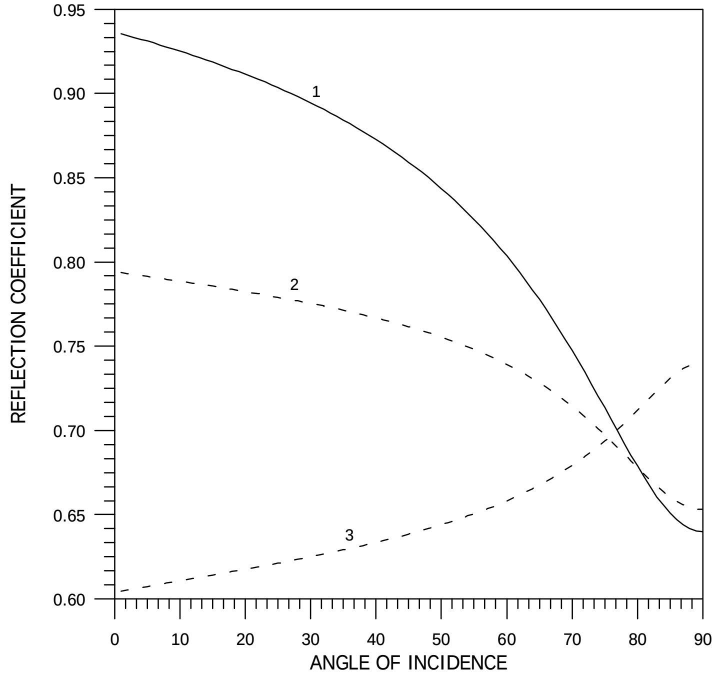

Fig.2. Effect of ε_H and ε'_H - variation of modulus of reflection coefficient (R) with angle of incidence (Curves – 1, 2 and 3 when $(\varepsilon_H, \varepsilon'_H) = (0, 0)$, $(0.02, 0.04)$ and $(0.1, 0.2)$ with $kd = 0.0$, $(\phi, \phi') = (30^{0}, 45^{0})$ and $c/\beta_1 = 1.20$).

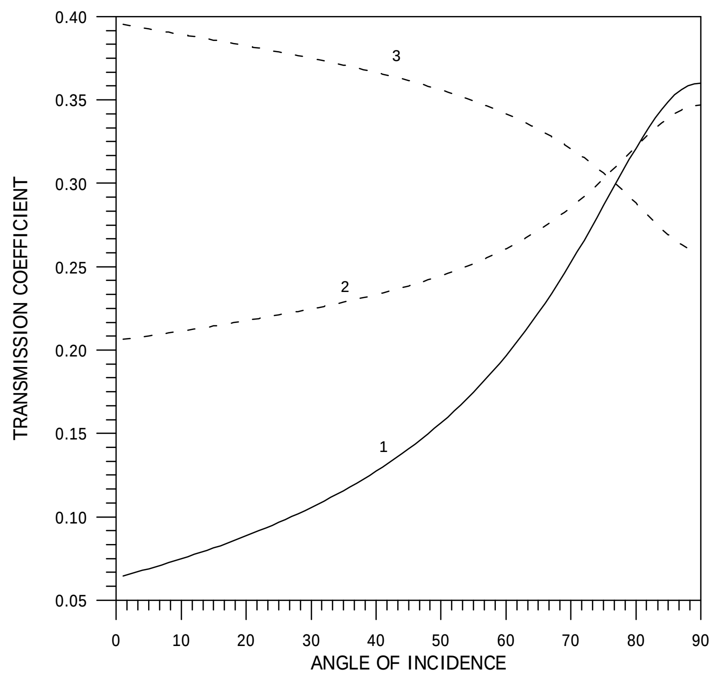

Fig.3. Effect of ε_H and ε'_H - variation of modulus of transmission coefficient (T) with angle of incidence (Curves – 1, 2 and 3 when $(\varepsilon_H, \varepsilon'_H) = (0, 0)$, $(0.02, 0.04)$ and $(0.1, 0.2)$ with $kd = 0.0$, $(\phi, \phi') = (30^{0}, 45^{0})$ and $c/\beta_1 = 1.20$).

