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PRESSURE DISTRIBUTION IN A SQUEEZE FILM BIOBEARING LUBRICATED BY A SYNOVIAL FLUID

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The flow of a synovial fluid in a squeeze film biological bearing is considered. The biobearing is modelled by two rotational surfaces and the porous layer is adheres to the curved non-porous surface. The flow in the biobearing clearance is considered with inertia and the Navier-Stokes and Poisson equations are uncoupled by using the Morgan-Cameron approximation. As an example the biobearing modelled by two disks and two spherical surfaces is discussed.

Key words: curvilinear bearing, porous layer, inertia effect, squeeze film, couple stress fluid.

1. Introduction

Biological bearings are lubricated by a synovial fluid which is – in general – a non-Newtonian fluid. Since the long chain hyaluronic acid molecules are found as additives in the synovial fluid, it is suggested to use a couple stress fluid to model the synovial fluid behaviour in human joints. From mechanical point of view the presence of small amounts of additives in a lubricant can improve the bearing performance by enhancing the lubricant viscosity and thus producing an increase of the pressure in the bearing clearance.

Now, rheological models are used in theoretical research to approach real lubricants. The model of a couple-stress fluid is an example. It may be a mathematical model of a synovial fluid, recognized as the most effective lubricant in nature. The theory of the couple-stress fluid was presented in 1966 (Stokes, 1966).

On the basis of the Stokes' couple-stress fluid model, the present paper considers the effects of couple-stresses and inertia effects on the characteristics of squeeze-film behaviour in thrust curvilinear bearings with reference to synovial joints. It is hoped that such an analysis will be useful in understanding the mechanism of human joint lubrication and the role of the long chain hyaluronic acid molecules in synovial fluids behaving like couple-stress fluids.

The study of the mechanism of synovial joints has recently become the object of scientific research. A human joint is a dynamically loaded bearing which employs articular cartilage as the bearing and synovial fluid as the lubricant

Depending on the joint location they may be the hip joints, shoulder joints, elbow or knee joints. This joints may be modelled by the spherical bearings, the quasi-spherical bearings, the quasi-cylindrical bearings or the plane bearings.

Biobearings are structured – similarly as any other bearing – from a pin which is a bone head and a sleeve which is an acetabulum. The pins and sleeves are covered with a cartilage which constitutes a porous structure with anti-friction properties. The cartilage structure on the pin surface is more compact than the one on the sleeve surface and – in the first approximation – it may be modelled as an impermeable wall, whereas the cartilage structure on the sleeve is less compact and – by reason of greater thickness – it should be modelled as a porous layer.

The purpose of the paper is to investigate an inertia effect on the pressure distribution in a squeeze flow between two surfaces of revolution with one porous layer, as shown in Fig.1.

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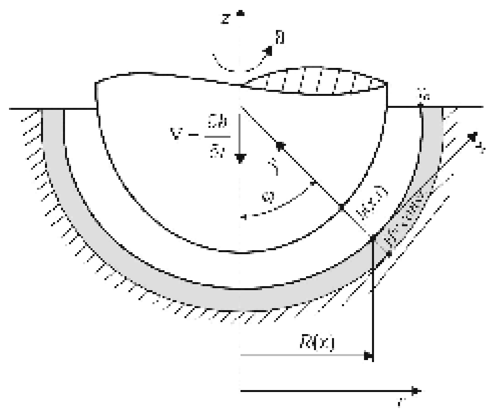


Fig.1. Configuration of a squeeze flow with one porous wall.

2. Analysis of a synovial fluid flow in a curvilinear clearance of a biological bearing

The equations of an unsteady motion of a synovial fluid – modelled as a couple stress fluid – in a biological bearing may be presented in the form (Jurczak, 2004; Walicka, 2002; Walicki and Walicka, 1998; Walicki, 2005)

$$\frac{1}{R}\frac{\partial(R\upsilon_x)}{\partial x}+\frac{\partial\upsilon_y}{\partial y}=0\,, \tag{2.1}$$

$$\rho\left[\frac{\partial\upsilon_x}{\partial t}+\left(\frac{R'}{R}+\frac{\partial}{\partial x}\right)\upsilon_x^2+\frac{\partial}{\partial y}\left(\upsilon_x\upsilon_y\right)\right]=-\frac{dp}{dx}+\mu\frac{\partial^2\upsilon_x}{\partial y^2}-\eta\frac{\partial^4\upsilon_x}{\partial y^4}\,. \tag{2.2}$$

