Int. J. of Applied Mechanics and Engineering, 2006, vol.11, No.4, pp.865-880

DEFORMATIONS DUE TO MECHANICAL SOURCES IN ELASTIC SOLID WITH VOIDS

R. KUMAR*
Department of Mathematics, Kurukshetra University
Kurukshetra, Haryana, INDIA, 136119
e-mail: rajneesh_kuk@rediffmail.com

P. AILAWALIA
Department of Applied Sciences, Institute of Engineering and Emerging Technologies
Makhnumajra, Baddi
Distt. Solan, H.P, INDIA, 173205
e-mail: praveen_2117@rediffmail.com

Void effects of a load applied normal to the boundary and moving at a constant velocity along one of the coordinate axis in an elastic half space is studied. The analytic expressions for displacement, force stress and volume fraction field for concentrated normal point force, uniformly distributed force, linearly distributed force and moving concentrated normal force are obtained by employing the eigen value approach after applying the integral transforms. A numerical inversion technique has been applied to obtain the solution in the physical domain. The numerical results are presented graphically. Some particular cases have been deduced.

Key words: voids, eigen value, integral transforms, volume fraction field.

1. Introduction

The theory of elasticity concerning the solid elastic material having a distribution of various pores, generally known as voids or vacuous pores, received greater attention due to its theoretical and practical relevance. The general theory in this respect has been formulated by Nunziato and Cowin (Nunziato and Cowin, 1979; Cowin and Nunziato, 1983). They also formulated the linearised version of the above theory Cowin and Nunziato (1983) where the voids were included as an additional kinematic variable. This theory reduces to the classical theory of elasticity in the limiting case when the void-volume vanishes. This theory can play an important role in practical problems of geological and synthetic porous media where the genuine elastic theory is inadequate. Some basic problems and a brief account of the theory on voids have been introduced by Iesan (1985) and Cowin (1984) respectively. Cowin (1984) presented the inter-relationship between this theory of voids and other theories of elasticity. The uniqueness theorem in the theory of elastic material with voids has been presented by Chandrasekharaiah (1987a). He investigated plane waves in a rotating elastic solid with voids (Chandrasekharaiah, 1987b). Ciarletta and Scalia (1991) discussed some theorems in the theory of viscoelastic materials with voids. Recently, different authors (Tvergaard, 1999; Scarpetta, 2002; Ciarletta *et al.*, 2003) discussed the problems of elastic materials with voids. However not much work has been done to study the effect of sources acting at the surface of elastic material with voids.

In the present problem we have obtained the closed form expressions for two dimensional displacement, stresses and volume fraction field due to normal point load and a moving normal point load. The deformation at any point of the medium due to normal point load is useful to analyze the deformation field around mining tremors and drilling into the crust of earth. It can also contribute to theoretical consideration of the seismic and volcanic sources since it can account for the deformation field in the entire volume surrounding the source region.

* To whom correspondence should be addressed

The dynamical response to moving loads is of considerable interest in a variety of technological and geophysical circumstances and several recent investigations are concerned with this problem. For instance, it is of great interest in solid dynamics where ground motions and stresses can be produced by blast waves (surface pressure waves due to explosions), or by a supersonic aircraft. This type of investigation is made in many branches of engineering, e.g., bridges and railways, beams subjected to pressure waves and piping systems subjected to two phase flow. Other applications are encountered within the context of contact mechanics like the problem of high velocity rocket sleds sliding over steel guide rails. Different authors (Halpern and Christiano, 1986; Nath and Sengupta, 1999; Katz, 2001; Verruijt and Cordova, 2001) have discussed the problems of moving load in the theory of elastic solids.

2. Formulation and solution of the problem

We consider a homogeneous elastic solid with voids in the undeformed state. We take the origin on the plane surface and z-axis normally into the medium, which is represented by $z \geq 0$. A normal point force is assumed to be acting at the origin of the rectangular Cartesian co-ordinates (Fig.1).

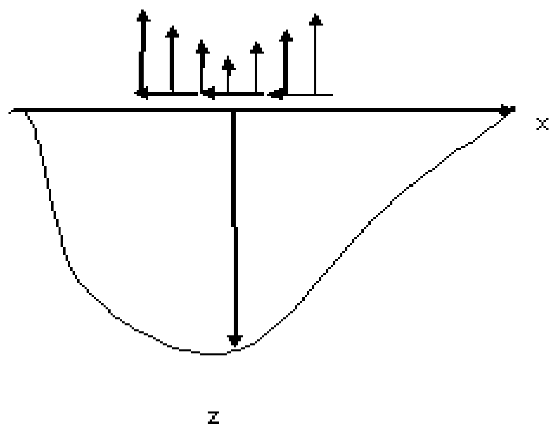

Fig.1. Geometry of the problem.

If we restrict our analysis to plane strain parallel to the x-z plane with the displacement vector $\boldsymbol{u} = (u_1, 0, u_3)$ then the field

$$\phi(x)\ \delta(t),$$

equations and constitutive relations for such a medium in the absence of body forces and equilibrated forces can be written (Cowin and Nunziato, 1983) as

$$(\lambda+\mu)\nabla(\nabla\cdot\boldsymbol{u})+\mu\nabla^2\boldsymbol{u}+\beta^*\nabla\psi=\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2}, \tag{2.1}$$

$$\alpha^*\nabla^2\psi-\varsigma^*\psi-\omega^*\frac{\partial\psi}{\partial t}-\beta^*\nabla\cdot\boldsymbol{u}=\rho\zeta^*\frac{\partial^2\psi}{\partial t^2}, \tag{2.2}$$

$$t_{ij}=\lambda u_{r,r}+\mu\left(u_{i,j}+u_{j,i}\right)+\beta^*\psi\,\delta_{ij}. \tag{2.3}$$