Figures 4 and 5 show the variation of the reflection and transmission coefficient respectively with angle of incidence for different values of *kd* when $(\varepsilon_H, \varepsilon'_H) = (0.02, 0.04)$, $(\phi, \phi') = (30^{\mathbf{o}}, 45^{\mathbf{o}})$ and $c/\beta_1 = 1.20$. We notice from Fig.4 that when $kd = 0.0$, the reflection coefficient starts with the value *0.79* approximately, then decreases slightly with an increasing value of angle of incidence attaining its minimum value *0.65* at the grazing incidence. The value of reflection coefficient increases near normal incidence with an increase in *kd*. For $kd \neq 0$, the reflection coefficient decreases and attains its minimum value near $\theta = 15^{\mathbf{o}}$ and then starts increasing rapidly attaining its maximum value at the grazing incidence. It is interesting to note that the value of reflection coefficient at $\theta = 35^{\mathbf{o}}$ and $\theta = 6^{\mathbf{o}}$ does not change with the change in '*kd*'. Thus the effect of the increasing values of '*kd*' is to increase the value of reflection coefficient before $\theta = 6^{\mathbf{o}}$ and after $\theta = 35^{\mathbf{o}}$, while it decreases rapidly in the range $6^{\mathbf{o}} < \theta < 35^{\mathbf{o}}$.

From Fig.5, it is clear that the effect of increase in *kd* is to decrease the transmission coefficients for all values of θ, however the magnitude is different for different θ. For $kd = 0.0$, the transmission coefficient increases with θ, while for $kd \neq 0$, the transmission coefficient first increases when the angle of incidence attains maximum value and then decreases.

Figures 6 and 7 show the variation of reflection and transmission coefficients with the angle of incidence for different values of c/β_1, namely *1.2, 1.6, 2* and *5* and $kd = 0.5$; $(\varepsilon_H, \varepsilon'_H) = (0.02, 0.04)$; $(\phi, \phi') = (30^{\mathbf{o}}, 45^{\mathbf{o}})$. It is clear from Fig.6 that the reflection coefficient starts decreasing from its maximum value at normal incidence attaining its minimum value and then increases monotonically with the increase in the angle of incidence attaining its maximum value at the grazing incidence. The effect of an increase in c/β_1 is different on the reflection coefficient at a given range of the angle of incidence, however the variation pattern of the reflection coefficient remains same. A comparison of Figs 6 and 7, shows that the variation trend is just reverse.

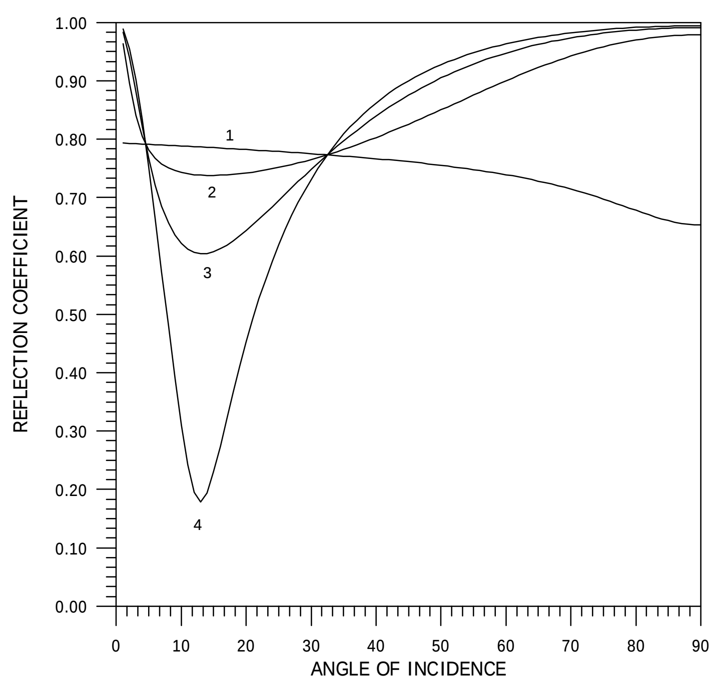

Fig.4. Effect of *kd* - variation of modulus of reflection coefficient (R) with angle of incidence (Curves – 1, 2, 3 and 4 when $kd = 0.0, 0.5, 1.0$ and 2.0 with $(\varepsilon_H, \varepsilon'_H) = (0.02, 0.04)$, $(\phi, \phi') = (30^{\mathbf{o}}, 45^{\mathbf{o}})$ and $c/\beta_1 = 1.20$).

