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SQUEEZE FILM BASED MAGNETIC FLUID IN BETWEEN POROUS CIRCULAR DISKS WITH SEALED BOUNDARY

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Efforts have been made to study the effect of the magnetic fluid lubricant and the sealing of the boundary for the squeeze film between two circular disks when the upper disk having a porous facing with its boundary sealed, approaches the non-porous lower disk normally. The modified Reynolds equations for the fluid region and the governing Laplacian equation for the pressure in a porous region are solved with appropriate boundary conditions. Expressions are obtained for pressure, load carrying capacity and the response time. The results are presented graphically. The combined effect of the magnetic fluid lubricant and sealing of the boundary increases the load carrying capacity significantly and hence the performance of the bearing can be enhanced considerably by sealing the boundary and taking a magnetic fluid as lubricant.

Key words: magnetic fluid, sealed boundary, slider bearing, load carrying capacity.

1. Introduction

It was Wu (1970) who analyzed the behavior of the squeeze film between two annular disks when the upper disk having a porous facing approaches the lower non porous disk. Murti (1974) considered this problem of the squeeze film behavior between the circular disks. In both the analyses it was assumed that the porous boundary was open resulting in the flowing out of the lubricant from the porous boundary. Ajwaliya (1984) studied this problem and improved the performance of the porous bearing while retaining its self lubrication property. Of course, he considered the squeeze film between two circular disks when the upper disk having a porous facing with its boundary sealed approaches the non porous lower disk normally. These types of bearings may have applications in lubricating clutch plates and automobile transmissions.

Verma (1986) and Bhat and Deheri (1993) investigated the squeeze film between porous plates. They proved that the application of a magnetic fluid lubricant improved the performance of the squeeze film.

The above mentioned improved performance of the bearing in the case of sealing the boundary and choosing the magnetic fluid lubricant makes it appealing to study the magnetic fluid based squeeze film between porous circular disks with a sealed boundary.

2. Analysis

The bearing configuration is shown in Fig.1. The bearing consists of two circular disks each of radius *a*. The upper disk has a porous facing of uniform thickness *H* and moves normally towards the non porous lower disk with a uniform velocity $\dot{h} = \dfrac{dh}{dt}$, *h* being the central film thickness. Assuming axially

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symmetric flow of the magnetic fluid between the disks under an oblique magnetic fluid $\overline{H}$, whose magnitude H is a function of r vanishing at $r = a$, the modified Reynolds equation governing the film pressure p for the fluid region (Wu, 1970) is

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d}{dr}\left(p - 0.5\mu_0\overline{\mu}H^2\right)\right] = \frac{12\mu}{h^3}\left[\dot{h} - \frac{\phi}{\mu}\left(\frac{\partial p^*}{\partial z}\right)_{z=h}\right] \tag{2.1}$$

where $H^2 = kr^2(a-r)/a$ (c.f. Bhat and Deheri, 1993; Prajapati, 1995), k is the permeability parameter of the porous matrix, μ_0 is the permeability of the free space, $\overline{\mu}$ is the magnetization permeability and μ is the viscosity of the fluid. The inclination ϕ of the external magnetic field $\overline{H}$ with the lower disc is determined from

$$\cot\phi\frac{\partial\phi}{\partial r} + \frac{\partial\phi}{\partial z} = \frac{2r-a}{2r(a-r)},$$