Introducing the dimensionless variables defined by the expressions

$$x' = \frac{x}{h}, \quad z' = \frac{z}{h}, \quad u_1' = \frac{u_1}{h}, \quad u_3' = \frac{u_3}{h}, \quad \psi' = \frac{\zeta^*}{h^2}\zeta,$$
$$t_{ij}' = \frac{t_{ij}}{\lambda}, \quad t' = \frac{1}{h}\sqrt{\frac{\lambda}{\rho}}\, t, \quad a' = \frac{a}{h} \tag{2.4}$$

(where h is the standard length and a is a half width of the strip load) in Eqs (2.1)-(2.2), we obtain (dropping the primes)

$$(\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_3}{\partial z}\right)+\mu\nabla^2 u_1+\frac{\beta^* h^2}{\zeta^*}\frac{\partial \psi}{\partial x}=\lambda\,\frac{\partial^2 u_1}{\partial t^2}, \tag{2.5}$$

$$(\lambda+\mu)\frac{\partial}{\partial z}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_3}{\partial z}\right)+\mu\nabla^2 u_3+\frac{\beta^* h^2}{\zeta^*}\frac{\partial \psi}{\partial z}=\lambda\,\frac{\partial^2 u_3}{\partial t^2}, \tag{2.6}$$

$$\alpha^*\nabla^2\psi-\varsigma^* h^2\psi-\omega^* h\sqrt{\frac{\lambda}{\rho}}\frac{\partial\psi}{\partial t}-\beta^*\zeta^*\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_3}{\partial z}\right)=\lambda\,\zeta^*\,\frac{\partial^2\psi}{\partial t^2}. \tag{2.7}$$

The initial and radiation conditions are given by

$$u_i(x,z,0)=\dot{u}_i(x,z,0)=0; \quad i=1,3,$$
$$\psi(x,z,0)=\dot{\psi}(x,z,0)=0, \tag{2.8}$$

and

$$u_1(x,z,t)=u_3(x,z,t)=\psi(x,z,t)=0, \quad \text{for} \quad t>0, \quad \text{when} \quad z\to\infty. \tag{2.9}$$

Applying the Laplace transform with respect to time 't' defined by

$$\{\bar{u}_i(x,z,p),\bar{\psi}(x,z,p)\}=\int_0^\infty\{u_i(x,z,t),\psi(x,z,t)\}e^{-pt}\,dt, \quad i=1,3, \tag{2.10}$$

and then the Fourier transform with respect to 'x' defined by

$$\{\tilde{u}_i(\xi,z,p),\tilde{\psi}(\xi,z,p)\}=\int_{-\infty}^{\infty}\{\bar{u}_i(x,z,p),\bar{\psi}(x,z,p)\}e^{i\xi x}\,dx, \quad i=1,3, \tag{2.11}$$

to Eqs (2.5)-(2.7) and with the help of initial conditions (2.8), we obtain

$$D^2\tilde{u}_1=b_{11}\,\tilde{u}_1+a_{12}\,D\tilde{u}_3+a_{13}\,\tilde{\psi}, \tag{2.12}$$

$$D^2\tilde{u}_3 = b_{22}\,\tilde{u}_3 + a_{21}\,D\tilde{u}_1 + b_{23}\,D\tilde{\psi}, \tag{2.13}$$

$$D^2\tilde{\psi} = b_{33}\,\tilde{u}_1 + a_{31}\,\tilde{\psi} + b_{32}\,D\tilde{u}_3, \tag{2.14}$$

where

$$b_{11} = \frac{\lambda p^2 + \xi^2(\lambda+2\mu)}{\mu}, \qquad b_{22} = \frac{\lambda p^2 + \mu\xi^2}{\lambda+2\mu}, \qquad b_{23} = -\frac{\beta^* h^2}{\zeta^*(\lambda+2\mu)},$$

$$b_{32} = \frac{\beta^*\zeta^*}{\alpha^*}, \quad b_{33} = -\frac{i\xi\beta^*\zeta^*}{\alpha^*}, \quad a_{12} = \frac{i\xi(\lambda+\mu)}{\mu}\ , \quad a_{13} = \frac{i\xi\beta^* h^2}{\mu\zeta^*}\ , \quad D = \frac{d}{dz}, \tag{2.15}$$

$$a_{21} = \frac{i\xi(\lambda+\mu)}{\lambda+2\mu}, \qquad a_{31} = \frac{1}{\alpha^*}\left(\alpha^*\xi^2 + \varsigma^* h^2 + \omega^* hp\sqrt{\frac{\lambda}{\rho}} + \lambda\zeta^* p^2\right).$$

Equations (2.12)-(2.14) may be written as

$$\frac{d}{dz}W(\xi,z,p) = A(\xi,p)\ W(\xi,z,p) \tag{2.16}$$

where

$$W = \begin{pmatrix} V \\ DV \end{pmatrix}, \qquad A = \begin{pmatrix} O & I \\ A_1 & A_2 \end{pmatrix}, \qquad V = \begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_3 \\ \tilde{\psi} \end{pmatrix},$$
$$A_1 = \begin{pmatrix} b_{11} & 0 & a_{13} \\ 0 & b_{22} & 0 \\ b_{33} & 0 & a_{31} \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 0 & a_{12} & 0 \\ a_{21} & 0 & b_{23} \\ 0 & b_{32} & 0 \end{pmatrix}. \tag{2.17}$$

O and I are respectively zero and identity matrix of order 3.