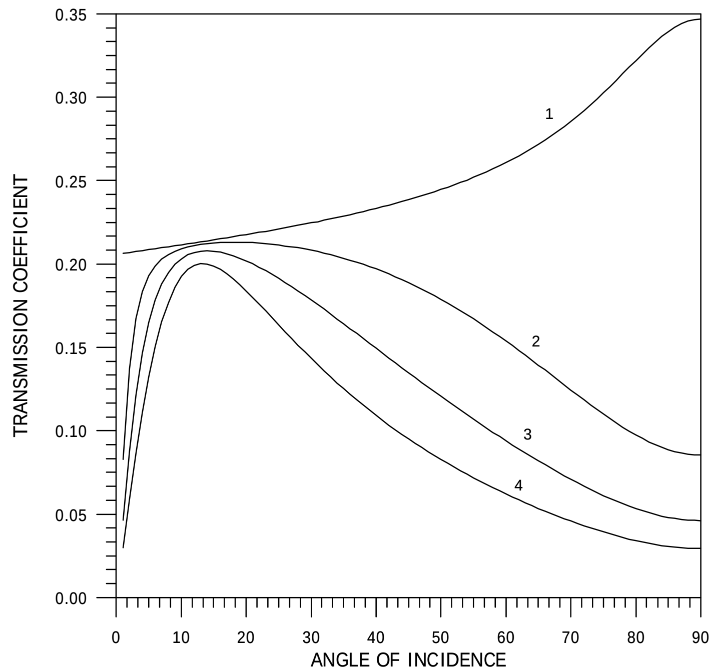

Fig.5. Effect of ***kd*** - variation of modulus of transmission coefficient (T) with angle of incidence (Curves – 1, 2, 3 and 4 when $kd = 0.0, 0.5, 1.0$ and 2.0 with $(\varepsilon_H, \varepsilon'_H) = (0.02, 0.04)$, $(\phi, \phi') = (30^{\circ}, 45^{\circ})$ and $c/\beta_1 = 1.20$).

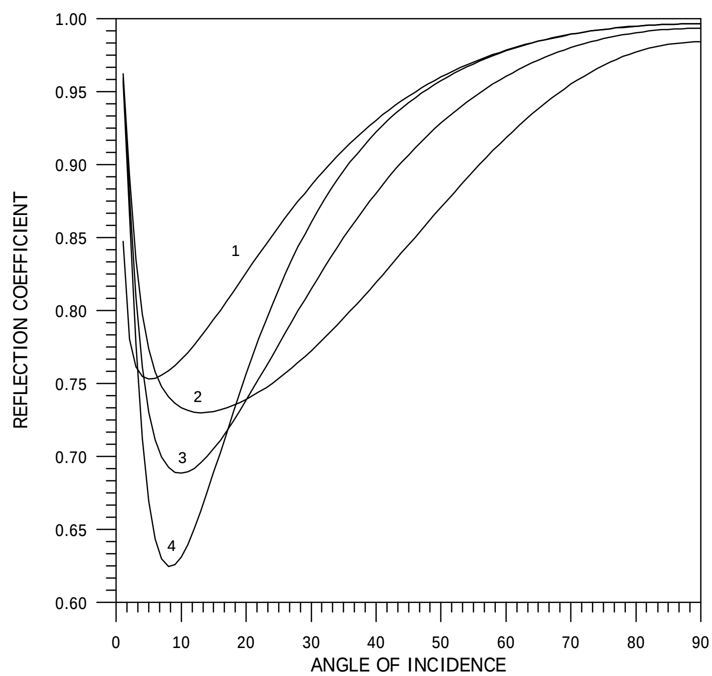

Fig.6. Effect of c/β_1 - variation of modulus of reflection coefficient (R) with angle of incidence (Curves – 1, 2, 3 and 4 when $c/\beta_1 = 1.2, 1.6, 2.0$ and 5.0 with $kd = 0.5$, $(\varepsilon_H, \varepsilon'_H) = (0.02, 0.04)$, $(\phi, \phi') = (30^{\circ}, 45^{\circ})$ and $c/\beta_1 = 1.20$).