whose solution is

$$c^2 \operatorname{cosec}^2\phi = r(a-r),$$

and

$$\sin\left(\frac{z}{c}\right) = \frac{2r-a}{\left[a^2 - 4c^2\right]^{1/2}}.$$

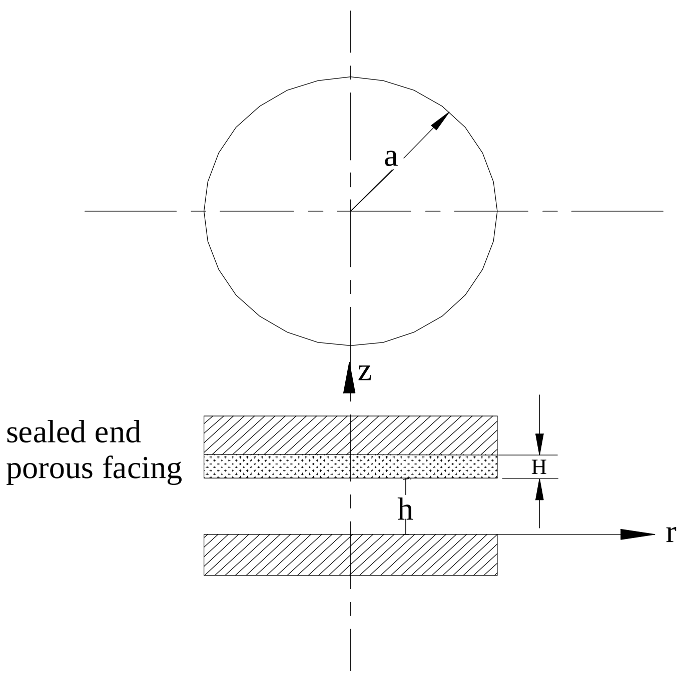

Fig.1. Squeeze film porous bearing; geometry and co-ordinates system.

Further, the governing Laplacian equation for the pressure p^* in porous region is

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial p^*}{\partial r}\right]+\frac{\partial^2 p^*}{\partial z^2}=0\,. \tag{2.2}$$

The boundary conditions for (2.1) and (2.2) are

$$p(a)=0\,, \tag{2.3}$$

$$(\partial p/\partial r)_{r=0}=0\,, \tag{2.4}$$

$$\left(\partial p^*/\partial r\right)_{r=a}=0\,, \tag{2.5}$$

$$\left(\partial p^*/\partial z\right)_{z=h+H}=0\,. \tag{2.6}$$

The boundary condition (2.5) results from Darcy's law and the requirement that there is no radial spilling of lubricant across the boundary of a porous matrix. It is to be noted that the matching condition is given by

$$p(r)=p^*(r,h)\,. \tag{2.7}$$

Equation (2.2) is solved by the method of separation of variables with the related boundary conditions (2.5) and (2.6). The solution is

$$p^*(r,z)=\sum_{n=0}^{N}C_n\exp(\alpha_n z)\left[1+\exp\left(2\alpha_n(h+H-z)\right)\right]J_0(\alpha_n r) \tag{2.8}$$

where α_0 and $\alpha_n(n>0)$ is the n^{th} eigen value satisfying $J_1(\alpha_n a)=0$. Equation (2.8) can be rearranged as

$$p^*(r,z)=2C_0\sum_{n=1}^{N}C_n\exp(\alpha_n z)\left[1+\exp\left(2\alpha_n(h+H-z)\right)\right]J_0(\alpha_n r)\,. \tag{2.9}$$

Substitution of p^* from Eq.(2.9) into Eq.(2.1) and then integration with respect to r with boundary condition (2.4)gives

$$r\frac{d}{dr}\left[p-0.5\mu_0\bar{\mu}H^2\right]=\frac{6\mu\dot{h}r^2}{h^3}-\frac{12\phi}{h^3}\sum_{n=1}^{N}C_n\exp(\alpha_n h)\left[1-\exp(2\alpha_n H)\right]r\,J_1(\alpha_n r)\,. \tag{2.10}$$

Again integrating (2.10) with boundary condition (2.3) finally yields

$$\begin{aligned}p=&\left[\left(0.5\mu_0\bar{\mu}k r^2(a-r)\right)/a\right]+\left[\left(3\mu\dot{h}\left(r^2-a^2\right)\right)/h^3\right]+\\&+\left(12\phi/h^3\right)\Big[\left(0.5\mu_0\bar{\mu}k/a\right)\left\{(a-r)\left(1.5h^3+hr^2\right)\right\}+\\&+\sum_{n=1}^{N}(C_n/\alpha_n)\left\{\exp(\alpha_n h)\left(1-\exp(2\alpha_n H)\right)\right\}\left\{J_0(\alpha_n a)-J_0(\alpha_n r)\right\}\Big].\end{aligned} \tag{2.11}$$