To solve Eq.(2.16), we assume

$$W(\xi,z,p) = X(\xi,p)e^{qz}, \tag{2.18}$$

which leads to the eigen value problem. The characteristic equation corresponding to matrix A is given by

$$|A - qI| = 0, \tag{2.19}$$

which on expansion yields

$$q^6 + \lambda_1 q^4 + \lambda_2 q^2 + \lambda_3 = 0 \tag{2.20}$$

where

$$\lambda_1 = -\left(a_{12}a_{21} + b_{23}b_{32} + b_{11} + b_{22} + a_{31}\right),$$

$$\lambda_2 = a_{21}\left(a_{31}a_{12} - a_{13}b_{32}\right) + b_{23}\left(b_{11}b_{32} - b_{33}a_{12}\right) + b_{11}a_{31} - a_{13}b_{33} + b_{22}\left(a_{31} - b_{11}\right), \tag{2.21}$$

$$\lambda_3 = b_{22}\left(b_{11}a_{31} - b_{33}a_{13}\right).$$

The eigen values of the matrix A are characteristic roots of Eq.(2.20). The vectors $X(\xi, p)$ corresponding to the eigen values q_s can be determined by solving the homogeneous equation

$$[A - qI]\, X(\xi, p) = 0. \tag{2.22}$$

The set of eigen vectors $X_s(\xi, p)$, $s = 1,2.........6$ may be obtained as

$$X_s(\xi, p) = \begin{pmatrix} X_{g1}(\xi, p) \\ X_{g2}(\xi, p) \end{pmatrix} \tag{2.23}$$

where

$$X_{g1}(\xi, p) = \begin{pmatrix} 1 \\ a_g q_g \\ b_g \end{pmatrix}, \qquad X_{g2}(\xi, p) = \begin{pmatrix} q_g \\ a_g q_g^2 \\ b_g q_g \end{pmatrix}, \qquad q = q_g, \qquad g = 1,2,3, \tag{2.24}$$

$$X_{\vartheta 1}(\xi, p) = \begin{pmatrix} 1 \\ -a_\vartheta q_\vartheta \\ b_\vartheta \end{pmatrix}, \qquad X_{\vartheta 2}(\xi, p) = \begin{pmatrix} -q_\vartheta \\ a_\vartheta q_\vartheta^2 \\ -b_\vartheta q_\vartheta \end{pmatrix}, \quad \vartheta = g + 3, \quad q = -q_g, \quad g = 1,2,3, \tag{2.25}$$

and

$$a_g = \frac{q_g^2 b_{23} - \left(b_{23}b_{11} - a_{21}a_{13}\right)}{\nabla_g},$$

$$b_g = \frac{q_g^2 a_g b_{32} + b_{33}}{q_g^2 - a_{31}}, \tag{2.26}$$

$$\nabla_g = q_g^2\left(a_{12}b_{23} + a_{13}\right) - a_{13}b_{22}.$$

The solution of Eq.(2.18) is given by

$$W(\xi, z, p) = \sum_{s=1}^{3}\left[D_s X_s(\xi, p)\exp(q_s z) + D_{s+3} X_{s+3}(\xi, p)\exp(-q_s z)\right]. \tag{2.27}$$

The transformed displacements and volume fraction field satisfying the radiation conditions (2.9) are given by

$$\tilde{u}_1 = D_4 \exp(-q_1 z) + D_5 \exp(-q_2 z) + D_6 \exp(-q_3 z),$$

$$\tilde{u}_3 = -a_1 q_1 D_4 \exp(-q_1 z) - a_2 q_2 D_5 \exp(-q_2 z) - a_3 q_3 D_6 \exp(-q_3 z), \tag{2.28}$$

$$\tilde{\psi} = b_1 D_4 \exp(-q_1 z) + b_2 D_5 \exp(-q_2 z) + b_3 D_6 \exp(-q_3 z).$$

3. Boundary conditions and application

3.1. Mechanical sources on the surface of the half-space

The boundary conditions in this case are

$$t_{33} = -F\,\phi(x)\delta(t), \qquad t_{31} = 0, \qquad \frac{\partial \psi}{\partial t} = 0 \qquad \text{at} \qquad z = 0. \tag{3.1}$$

Using Eq.(2.4) and then applying the Laplace and Fourier transforms from Eqs (2.10) and (2.11) to system of Eq.(3.1) and with the help of Eq.(2.28), we get the transformed displacement, stresses and volume fraction field as

$$\tilde{u}_3 = -\frac{1}{\Delta}\left[a_1 q_1 \Delta_1 e^{-q_1 z} + a_2 q_2 \Delta_2 e^{-q_2 z} + a_3 q_3 \Delta_3 e^{-q_3 z}\right], \tag{3.2}$$

$$\tilde{t}_{33} = \frac{1}{\Delta}\left[r_1 \Delta_1 e^{-q_1 z} + r_2 \Delta_2 e^{-q_2 z} + r_3 \Delta_3 e^{-q_3 z}\right], \tag{3.3}$$

$$\tilde{\psi} = \frac{1}{\Delta}\left[b_1 \Delta_1 e^{-q_1 z} + b_2 \Delta_2 e^{-q_2 z} + b_3 \Delta_3 e^{-q_3 z}\right] \tag{3.4}$$

where

$$\Delta = -\frac{1}{F\tilde{\phi}(\xi)}\left[r_1\Delta_1 + r_2\Delta_2 + r_3\Delta_3\right],$$

$$\Delta'_{1,2,3} = (-1)^{\Theta} F\tilde{\phi}(\xi)\left[s_{2,1,1} b_{3,3,2} q_{3,3,2} - s_{3,3,2} b_{2,1,1} q_{2,1,1}\right],$$

$$r_\Theta = -i\xi + \left(\frac{\lambda + 2\mu}{\lambda}\right) q_\Theta^2 a_\Theta + \frac{\beta^* h^2}{\zeta^* \lambda} b_\Theta, \qquad s_\Theta = -i\xi + q_\Theta^2 a_\Theta + i\xi \frac{\mu}{\lambda} q_\Theta a_\Theta - \frac{\mu}{\lambda} q_\Theta \tag{3.5}$$