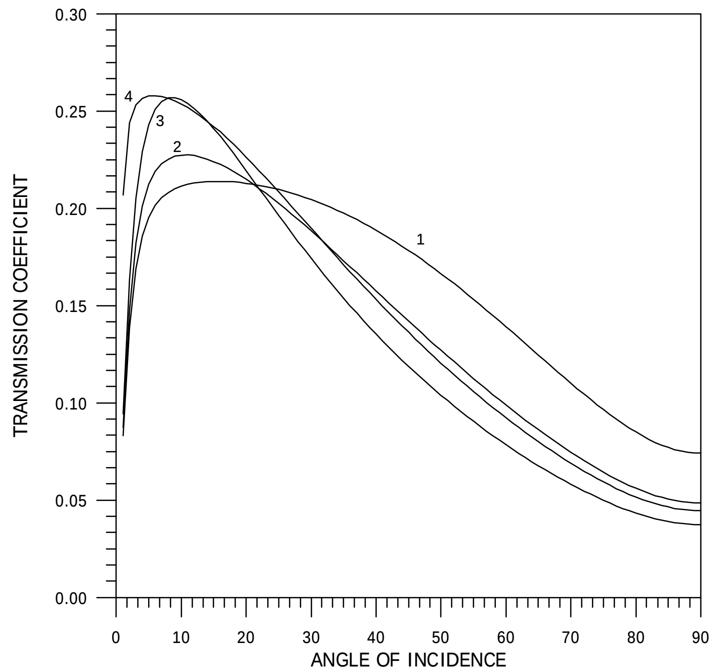

Fig.7. Effect of c/β_1 - variation of modulus of transmission coefficient (T) with angle of incidence (Curve – 1, 2, 3 and 4 when $c/\beta_1 = 1.2, 1.6, 2.0$ and 5.0 with $kd = 0.5$, $(\varepsilon_H, \varepsilon'_H) = (0.02, 0.04)$, $(\phi, \phi') = (30^0, 45^0)$).

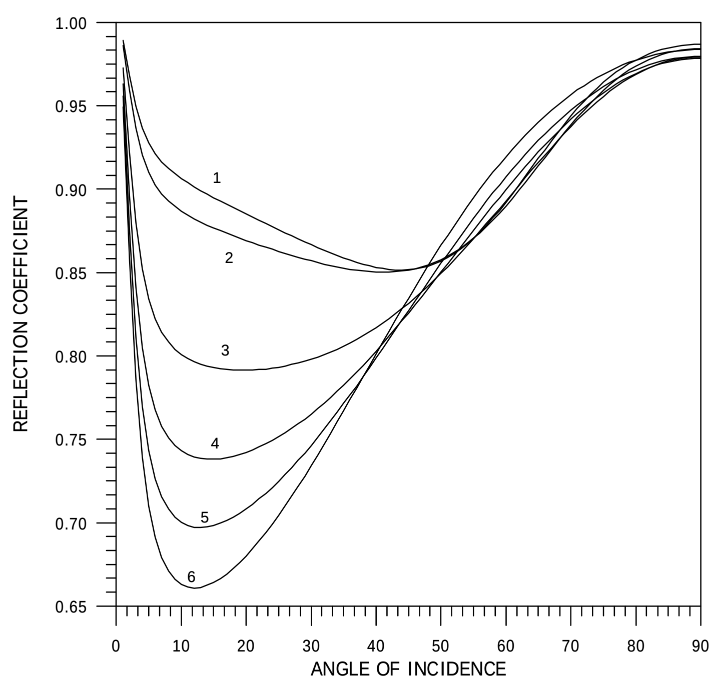

Fig.8. Effect of ϕ and ϕ' - variation of modulus of reflection coefficient (R) with angle of incidence (Curve – 1, 2, 3, 4, 5 and 6 when $\phi = \phi' = 0^0, 10^0, 30^0, 45^0, 60^0$ and 90^0 respectively with $kd = 0.5$, $(\varepsilon_H, \varepsilon'_H) = (0.02, 0.04)$, and $c/\beta_1 = 1.20$).