The problem statement is complete after specification of boundary conditions which are (Walicka, 2002; Walicki and Walicka, 1998; Walicki, 2005)

$$\begin{aligned}
&\upsilon_x=0, \qquad \upsilon_y=V_H, \qquad \frac{\partial^2\upsilon_x}{\partial y^2}=0 \qquad \text{for} \qquad y=0\,,\\
&\upsilon_x=0, \qquad \upsilon_y=\frac{\partial h}{\partial t}=\dot{h}, \qquad \frac{\partial^2\upsilon_x}{\partial y^2}=0 \qquad \text{for} \qquad y=h\,,\\
&\left.\frac{\partial p}{\partial x}\right|_{x=0}=0, \qquad p(x_o)=p_o\,.
\end{aligned} \tag{2.3}$$

3. Modified Reynolds equation

Averaging the left-hand side of Eq.(2.2) across the clearance thickness and introducing the notation (Jurczak, 2004; Walicka, 2002; Walicki and Walicka, 1998; Walicki, 2005)

$$f(x,t)=\frac{1}{\mu}\frac{dp}{dx}+\frac{\rho}{\mu h}\frac{\partial}{\partial t}\int_0^h\upsilon_x dy+\frac{\rho}{\mu h}\left(\frac{R'}{R}+\frac{\partial}{\partial x}\right)\int_0^h\upsilon_x^2 dy\,, \tag{3.1}$$

we obtain the following equation

$$\frac{\partial^2 \upsilon_x}{\partial y^2} - l^2 \frac{\partial^4 \upsilon_x}{\partial y^4} = f(x,t) \tag{3.1}$$

where

$$l^2 = \frac{\eta}{\mu}.$$

Its solution takes the form

$$\upsilon_x = \frac{f}{2}\left[y^2 - hy + 2l^2\left(1 - \mathrm{ch}\frac{y}{l} - \frac{1 - \mathrm{ch}\frac{h}{l}}{\mathrm{sh}\frac{h}{l}}\mathrm{sh}\frac{y}{l}\right)\right]. \tag{3.3}$$

By substituting Eq.(6) into Eq.(4) one obtains

$$\frac{1}{R}\frac{\partial}{\partial x}\left(Rh^3 f\right) = 12\left(\frac{\partial h}{dt} - V_H\right). \tag{3.4}$$

The flow in the porous matrix is given by the Darcy law (Jurczak, 2004; Walicki, 2005):, therefore one has

$$\frac{1}{R}\frac{\partial}{\partial x}\left(Rh^3 f\right) = 12\left[\frac{\partial h}{\partial t} + \frac{\Phi}{\mu}\left(\frac{\partial \overline{p}}{\partial y}\right)_{y=0}\right] \tag{3.5}$$

where $\overline{p}$ is the pressure in the porous matrix.

Using the Morgan-Cameron approximation (Jurczak, 2004; Morgan and Cameron, 1957; Walicki, 2005): one obtains

$$\left(\frac{\partial \overline{p}}{\partial y}\right)_{y=0} = -\frac{\mu H}{R}\frac{\partial}{\partial x}(Rf), \tag{3.6}$$

but the Reynolds equation for the flow in a squeeze film takes the form

$$\frac{1}{R}\frac{\partial}{\partial x}\left(Rh^3 f\right) = 12\left[\frac{\partial h}{\partial t} + \frac{\Phi H}{R}\frac{\partial}{\partial x}(Rf)\right]. \tag{3.7}$$

Solving the Reynolds equation one finds f

$$f=\frac{12}{R\left\{h^3-12\left[\left(\frac{l}{h}\right)^2-\Phi H\right]\right\}}\int R\dot{h}dx\,, \tag{3.8}$$

and next the pressure distribution p

$$p(x,t)=p_o-12\mu\left[S_o-S(x,t)\right]-\rho\left[I_o-I(x,t)\right]+\frac{6\rho}{5}\left[T_o-T(x,t)\right] \tag{3.9}$$

where

$$A(x,t)=\int R\dot{h}dx, \qquad B(x,t)=\frac{A(x,t)}{R\left\{h^3-12\left[\left(\frac{l}{h}\right)^2-\Phi H\right]\right\}},$$

$$S(x,t)=\int B(x,t)dx, \qquad C(x,t)=h^3B(x,t), \tag{3.10}$$

$$I(x,t)=\int\frac{1}{h}\frac{\partial}{\partial t}C(x,t)f(l,h)dx, \qquad T(x,t)=\int\frac{1}{h}\left(\frac{R'}{R}+\frac{\partial}{\partial x}\right)\frac{C^2(x,t)}{h}g(l,h)dx,$$

and

$$f(l,h)=1-12\left(\frac{l}{h}\right)^2+24\left(\frac{l}{h}\right)^3\mathrm{th}\left(\frac{h}{2l}\right)\approx 1-12\left(\frac{l}{h}\right)^2,$$

$$g(l,h)=1+40\left(\frac{l}{h}\right)^3\left[\mathrm{cth}\left(\frac{h}{l}\right)-\frac{h}{2l}\right]\approx 1-20\left(\frac{l}{h}\right)^2. \tag{3.11}$$