$$\Theta = 1,2,3.$$

Particular case

Neglecting the material constants due to the presence of voids $\left(i.e,\ \alpha^* = \beta^* = \zeta^* = \varsigma^* = \omega^* = 0\right)$, we obtain the expressions for normal displacement and force stress for an elastic solid as

$$\tilde{u}_3 = -\frac{1}{\Delta_0}\left[p_1 d_1 \Delta_{10}\, e^{-p_1 z} + p_2 d_2 \Delta_{20}\, e^{-p_2 z}\right], \tag{3.6}$$

$$\tilde{t}_{33} = \frac{1}{\Delta_0}\left[r_1 \Delta_{10}\, e^{-p_1 z} + r_2 \Delta_{20}\, e^{-p_2 z}\right] \tag{3.7}$$

where

$$\Delta = -\frac{1}{F\tilde{\phi}(\xi)}\left[r_1' \Delta_{10} + r_2' \Delta_{20}\right], \qquad \Delta_{10,20} = (-1)^{\Pi} F\tilde{\phi}(\xi) s_{2,1},$$

$$r_{\Theta} = -i\xi + \left(\frac{\lambda + 2\mu}{\lambda}\right) p_{\Pi}^2 d_{\Pi}, \qquad s_{\Pi} = -i\xi + p_{\Pi}^2 d_{\Pi} + i\xi \frac{\mu}{\lambda} p_{\Pi} d_{\Pi} - \frac{\mu}{\lambda} p_{\Pi}, \tag{3.8}$$

$$d_{\Pi} = \frac{p_{\Pi}^2 - b_{11}}{e_{12}}.$$

The eigen values $\pm p_{\Pi} (\Pi = 1,2)$ for an elastic solid are given by the equation

$$p^4 + \lambda_4 p^2 + \lambda_5 = 0 \tag{3.9}$$

where

$$\lambda_4 = -(e_{12} e_{21} + b_{11} + b_{22}), \qquad \lambda_5 = b_{11} b_{22}. \tag{3.10}$$

3.1.1. Concentrated normal force

In order to determine displacements and stresses due to the concentrated normal force described as Dirac delta function $\phi(x) = \delta(x)$ must be used. The Fourier transform of $\phi(x)$ with respect to the pair (x, ξ) will be $\tilde{\phi}(\xi) = 1$.

3.1.2. Uniformly distributed force

The solution due to distributed force applied on the half space are obtained by setting

$$\phi(x) = \begin{cases} 1 & \text{if} \quad |x| \le a, \\ 0 & \text{if} \quad |x| > a, \end{cases}$$

in Eq.(3.1). The Fourier transform with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width $2a$ applied at the origin of the coordinate system $(x = z = 0)$ in dimensionless form after suppressing the primes becomes

$$\tilde{\phi}(\xi) = [2\sin(\xi h a)/\xi], \qquad \xi \ne 0. \tag{3.11}$$

3.1.3. Linearly distributed force

The solutions due to linearly distributed force are obtained by substituting

$$\phi(x)=\begin{cases}1-\dfrac{|x|}{a} & \text{if} \quad |x|\le a, \\ 0 & \text{if} \quad |x|>a.\end{cases} \tag{3.12}$$

The Fourier transform in case of linearly distributed force applied at the origin of the system in dimensionless form are

$$\tilde{\phi}(\xi)=\frac{2[1-\cos(\xi ha)]}{\xi^2 ha}. \tag{3.13}$$

The expressions for displacement, stress and volume fraction field may be obtained as in Eqs (3.2)-(3.4) and (3.6)-(3.7), by replacing $\tilde{\phi}(\xi)$ by 1, $[2\sin(\xi ha)/\xi]$ and $\dfrac{2[1-\cos(\xi ha)]}{\xi^2 ha}$ in the case of the concentrated normal point force, uniformly distributed force and linearly distributed force respectively.

3.2. Problem II: Moving concentrated normal point load

We consider a concentrated normal point load moving along the surface of an elastic solid with voids. The rectangular Cartesian coordinates are introduced having origin on the surface $z=0$ and the z-axis pointing vertically into the medium. Let us consider a pressure pulse $P(x+Ut)$ which is moving with a constant velocity in the negative x direction for an infinitely long time so that a steady state prevails in the neighbourhood of the loading as seen by the observer moving with the load (Fig.2).

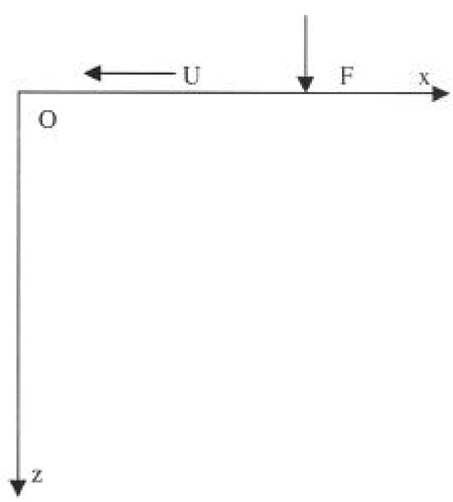

Fig.2. Moving concentrated normal point load.

Using Galilean transformations (Fung, 1968) $x^* = x + Ut$, $z^* = z$, $t^* = t$ and introducing dimensionless quantities defined by Eq.(2.4) and applying the Fourier transforms defined by Eq.(2.11) in Eqs (2.1)-(2.2), we obtain the results in the case of moving load at the surface of elastic solid with voids.