Substitution from Eqs (2.8) and (2.11) into Eq.(2.7) and rearrangement of terms yields

$$\begin{aligned}&\left(\frac{6\mu\dot{h}a^2}{\alpha_n^2 h^3}\right)+\frac{0.5\mu_0\bar{\mu}k}{a\alpha_n^2}\left\{2ar^2-3r^2+4.5rh^2+(3/8)\left(h^4/r\right)\right\}+\\&+\left(12\phi/h^3\right)\left\{\left(0.5\mu_0\bar{\mu}k\right)/\left(2a\alpha_n^2\right)\right\}\left[\left(4ar^2h-3rh^3-6r^3h\right)\right]=\\&=2C_0+\sum_{n=1}^{N}C_n\exp(\alpha_n h)\left[\left\{1+\exp(2\alpha_n H)\right\}+\left(12\phi/h^3\alpha_n\right)\left\{1-\exp(2\alpha_n H)\right\}\right]J_0(\alpha_n r)+\\&-\left(12\phi/h^3\right)\sum_{n=1}^{N}\left(C_n/\alpha_n\right)\left\{\exp(\alpha_n h)\left(1-\exp(2\alpha_n H)\right)\right\}J_0(\alpha_n a).\end{aligned} \tag{2.12}$$

To determine the Fourier-Bessel coefficients C_n we invoke the orthogonality of the eigen function $J_1(\alpha_n r)$ over the interval $[0, a]$. Thus for, $n>0$ we can have the expression for C_n as

$$C_n=\frac{12\mu\dot{h}+X+Y}{J_0(\alpha_n a)\alpha_n^2 h^3\exp(\alpha_n h)\{S\}} \tag{2.13}$$

where

$$X=\left(h^3\mu_0\bar{\mu}k/a^3\right)\left\{-a^3+4.5ah^2+(3/8)\left(h^4/a\right)\right\},$$

$$Y=\left(6\phi\mu_0\bar{\mu}k/a^3\right)\left\{10ha^3+3ah^3\right\},$$

$$S=\left[1+\exp(2\alpha_n H)-\left(12\phi/h^3\alpha_n\right)\left\{1-\exp(2\alpha_n H)\right\}\right].$$

Hence the pressure distribution which is given by (2.11) now assumes the form

$$\begin{aligned}p=&\left[\left(0.5\mu_0\bar{\mu}kr^2(a-r)\right)/a\right]+\left[\left(3\mu\dot{h}\left(r^2-a^2\right)\right)/h^3\right]+\\&+\left(6\phi\mu_0\bar{\mu}k/h^3a\right)\left\{(a-r)\left(1.5h^3+hr^2\right)\right\}-\left[\left(12\mu h/h^3\right)+\right.\\&+\left(\mu_0\bar{\mu}k/a^3\right)\left\{-a^3+4.5ah^2+(3/8)\left(h^4/a\right)\right\}+\left(6\phi\mu_0\bar{\mu}k/h^3a^3\right)\left.\left\{10ha^3-3ah^3\right\}\right]\times\\&\times\sum_{n=1}^{N}\frac{J_0(\alpha_n a)-J_0(\alpha_n r)}{J_0(\alpha_n a)\alpha_n^2\left[1+\left(h^3\alpha_n/12\phi\right)\left\{\left(\exp(2\alpha_n H)+1\right)/\left(\exp(2\alpha_n H)-1\right)\right\}\right]}.\end{aligned} \tag{2.14}$$