4. Examples of application

4.1. The flow of a synovial fluid in a clearance of a plane biobearing

Let us consider a plane biobearing modelled by two disks as shown in Fig.2. Introducing the following nondimensional parameters

$$\tilde{x}=\frac{x}{R_o}, \qquad \tilde{R}=\frac{R}{R_o}, \qquad \tilde{h}=E(t)=\frac{h}{h_o}, \qquad E(t)=1-\varepsilon(t), \qquad K=\left(\frac{12\Phi H}{h_o^3}\right)^{\frac{1}{3}},$$

$$\mathrm{Re}=\frac{\rho h_o^2\dot{\varepsilon}}{\mu}, \qquad A=\frac{\ddot{\varepsilon}E}{\dot{\varepsilon}^2}, \qquad l^*=\frac{l}{h_o}, \qquad \dot{\varepsilon}=\frac{d\varepsilon}{dt}, \qquad \ddot{\varepsilon}=\frac{d^2\varepsilon}{dt^2}. \tag{4.1}$$

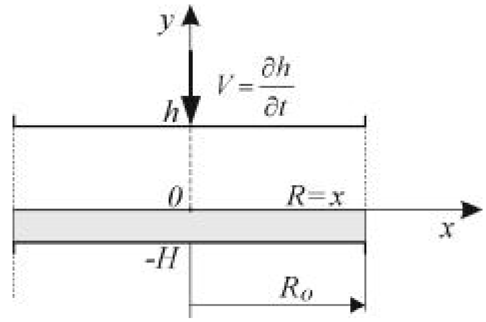


Fig.2. Thrust plane biobearing modelled by two disks.

we obtain from Eq.(3.9) the following formula for the dimensionless pressure distribution in the clearance of the plane biobearing with squeeze fluid film (for $K << 1$)

$$\tilde{p}=\frac{p-p_o}{\mu\dot{\varepsilon}}\left(\frac{h_o}{x_o}\right)^2=\left\{\frac{3}{E^3}\left[1+12\left(\frac{l^*}{E}\right)^2-\left(\frac{K}{E}\right)^3\right]+\frac{\mathrm{Re}}{2E^2}\left[A+(3+A)\left(\frac{K}{E}\right)^3+\right.\right.$$

$$\left.+72(4+A)\left(\frac{l^*}{E}\right)^4-6(5+A)\frac{l^{*2}K^3}{E^5}\right]+\frac{3\,\mathrm{Re}}{10E^2}\left[1+4\left(\frac{l^*}{E}\right)^2-2\left(\frac{K}{E}\right)^3-336\left(\frac{l^*}{E}\right)^4+\right. \tag{4.2}$$

$$\left.\left.+\left(\frac{K}{E}\right)^6+16\frac{l^{*2}K^3}{E^5}-2880\left(\frac{l^*}{E}\right)^6-20\frac{l^{*2}K^6}{E^8}+480\frac{l^{*4}K^3}{E^7}\right]\right\}\left(1-\tilde{x}^2\right),$$

Here Re denotes the modified Reynolds number, and A denotes the acceleration squeeze number.

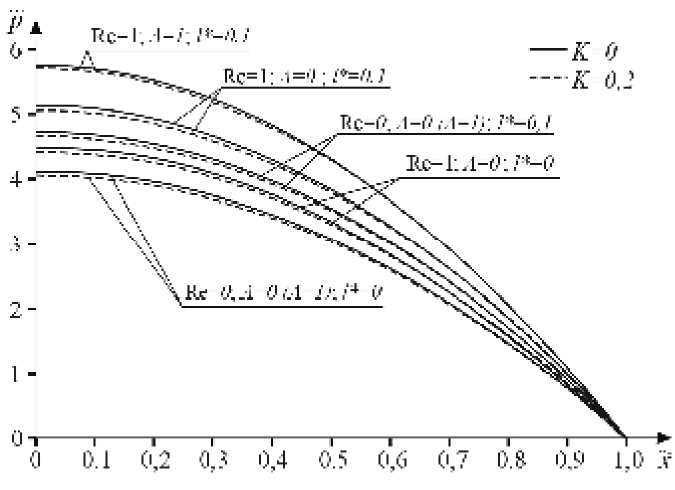


Fig.3. Dimensionless pressure distribution in the clearance of the plane biobearing for $\varepsilon = 0,1$.