The boundary conditions in this case are

$$t_{33} = -F\delta\left(x^*\right), \qquad t_{31} = 0, \qquad \frac{\partial\psi}{\partial t} = 0$$

where

$$P(x + Ut) = F\delta\left(x^*\right).$$

The expressions for displacement, stress and volume fraction field for an elastic solid with voids and an elastic solid in the case of moving normal point load are given by Eqs (3.2)-(3.4) and (3.6)-(3.7) by changing $\lambda p^2 \to -\rho U^2 \xi^2$ and $p\sqrt{\frac{\lambda}{\rho}} \to -i\xi U$ in the expressions (2.15) and $\tilde{\phi}(\xi) \to 1$ in Eqs (3.5) and (3.8).

4. Inversion of the transform

The transformed displacements and stresses are functions of z, the parameters of the Laplace and Fourier transforms p and ξ respectively, and hence are of the form $\tilde{f}(\xi, z, p)$. To get the function in the physical domain, first we invert the Fourier transform using

$$\begin{aligned} \overline{f}(x, z, p) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \tilde{f}(\xi, z, p) d\xi, \\ &= \frac{1}{\pi} \int_{0}^{\infty} \left\{\cos(\xi x) f_e - i \sin(\xi x) f_o\right\} d\xi \end{aligned} \tag{4.1}$$

where f_e and f_o are even and odd parts of the function $\tilde{f}(\xi, z, p)$ respectively. Thus, expressions (3.2)-(3.4) and (3.6)-(3.7) yield the transform $\overline{f}(x, z, p)$ of the function $f(x, z, t)$.

Following Honig and Hirdes (1984) the Laplace transform function $\overline{f}(x, z, p)$ can be inverted to $f(x, z, t)$.

The last step is to evaluate the integral in Eq.(4.1). The method for evaluating this integral by Press *et al*. (1986) involves the use of Rhomberg's integration with an adaptive step size.

5. Numerical results and discussions

We take magnesium (Eringen, 1984) as an elastic solid

$$\rho = 1.74 \times 10^3 \ Kg/m^3, \qquad \lambda = 9.4 \times 10^{10} \ N/m^2, \qquad \mu = 4.0 \times 10^{10} \ N/m^2.$$

The void parameters are taken as

$$\alpha^* = 3.668\times 10^{-9}\,N\,,\qquad \beta^* = 1.13849\times 10^{10}\,N/m^2\,,\qquad \varsigma^* = 1.475\times 10^{10}\,N/m^2\,,$$

$$\omega^* = 0.0787\times 10^{-3}\,N\,\mathrm{sec}/m^2\,,\qquad \zeta^* = 1.753\times 10^{-19}\,m^2\,.$$

The variations of normal displacement $U_3 = (u_3/F)$ and normal force stress $T_{33} = (t_{33}/F)$ for an elastic solid with voids (ESWV) and an elastic solid (ES) have been studied and the variations of these components with distance *x* have been shown by (a) solid line (———) for ESWV and dashed line (- - - -) for ES at $t = 0.05$ (b) solid line with centered symbol (x—x—x—x) for ESWV and dashed line with centered symbol (x- - x- -x) for ES at $t = 0.1$ and (c) solid line with centered symbol (o—o—o—o) for ESWV and dashed line with centered symbol (o- - o- -o) for ES at $t = 0.2$. These variations are shown in Figs 3-8. The computations are carried out for $z = 1.0$ in the range $0 \le x \le 10.0$ and $h^2 = 1.0\times 10^{-15}\,cm^2$ and for one value of dimensionless width $a = 1.0$.

6. Discussions for various cases

(a) Concentrated force: At a particular time, the variations of normal displacement and normal force stress are greater for ES as compared to the variations for ESWV. Moreover, the variations of all the quantities decrease with an increase in time. It is observed that the values of normal displacement for ESWV, very close to the point of application of the source, increase with an increase in time but the values of normal force stress at the same point decrease with increase in time. However, the values of both the quantities for an ES decrease with time, near the point of application. The variations of normal displacement and normal force stress for both ES and ESWV at different times are shown in Figs 3 and 4 respectively.

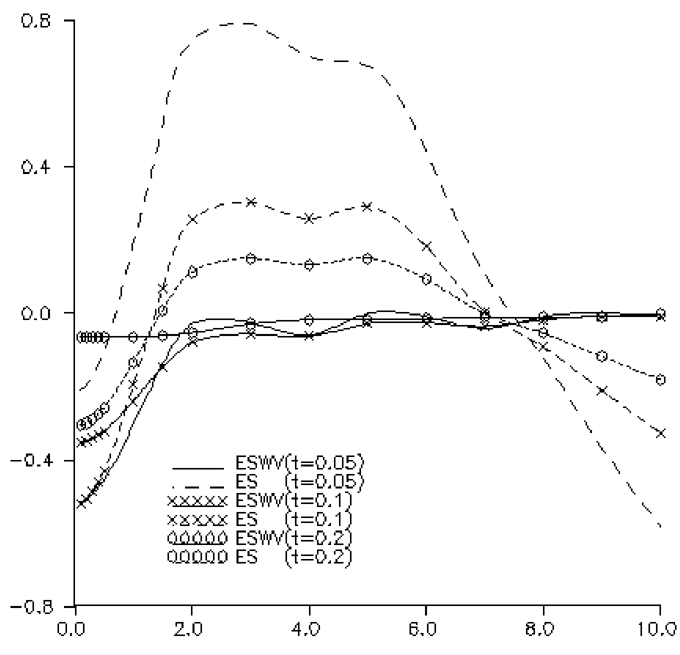

Fig.3. Variation of normal displacement $U_3 (= u_3/F)$ with distance *x* for concentrated normal force.