Using the dimensionless quantities

$$\mu^*=\left(-\mu_0\bar{\mu}kh^3/\mu\dot{h}\right),\qquad \phi=\left(\psi h^3/H\right),\qquad R=(r/a),\qquad \bar{h}=(h/a),\qquad \bar{H}=(H/a),$$

we get the dimensionless pressure from (2.14) in the form

$$P=-\left(h^3 p/\mu a^2 \dot{h}\right)=0.5\mu^* R^2(1-R)+3\left(1-R^2\right)+\left\{6\mu^*\psi(1-R)/\overline{H}\right\}\left(1.5\bar{h}^3+\bar{h}R^2\right)+$$
$$+\left[12+\mu^*\left(1-4.5\bar{h}^2-0.375\bar{h}^4\right)+\left(6\mu^*\psi/\overline{H}\right)\left(-10\bar{h}+3\bar{h}^3\right)\right]\times \tag{2.15}$$
$$\times\sum_{n=1}^{N}\frac{J_0(\overline{\alpha}_n R)-J_0(\overline{\alpha}_n)}{J_0(\overline{\alpha}_n)\overline{\alpha}_n^2\left[1+\left(\overline{H}\overline{\alpha}_n/12\psi\right)\left\{\left(\exp\left(2\overline{\alpha}_n\overline{H}\right)+1\right)/\left(\exp\left(2\overline{\alpha}_n\overline{H}\right)-1\right)\right\}\right]}.$$

The load carrying capacity W in a dimensionless form is given by

$$\overline{W}=\left(-h^3 W/\mu a^4 \dot{h}\right)=(3\pi/2)+\pi\left[0.05\mu^*+\left(0.6\mu^*\bar{h}\psi/\overline{H}\right)\left(5\bar{h}^2+1\right)\right]+$$
$$-\pi\sum_{n=1}^{N}\frac{\left[-12+\mu^*\left(-1+4.5\bar{h}^2+0.375\bar{h}^4\right)+\left(6\bar{h}\psi\mu^*/\overline{H}\right)\left(10-3\bar{h}^2\right)\right]}{\overline{\alpha}_n^2\left[1+\left(\overline{\alpha}_n\overline{H}/12\psi\right)\left\{\left(\exp\left(2\overline{\alpha}_n\overline{H}\right)+1\right)/\left(\exp\left(2\overline{\alpha}_n\overline{H}\right)-1\right)\right\}\right]}. \tag{2.16}$$

The time taken by the upper disk to reach a film thickness h_1 starting from h_0 is given by (for constant load W)

$$\Delta t=\int_{t_0}^{t_1}dt=D\int_1^{\bar{h}}\left(1/\bar{h}^3\right)d\bar{h}+\sum_{n=1}^{N}\int_1^{\bar{h}}\left[A_n/\left\{\bar{h}^3\left(1+B_n^3 h^3\right)\right\}\right]d\bar{h} \tag{2.17}$$

where

$$D=\left(-3\pi\mu a^4/2h_0^2 W\right),\qquad A_n=\left(12\pi\mu a^4/\overline{\alpha}_n^2 h_0^2 W\right),\qquad \bar{h}=(h_1/h_0),$$

$$B_n^3=\left(\overline{\alpha}_n\overline{H}/12\psi_0\right)\left[\left(\exp\left(2\overline{\alpha}_n\overline{H}\right)+1\right)/\left(\exp\left(2\overline{\alpha}_n\overline{H}\right)-1\right)\right].$$

Now the dimensionless response time ΔT is given by

$$\Delta T=\left(Wh_0^2\Delta t/\mu a^4\right)=(-3\pi/4)\left\{1-\left(1/\bar{h}^2\right)\right\}+\sum_{n=1}^{N}\left(12\pi/\overline{\alpha}_n^2\right)\left[0.5\left\{1-\left(1/\bar{h}^2\right)\right\}+\right.$$
$$+\left(B_n^2/6\right)\log\left\{\left((1+B_n)^2\left(1-B_n\bar{h}+B_n^2 h^2\right)\right)\Big/\left(\left(1+B_n\bar{h}\right)^2\left(1-B_n+B_n^2\right)\right)\right\}+ \tag{2.18}$$
$$\left.+\left(B_n^2/\sqrt{3}\right)\left\{\tan^{-1}\left((2B_n-1)/\sqrt{3}\right)-\tan^{-1}\left(\left(2B_n\bar{h}-1\right)/\sqrt{3}\right)\right\}\right].$$