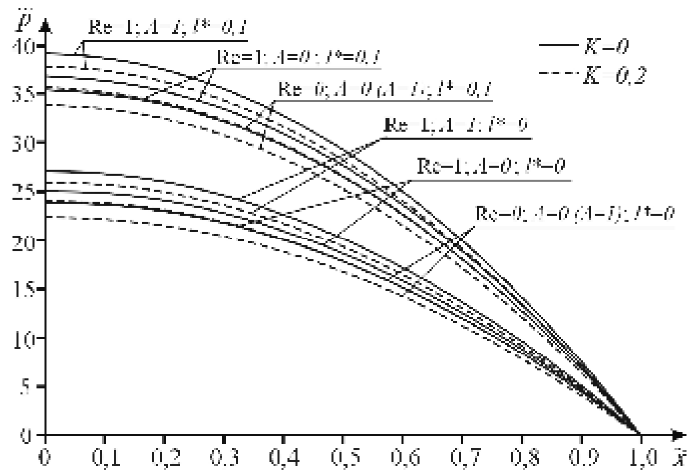


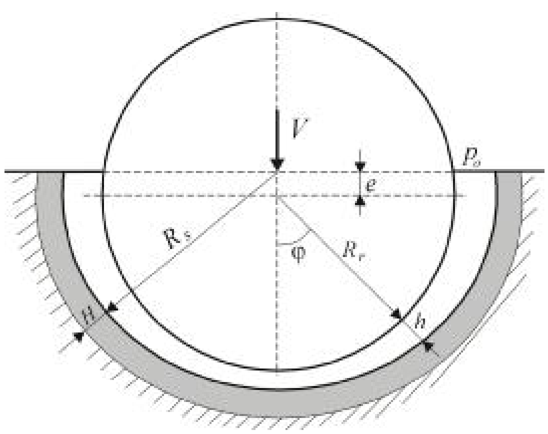
Fig.4. Dimensionless pressure distribution in the clearance of the plane biobearing for $\varepsilon = 0{,}5$.

4.2. The flow of a synovial fluid in a clearance of a spherical biobearing

Let us consider the spherical bearing lubricated by a couple stress fluid with a squeeze film shown in Fig.5. Introducing the following nondimensional parameters

$$\tilde{h} = \frac{h}{C} = 1 - \varepsilon\cos\varphi, \qquad K = \left(\frac{12\Phi H}{C^3}\right)^{\frac{1}{3}}, \qquad \mathrm{Re} = \frac{\rho C^2 \dot{\varepsilon}}{\mu},$$

$$A = \frac{\ddot{\varepsilon}\varepsilon}{\dot{\varepsilon}^2}, \qquad l^* = \frac{l}{h_o}, \qquad \tilde{p} = \frac{(p - p_o)C^2}{\mu\dot{\varepsilon}R_r^2}, \tag{4.3}$$



$$C = R_s - R_r,$$

$$h = C(1 - \varepsilon\cos\varphi) = C\Phi_s,$$

$$\frac{\partial h}{\partial t} = \dot{h} = -C\dot{\varepsilon}\cos\varphi, \qquad \varepsilon = \frac{e}{C},$$

$$R = R_r \sin\varphi,$$

$$\varphi = \frac{x}{R_r}, \qquad \dot{\varepsilon} = \frac{d\varepsilon}{dt}$$

Fig.5. Spherical biobearing with a squeeze film.

we obtain from Eq.(3.9) the following formula for the dimensionless pressure distribution in the clearance of the spherical biobearing with squeeze fluid film (for $K << 1$)

$$\tilde{p}=-\frac{3}{\varepsilon}\left[\left(\frac{1}{F^2}-\frac{1}{\Phi_s^2}\right)+6l^{*2}\left(\frac{1}{F^4}-\frac{1}{\Phi_s^4}\right)-\frac{2K^3}{5}\left(\frac{1}{F^5}-\frac{1}{\Phi_s^5}\right)\right]+\frac{\mathrm{Re}}{2\varepsilon^2}\left\{A\left(\ln E-\ln\Phi_s\right)+\right.$$
$$\left.-K^3\left[\frac{3}{4}\left(\frac{1}{E^4}-\frac{1}{\Phi_s^4}\right)-(A+3)\left(\frac{1}{E^3}-\frac{1}{\Phi_s^3}\right)\right]\right\}+\frac{3\,\mathrm{Re}}{10\varepsilon^2}\left\{4\left(\ln E-\ln\Phi_s\right)-\left(\frac{1}{E}-\frac{1}{\Phi_s}\right)+\right.$$
$$-\frac{1-\varepsilon^2}{2}\left(\frac{1}{E^2}-\frac{1}{\Phi_s^2}\right)+4l^{*2}\left[\left(\frac{1}{E^3}-\frac{1}{\Phi_s^3}\right)-\frac{3\left(1-\varepsilon^2\right)}{4}\left(\frac{1}{E^4}-\frac{1}{\Phi_s^4}\right)\right]+ \tag{4.4}$$
$$\left.-2K^3\left[\frac{5}{4}\left(\frac{1}{E^4}-\frac{1}{\Phi_s^4}\right)-\frac{1}{3}\left(\frac{1}{E^3}-\frac{1}{\Phi_s^3}\right)-\frac{4\left(1-\varepsilon^2\right)}{5}\left(\frac{1}{E^5}-\frac{1}{\Phi_s^5}\right)\right]\right\},$$