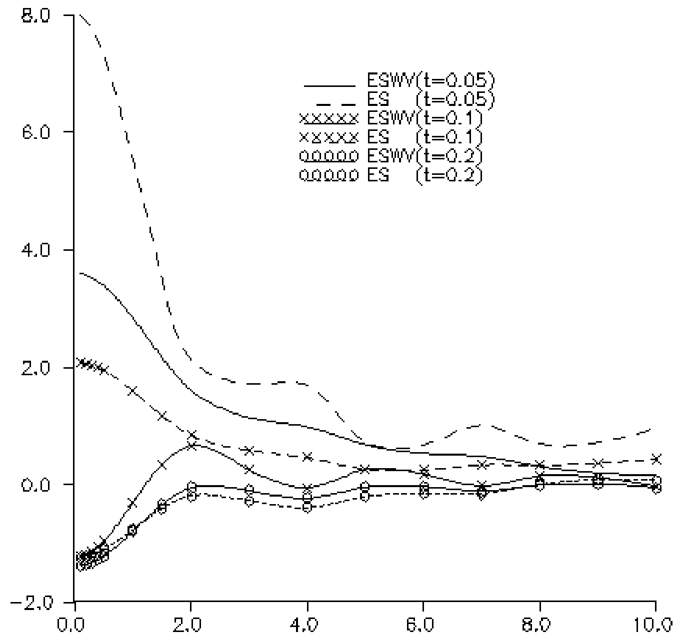

Fig.4. Variation of normal force stress $T_{33}\left(=t_{33}/F\right)$ with distance *x* for concentrated normal force.

Figure 5 shows that the variations of volume fraction field for ESWV is oscillatory in nature but the magnitude of oscillations decreases with an increase in horizontal distance and time.

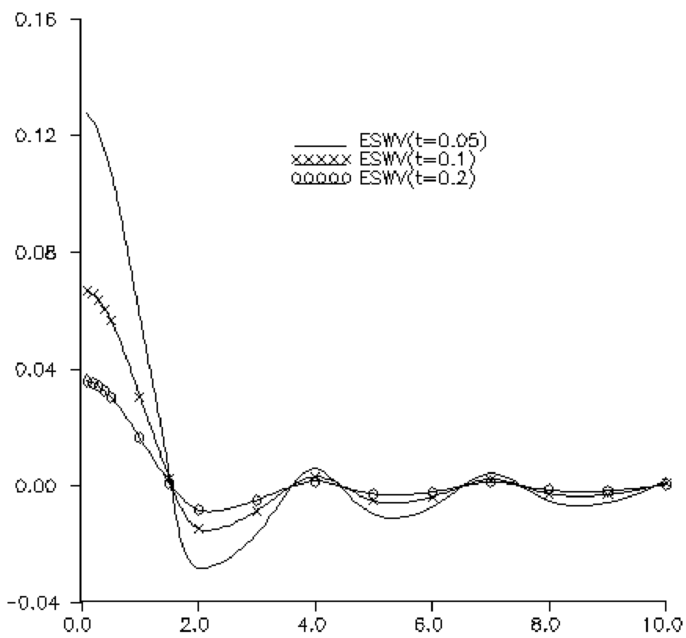

Fig.5. Variation of volume fraction field $V\left(=v/F\right)$ with distance *x* for concentrated normal force.

(b) Strip loading: The values of all the quantities are larger when strip loads are applied on the surface of a solid as compared to the values on the application of concentrated load. However, the variations of all the quantities for uniformly distributed load and linearly distributed load are similar in nature with difference in magnitudes. The variations of normal displacement, normal force stress and volume fraction field in the case of uniformly distributed load are shown in Figs 6-8 respectively.

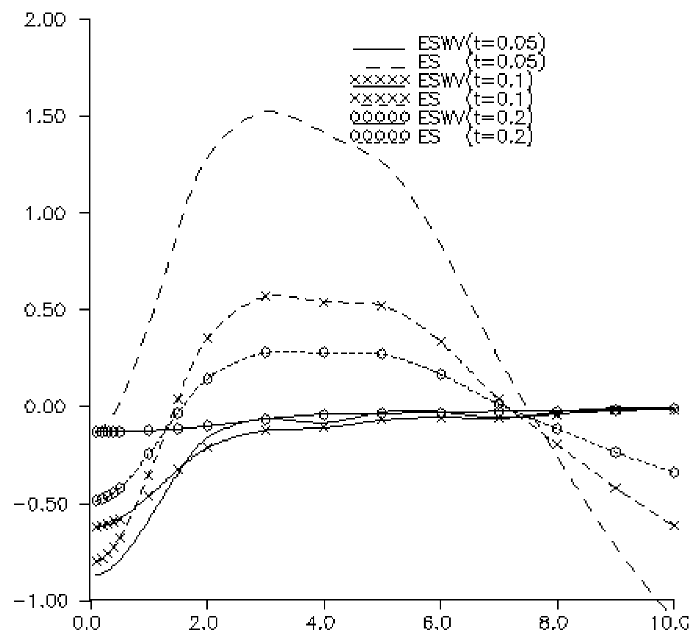

Fig.6. Variation of normal displacement $U_3(=u_3/F)$ with distance *x* for uniformly distributed force.

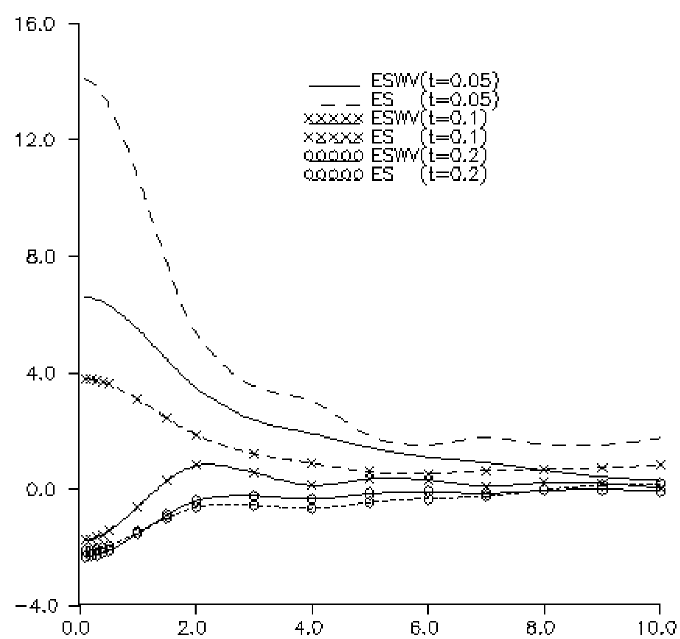

Fig.7. Variation of normal force stress $T_{33}(=t_{33}/F)$ with distance *x* for uniformly distributed force.