3. Results and discussion

Expressions for pressure distribution, load carrying capacity and response time are presented in Eqs (2.15), (2.16) and (2.18) respectively. The results are presented graphically. All these results indicate that pressure distribution, load carrying capacity and response time increase significantly in the case of a magnetic fluid based squeeze film with sealed boundary in comparison to that of an open end porous bearing with a conventional lubricant. For larger values of permeability parameter the effect of sealing the boundary of the porous matrix in the presence of the magnetic fluid as lubricant results in a substantial increase in the response time. However, the effect of sealing the boundary on the response time is independent of the magnetization parameter which is the essence of expression of response time.

As special cases these results yield the results of Ajwaliya (1984) and Bhat and Deheri (1993). The combined effect of the magnetic fluid lubricant and sealing of the boundary increases the load carrying capacity significantly and hence the performance of the bearing can be enhanced considerably by sealing properly the boundary and choosing a magnetic fluid as lubricant.

Appendix

Normally the following assumptions are made.

1. The lubricant flow is considered laminar and lubricant film is assumed to be isoviscous.
2. There are no external fields of force acting on the fluid. While magnetic and electric forces are not present in the flow of non conducting lubricants, forces due to gravitational attraction are always present. However, these forces are small compared to the viscous force involved.
3. The flow is considered steady and temperature changes of the lubricant are neglected.
4. The bearing surfaces are assumed to be perfectly rigid so that elastic deformations of the bearing surfaces may be neglected.
5. Bearing surfaces are assumed to be perfectly smooth or even when there is surface roughness it is of very small order of magnitude in comparison with the minimum film thickness.
6. The thickness of the lubricant film is very small when compared to the dimensions of the bearing.
7. The lubricant velocity along the transverse direction to the film is considered small enough.
8. Velocity gradients and indeed the second derivatives along the direction transverse to the film are predominant as compared to those in the plane of the film.
9. The lubricant inertia is considered negligible.
10. The porous matrix of the bearing surface is assumed to be homogeneous and isotropic.
11. Darcy's law is assumed to govern the lubricant flow within the porous matrix, while no slip condition is taken at the porous matrix-film interface.

In cylindrical polar coordinates (r, θ, z) assuming axially symmetry the following two equations were obtained

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial p}{\partial r}\right)=12\mu\left[\frac{dh}{dt}-\frac{\phi}{\mu}\left(\frac{\partial p^*}{\partial z}\right)_{z=h}\right], \tag{*1}$$

and

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p^*}{\partial r}\right)+\frac{\partial^2 p^*}{\partial z^2}=0\,. \tag{*2}$$

Derivation of magnetohydrodynamic equations

Let $\overline{q}=(u, 0, w)$ be the fluid velocity in the film region, $\overline{M}=(M_r, 0, M_z)$ the magnetization vector, $\overline{H}=(H_r, 0, H_z)$ the external magnetic field, μ the viscosity of the magnetic fluid lubricant and μ_0 is the permeability of the free space. Then the governing equations for the pressure p in the film region are as in Verma (1986), Bhat and Deheri (1993; 1991)

$$-\frac{\partial p}{\partial r}+\mu\frac{\partial^2 u}{\partial z^2}+\mu_0\left(M_r\frac{\partial H_r}{\partial r}+M_z\frac{\partial H_r}{\partial z}\right)=0\,, \tag{A.1}$$

$$-\frac{\partial p}{\partial z}+\mu_0\left(M_r\frac{\partial H_z}{\partial r}+M_z\frac{\partial H_z}{\partial z}\right)=0\,, \tag{A.2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial w}{\partial z}=0\,. \tag{A.3}$$