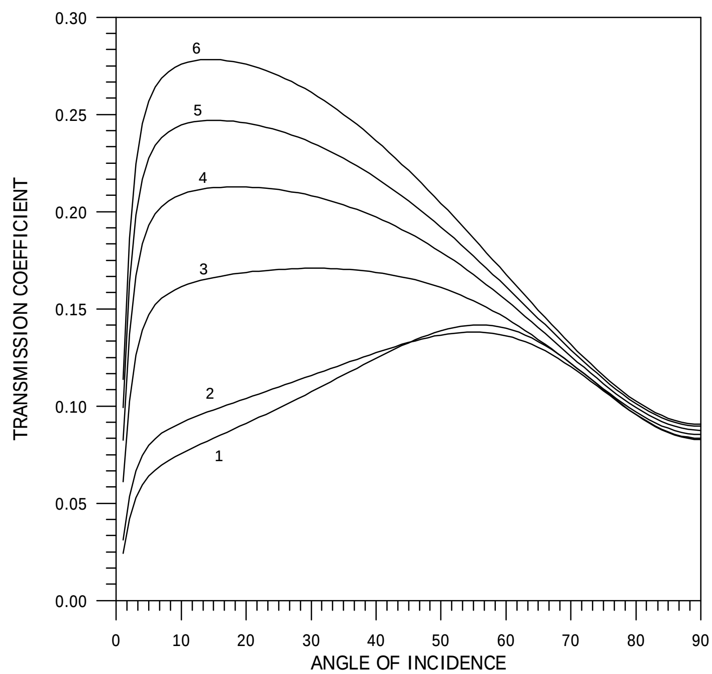

Fig.9. Effect of ϕ and ϕ' -variation of modulus of transmission coefficient (T) with angle of incidence (Curves – 1, 2, 3, 4, 5 and 6 when $\phi = \phi' = 0^0, 10^0, 30^0, 45^0, 60^0$ and 90^0 respectively with $kd = 0.5$, $(\varepsilon_H, \varepsilon'_H) = (0.02, 0.04)$, and $c/\beta_1 = 1.20$).

6. Conclusions

1. Closed form expressions of reflection and transmission coefficients are derived when a plane SH-wave is incident upon a uniform elastic layer interposed between two different self-reinforced elastic solid half-spaces.
2. Analytical formulae show that the reflection and transmission coefficients are continuous functions of the angle of incidence, thickness parameter and that of the reinforced elastic parameters.
3. Numerical study predicts that the variation of reflection and transmission coefficients with the angle of incidence is reverse in general.
4. It was also noticed that an increase in the value of '*kd*' results in an increase in the reflection coefficient near the normal and grazing incidences. On the other hand, the transmission coefficient decreases with the angle of incidence as the values of '*kd*' are increased.
5. It was also observed that the parameters under consideration have a significant effect on reflection and transmission coefficients.
6. The effect of ϕ and ϕ' is observed on reflection and transmission coefficients, however these coefficients are least affected at normal and grazing incidences.

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Nomenclature

a_i – components of unit vector $\boldsymbol{a}$ in the direction of self-reinforcement
$\boldsymbol{B}$ – magnetic induction
c – phase velocity
d – thickness of the interposed layer H_2
$\boldsymbol{E}$ – induced electric current
e_{ij} – components of infinitesimal strain
$\boldsymbol{H}$ – total applied and induced magnetic field
h_2 – perturbation in the magnetic field
$i = \sqrt{-1}$
$\boldsymbol{J}$ – electric current
k – wave number
R, T – reflection and transmission coefficient
t – time
u_i – components of displacement vector $\boldsymbol{u}$
$V_m - y$ – component of displacement in medium H_m
x, z – Cartesian coordinates
β_1 – velocity of SH-wave in medium H_1
β'_1 – velocity of SH-wave in medium H_3
β_2 – velocity of SH-wave in medium H_2
$\varepsilon_H, \varepsilon'_H$ – reinforcement parameter in medium H_1, H_3
θ_1 – angle made by reflected SH-wave with vertical in medium H_1
θ_0, θ_2, θ_3 – angle made by SH-wave with vertical in medium H_1, H_2, H_3 respectively
$\lambda, \mu_T, \mu_L, \alpha, \beta$ – material constants in self-reinforced medium
μ_2 – rigidity of medium H_2
μ_e – magnetic permeability
ρ – density of half-space H_1
ρ' – density of half-space H_3
ρ_2 – density of interposed layer H_2
σ – electrical conductivity
τ_{ij} – stress tensor
ϕ, ϕ' – angle between wave propagation and magnetic field in medium H_1, H_3
ω – angular frequency

References

Belfield A.J., Rogers T.G. and Spencer A.J.M. (1983): *Stress in elastic plates reinforced by fibers lying in concentric circles.* – J. Mech. Phys. Solids, vol.31, No.1, pp.25-54.