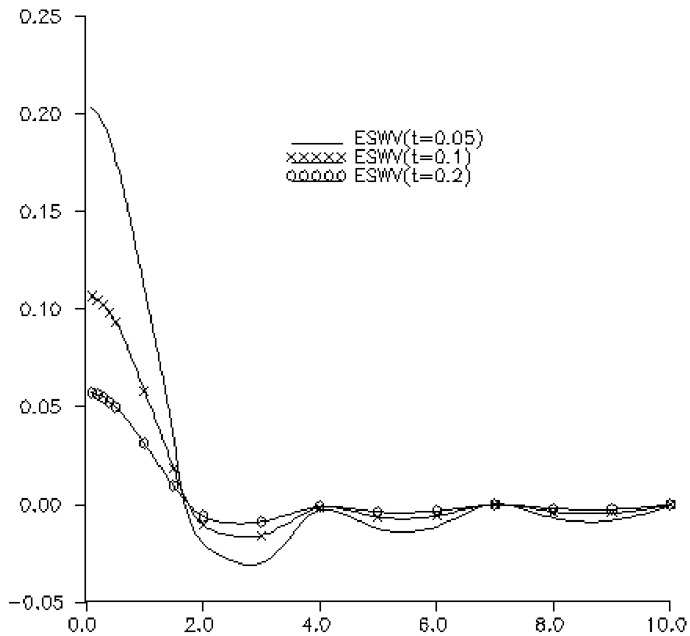

Fig.8. Variation of volume fraction field $V(=v/F)$ with distance *x* for uniformly distributed force.

(c) Steady state response due to moving load: Although the variations of all the quantities are oscillatory in nature but the magnitude of oscillations varies with the magnitude of moving load velocity. The variations increase with an increase in magnitude of moving load velocity applied on the surface. Also the values of normal displacement and normal force stress for ESWV lie in a short range as compared to the values for ES for a particular value of moving load velocity. These variations of normal displacement, normal force stress and couple stress for different magnitudes of moving load velocity are shown in Figs 9-11.

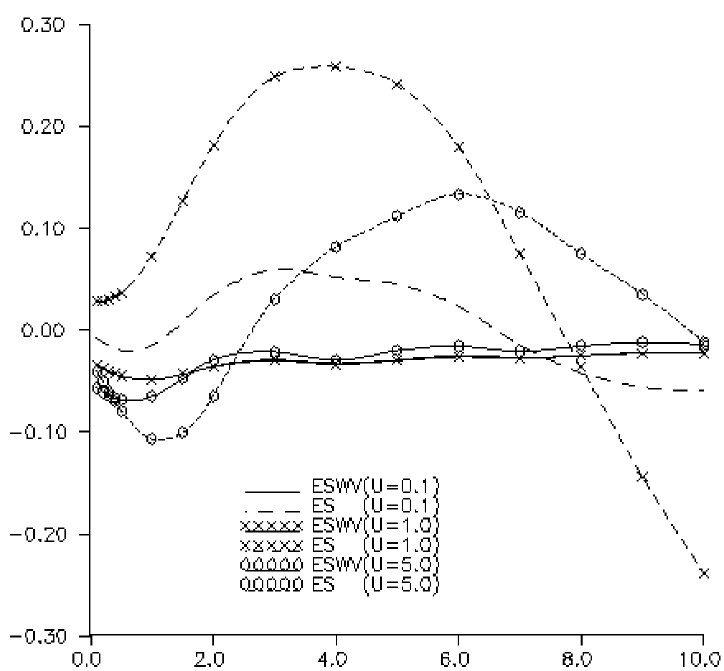

Fig.9. Variation of normal displacement $U_3(=u_3/F)$ with distance *x*.

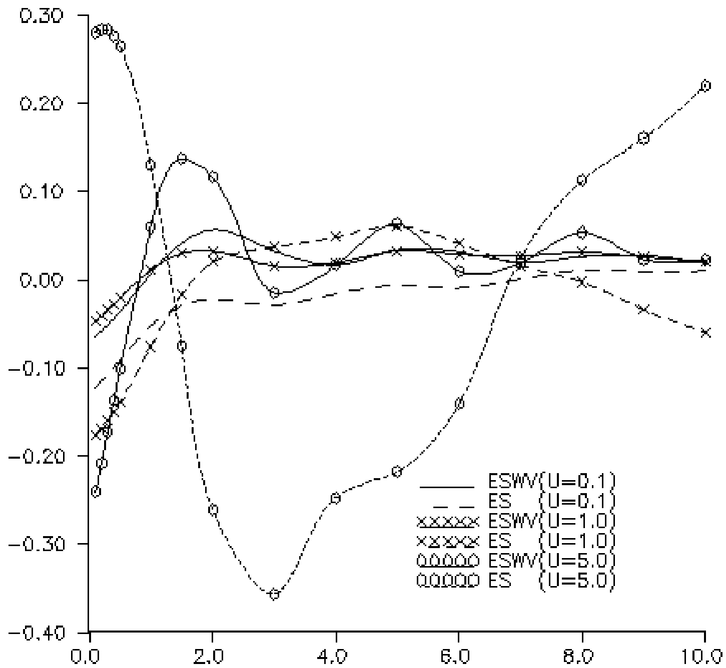

Fig.10. Variation of normal force stress $T_{33}\left(=t_{33}/F\right)$ with distance *x*.