If $\overline{Q}=(\overline{u},0,\overline{w})$ is the velocity and $\overline{p}$ the pressure in the porous region

$$\overline{u}=-\frac{\overline{k}}{\mu}\left\{\frac{\partial\overline{p}}{\partial r}-\mu_0\left(M_r\frac{\partial H_r}{\partial r}+M_z\frac{\partial H_r}{\partial z}\right)\right\}, \tag{A.4}$$

$$\overline{w}=-\frac{\overline{k}}{\mu}\left\{\frac{\partial\overline{p}}{\partial z}-\mu_0\left(M_r\frac{\partial H_z}{\partial r}+M_z\frac{\partial H_z}{\partial z}\right)\right\}, \tag{A.5}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\overline{u})+\frac{\partial\overline{w}}{\partial z}=0\,, \tag{A.6}$$

$\overline{k}$ being the permeability of the porous material. Assuming Φ to be the potential of $\overline{H}$ and $\overline{M}=\overline{\mu}\,\overline{H},\overline{\mu}$ being the magnetic permeability, we can show that

$$M_r\frac{\partial H_r}{\partial r}+M_z\frac{\partial H_r}{\partial z}=\frac{\overline{\mu}}{2}\frac{\partial H^2}{\partial r}\,, \tag{A.7}$$

$$M_r\frac{\partial H_z}{\partial r}+M_z\frac{\partial H_z}{\partial z}=\frac{\overline{\mu}}{2}\frac{\partial H^2}{\partial z}\,. \tag{A.8}$$

H being the magnitude of $\overline{H}$. Substituting Eqs (A.7) and (A.8) into Eqs (A.1), (A.2), (A.4) and (A.5) and using Eqs (*1) and (*2) we get the Reynolds' equation as in (2.1), wherein the associated boundary conditions are as in Eqs (2.3) to (2.6).

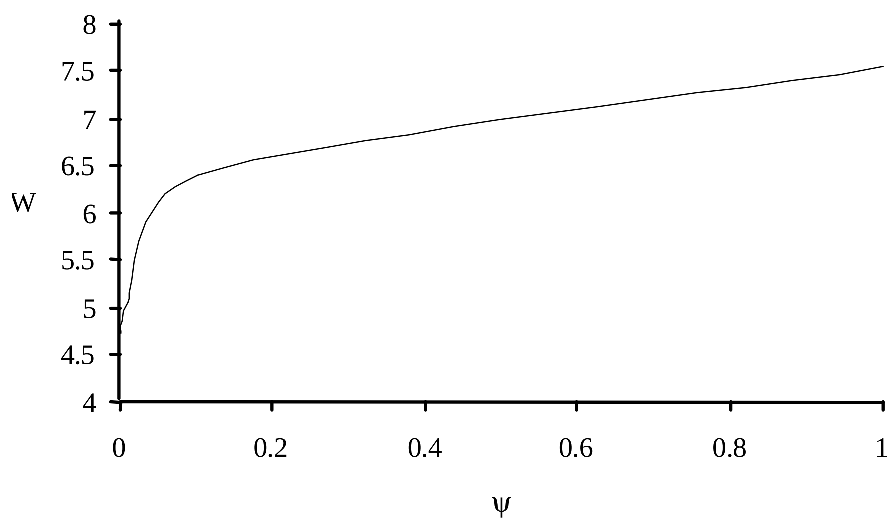

Fig.2. Load carrying capacity in the absence of the magnetic fluid.

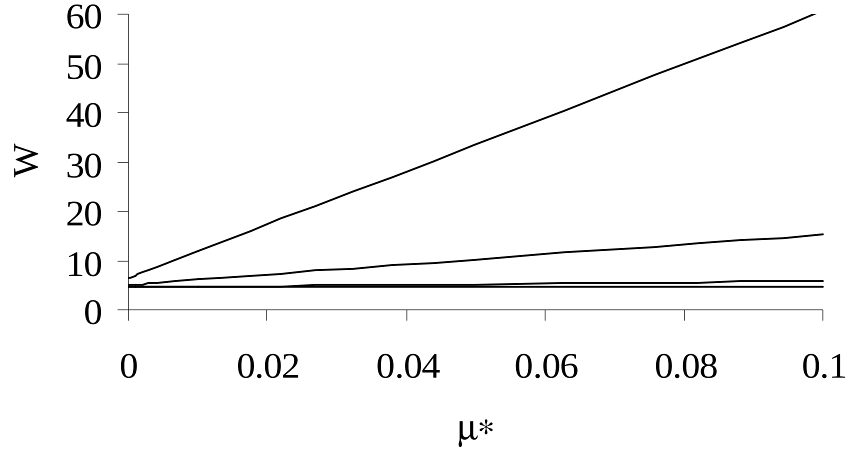

Fig.3. Load carrying capacity due to the combined effect of the magnetic fluid and sealing the boundary.