Bullen K.E. and Bolt B.A. (1985): *An Introduction to the Theory of Seismology.* – London: Cambridge University Press.

Cerveny V. (1974): *Reflection and transmission coefficients for transition layers.* – Studia Geophys. Geodt., vol.17, pp.59-68.

Chattopadhyay A. and Venkateswarul R.L.K. (1998): *Stresses produced in a fiber reinforced half-space due to moving load.* – Bull. Cal. Math. Soc., vol.90, pp.337-342.

Chattopadhyay A. and Choudhury S. (1990): *Propagation, reflection and transmission of magnetoelastic shear waves in a self-reinforced medium.* – Int. J. Eng. Sci., vol.28, No.6, pp.485-495.

Chattopadhyay A. and Choudhury S. (1995): *Magnetoelastic shear waves in an infinite self-reinforced plate.* – Int. J. Num. Anal. Methods in GeoMech., vol.19, pp.289-304.

Caviglia G. and Morro A. (2002): *Wave reflection and transmission from anisotropic layers through Riccati equations.* – Q. J. Mech. Appl. Math., vol.55, No.1, pp.93-107.

Chaudhary S., Tomar S.K. and Kaushik V.P. (2005): *Transmission of plane SH-wave through a self-reinforced elastic layer sandwiched between two anisotropic inhomogeneous elastic solid half-spaces.* – Int. J. Appl. Mech. Eng., vol.9, No.1, pp.131-146.

Chaudhary S., Kaushik V.P. and Tomar S.K. (2004): *Reflection/Transmission of plane SH-wave through a self-reinforced elastic layer sandwiched between two homogeneous visco-elastic solid half-spaces.* – Acta Geophysica Polonica, vol.52, No.2, pp.219-235.

Gupta R.N. (1962): *Reflection of elastic waves from a transition layer.* – Bull. Seism. Soc. Am., vol.56, pp.511-526.

Gupta R.N. (1966): *Reflection of plane elastic waves from a transition layers with arbitrary variation of velocity and density.* – Bull. Seism. Soc. Am., vol.56, pp.633-642.

Kaushik V.P. and Rana A. (1997): *Transmission of SH-waves through a linear viscoelastic layer sandwiched between two anisotropic inhomogeneous elastic half-spaces.* – Indian J. Pure Appl. Math, vol.28, pp.851-863.

Malhotra S., Chopra S.D. and Gogna M.L. (1983): *Reflection and transmission of SH-waves in a laterally and vertically heterogeneous anisotropic stratum.* – Acta Geophysica Polonica, vol.31, pp.267-280.

Sengupta P.R. and Nath S. (2001): *Surface waves in fibre-reinforced anisotropic elastic media.* – Sadhana, vol.26, No.4, pp.363-370.

Singh B.M., Singh S.J. and Chopra S.D. (1978): *Reflection and refraction of SH-waves at the plane boundary between two laterally and vertically heterogeneous solids*. – Acta Geophysica Polonica, vol.26, pp.209-216.

Singh V.P. (1977): *SH-waves in multilayered laterally heterogeneous media.* – Bull. Seism. Soc. Am., vol.67, pp.331-345.

Sinha S.B. (1964): *Transmission of elastic waves through a homogeneous layer sandwiched in homogeneous media.* – J. Phys. Earth., vol.12, pp.1-4.

Thapliyal V. (1974): *Reflection of SH-waves from anisotropic transition layer.* – Bull. Seism. Soc. Am., vol.64, 1979-1991.

Verma P.D.S. and Rana O.H. (1983): *Rotation of a circular cylindrical tube reinforced by fibers lying along helices*. – Mech. Materials, vol.2, pp.353-359.

Verma P.D.S. (1986): *Magnetoelastic shear waves in self-reinforced bodies.* – Int. J. Eng. Sci., vol.24, pp.1067-1073.

Verma P.D.S., Rana O.H. and Verma M. (1988): *Magnetoelastic transverse surface waves in self-reinforced elastic bodies.* – Indian J. Pure Appl. Math., vol.19, No.7, pp.713-716.

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