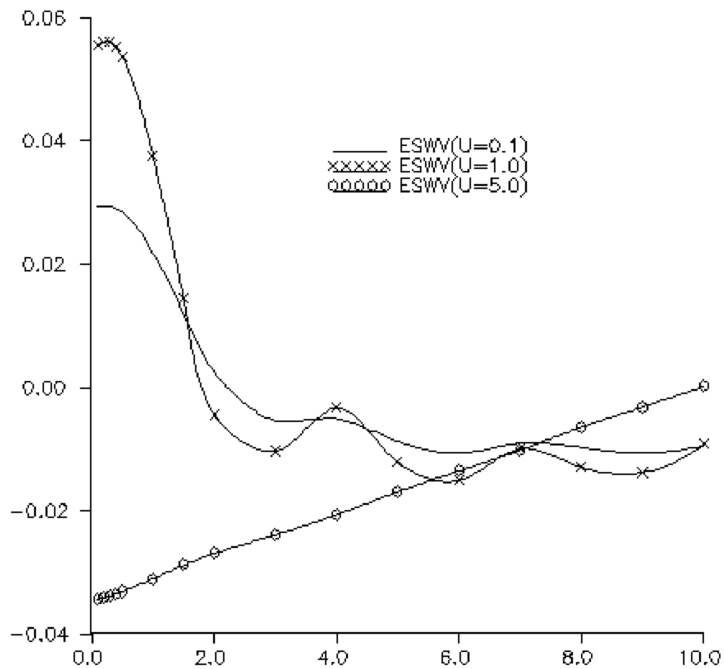

Fig.11. Variation of volume fraction field $V\left(=v/F\right)$ with distance *x*.

7. Conclusion

It is observed that the presence of voids plays a significant role in the study of deformation of a body. The values of normal components of displacement and force stress are smaller due to the presence of voids. Moreover, it is also observed that the body is deformed to a greater extent on the application of strip loading. Also in the case of steady state response the variations of all the quantities increase with an increase in magnitude of moving load velocity on the surface of the solid. The problem also finds its application in various engineering problems involving elastic solids with voids.

Nomenclature

F – magnitude of force applied
t_{ij} – force stress
$\boldsymbol{u}$ – displacement vector
U – magnitude of moving load velocity at the surface of elastic solid with voids
$\alpha^*, \beta^*, \varsigma^*, \omega^*$ and ζ^* – material constant due to the presence of voids
$\delta(t)$ – Dirac delta function
λ, μ – material constants
ρ – density of solid
$\phi(x)$ – vertical traction distributed function along x-axis
ψ – volume fraction field

References

Chandrasekharaiah D.S. (1987a): *A uniqueness theorem in the theory of elastic materials with voids*. – J. Elast., vol.18, pp.173-179.

Chandrasekharaiah D.S. (1987b): *Plane waves in a rotating elastic solid with voids*. – Int. J. Eng. Sci., vol.25, pp.591-596.

Ciarletta M., Iovane G. and Sumbatyan M.A. (2003): *On stress analysis for cracks in elastic materials with voids*. – Int. J. Eng. Sci., vol.41, pp.2447-2461.

Ciarletta M. and Scalia A. (1991): *On some theorems in the linear theory of viscoelastic materials with voids*. – J. Elast, vol.25, pp.149-158.

Cowin S.C. (1984): *The stresses around a hole in a linear elastic material with voids*. – Q. J. Mech. Appl. Math., vol.37, pp.441-465.

Cowin S.C. and Nunziato J. W. (1983): *Linear elastic materials with voids*. – J. Elast., vol.13, pp125-147.

Eringen A.C. (1984): *Plane waves in non-local micropolar elasticity*. – Int. J. Eng. Sci., vol.22, pp.1113-1121.

Fung Y.C. (1968): *Foundations of solid mechanics*. – New Delhi: Prentica Hall.

Halpern M.R. and Christiano P. (1986): *Steady state harmonic response of a rigid plate bearing on a liquid saturated poroelastic half space*. – Earthquake. Eng. and Structural Dynamics, vol.14, pp.439-454.

Honig G. and Hirdes V. (1984): *A method for the numerical inversion of the Laplace transform*. – J. Comp. Appl. Math., vol.10, pp.113-132.

Iesan D. (1985): *Some theorems in the theory of elastic materials with voids*. – J. Elast., vol.15, pp.215-224.

Katz R. (2001): *The dynamic response of a rotating shaft subject to an axially moving and rotating load*. – J. of Sound and Vibration., vol.246, No.5, pp.757-775.

Nath S. and Sengupta P.R. (1999): *Steady state response to moving loads in an elastic solid media*. – Indian. J. Pure. Appl. Math., vol.30, pp.317-327.

Nunziato J.W. and Cowin S.C. (1979): *A nonlinear theory of elastic materials with voids*. – Arch. Ration. Mech. Anal., vol.72, pp.175-201.

Press W.H., Teukolsky S.A., Vellerling W.T. and Flannery B.P. (1986): *Numerical Recipes*. – (Cambridge: Cambridge University Press).

Scarpetta E. (2002): *Minimum principles for the bending problem of elastic plates with voids*. – Int. J. Eng. Sci., vol.40, pp.1317-1327.

Tvergaard V. (1999): *Effect of large elastic strains on cavitation instability predictions for elastic plastic solids*. – Int. J. Solids. Struct., vol.36, pp.5453-5466.

Verruijt A. and Cordova C.C. (2001): *Moving loads on an elastic half plane with hysteretic damping*. – J. Appl. Mechanics, vol.68, pp.915-922.

Received: April 23, 2005

Revised: August 18, 2006