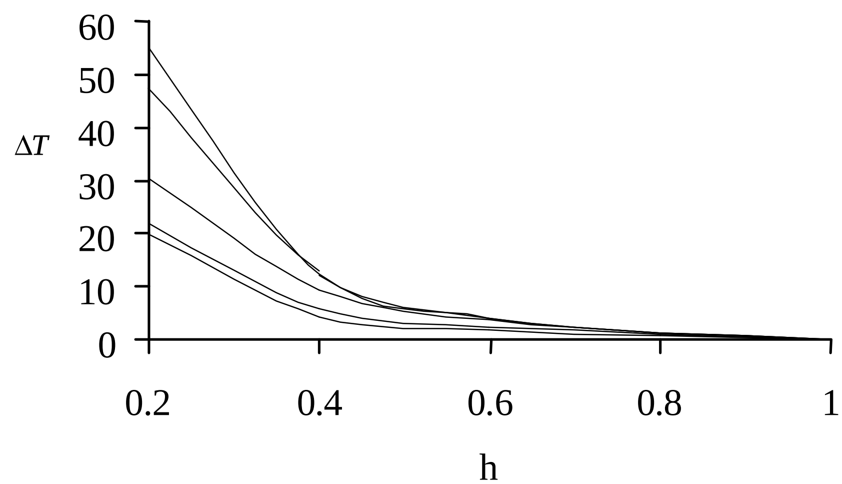

Fig.4. Variation of response time with respect to height (open end).

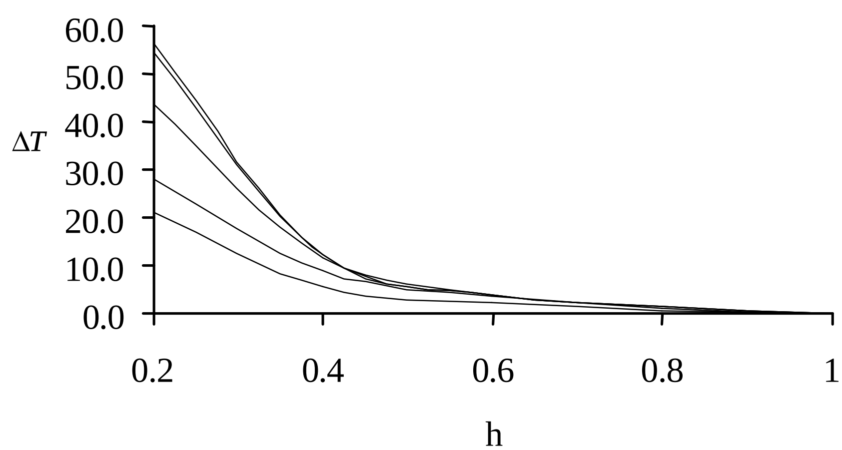

Fig.5. Variation of response time with respect to height (sealed end).

Nomenclature

a – radius of the circular disk
C_n – Fourier Bessel coefficients
h – central film thickness
H – thickness of the upper disk
J_n – Bessel's function
k – permeability of the porous matrix
p – pressure distribution
P – dimensionless pressure
p^* – pressure in the porous region
W – load carrying capacity
$\overline{W}$ – dimensionless load carrying capacity
α_n – nth eigen value satisfying $J_1(\alpha_n a)=0$
Δt – response time
ΔT – dimensionless response time
μ – viscosity of the fluid
$\overline{\mu}$ – magnetization permeability
μ_0 – permeability of the free space
ψ – porosity parameter

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