where $\Phi_s=1-\varepsilon\cos\varphi,\qquad E=1-\varepsilon,\qquad F=1-\varepsilon\cos\varphi_o.$

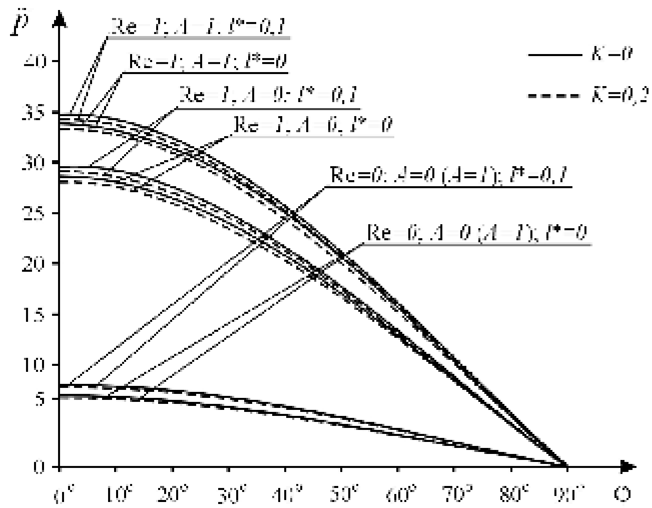


Fig.6. Dimensionless pressure distribution in the clearance of the spherical biobearing for $\varepsilon=0{,}1$.

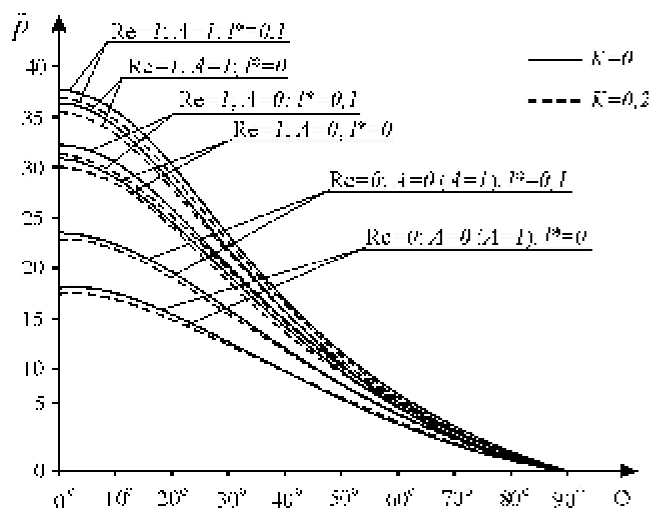


Fig.7. Dimensionless pressure distribution in the clearance of the spherical biobearing for $\varepsilon=0{,}5$.

5. Conclusions

From the general considerations, formulae and graphs presented here for particular cases relating to the studies of biobearings one may conclude:

- the couple-stress fluid $\left(l^* \neq 0\right)$ is characterized by larger pressures than those of the Newtonian fluid $\left(l^* = 0\right)$,
- the porosity reduces the pressure values;
- pressure values increase with a decrease in the relative film thickness E,
- pressure values also increase with an increase in the values of Re and A.

One should note that the values of pressures and load-capacities obtained here for $\mathrm{Re} = 0$, $K = 0$ are almost the same as those obtained by Lin (1996) who used the analytical-numerical method to solve the Reynolds equation (for hemispherical bearings without porous wall).

Nomenclature

h – the film thickness
H – thickness of the porous layer
K – porosity
p – pressure
R – radius of the bearing surface
Φ – permeability of porous layer
μ – coefficient of plastic viscosity
η – couple-stress viscosity
ρ – density
ω – the angular velocity
~ – dimensionless values

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