WEIGHT FUNCTION FOR A CRACK IN AN ORTHOTROPIC MEDIUM UNDER NORMAL IMPACT LOADING

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The paper deals with the investigation of an elastodynamic response of an infinite orthotropic medium containing a central crack under normal impact loading. Laplace and Fourier integral transforms are employed to reduce the dimensional wave propagation problem to the solution of a pair of dual integral equations in the Laplace transform plane. These integral equations are then reduced to integral differential equations which have been solved in the low frequency domain by method of iteration. To determine time dependence of the parameters, these equations are inverted to yield the dynamic stress intensity factor (SIF) for normal point force loading. These results have been used to obtain the SIF at the crack tip which corresponds to the weight function for the crack under normal loading. Analytical expressions of the weight function are used to derive SIF for polynomial loading. Numerical results of normalized SIF for a large normalized time variable and for different concentrated point force loading at an arbitrary location of the crack surface have been calculated for different orthotropic materials. In the present paper, a numerical Laplace inversion technique is used to recover the time dependence of the solution. Finally, the results obtained are displayed graphically.

Key words: orthotropy, impact loading, integral transforms, crack, stress intensity factor, weight function.

1. Introduction

It goes beyond mention that the presence of stress concentration in structural members is of prime importance to a design engineer. Geometrical discontinuities such as holes, notches, fillets, grooves and load discontinuities are within control of the designer, whereas inherent flaws such as cracks, segregations and voids, which are 'metallurgical or fabrication discontinuities', cannot be easily controlled. In case of sharp cracks, the analyses become formidable.

However, the increased usage of composite materials has created greater interest in the problems with cracks. Composite materials are by nature anisotropic in the gross sense. Thus the study of an anisptropic medium with a crack or cracks is of great importance in fracture analysis. Problems with Griffith cracks in orthotropic elastic material were considered by Satpathy and Parhi (1978), Cinar and Erdogan (1983), Kassir and Tse (1983), Itou (1989), and many others. The dynamic problems of singular stresses around cracks in an orthotropic medium are few in number. This may be due to mathematical complexities of such problems. Elastodynamic crack problems were solved by Chen and Sih (1977), Kassir and Bandopadhyay (1983), Shindo (1985), Viola and Piva (1986), Gonzalez and Mason (1999).

The concept of weight function was introduced by Bueckner (1970; 1973) and Rice (1989). One of the main advantages of the weight function theory is that once the weight function for a crack geometry is known, the stress intensity factor for the crack under an arbitrary loading system can be obtained simply by the weighted average of the loading system with the weight function. This simple and widely applicable result has promoted the search for weight functions for different geometries both analytically and numerically.

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The weight function method allows the calculations of stress intensity factor k for arbitrarily loaded cracks. It the weight function is known, the stress intensity factor for a crack whose surfaces are loaded by a stress distribution $\sigma(x)$, can be calculated as

$$k=\int_{x_B}^{x_A} k^*(x)\sigma(x)dx$$

where x_A and x_B are the location of the crack tips and $k^*(x)$ is the weight function to be determined for a given crack and specimen geometry.

In the present paper, the problem under normal impact response of an orthotropic medium with a central crack has been investigated. Laplace and Fourier integral transforms are employed to reduce the two dimensional wave propagation problem to the solution of a pair of dual integral equations in the Laplace transform plane. These integral equations have been reduced to the solution of a set of integral equations which have further been reduced to the solution of an integro-differential equation. The iteration method has been used to obtain the low frequency solution of the problem. To determine the time dependence of the solution the expressions are inverted to yield the dynamic stress intensity factor for normal point force loading. Numerical results for the normalized stress intensity factor for normal point loading and for a large normalized time variable have been calculated for Graphite-Epoxy and Glass-Epoxy composite materials for different particular cases. These results are displayed graphically. As a byproduct the weight function of the crack has been obtained. It is also shown that the weight function is really useful to obtain the SIF's for polynomial loadings.

2. Formulation of the problem

Consider a plain problem of a central crack of length $2a$ situated at the mid plane of an infinite orthotropic medium subjected to a sudden state of loading. Let E_i, μ_{ij} and $\nu_{ij}\,(i,\ j=1, 2, 3)$ denote the engineering constants of the material where indices *1, 2, 3* correspond to the directions of a system of Cartesian co-ordinates chosen to coincide with the axes of material orthotropy. In the system of co-ordinate the crack is defined by the relations $|x|\le a$, $y=\pm 0$. Since the problem under discussion is restricted to the propagation in the plane, it is readily shown by setting the displacement component along the *z*-direction and the derivative with respect to z to be zero, that the displacement equations of motion reduce to

$$C_{11}\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+(1+C_{12})\frac{\partial^2 v}{\partial x\partial y}=\frac{1}{C_S^2}\frac{\partial^2 u}{\partial t^2}, \tag{2.1}$$

$$C_{22}\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}+(1+C_{12})\frac{\partial^2 u}{\partial x\partial y}=\frac{1}{C_S^2}\frac{\partial^2 v}{\partial t^2}, \tag{2.2}$$

at which *u*, *v* are the *x*, *y* components of the displacement vector, $C_S=\sqrt{\frac{\mu_{12}}{\rho}}$ with μ_{12} being the shear modulus and ρ is the density of the material, $C_{ij}\,(i,\ j=1, 2)$ are non-dimensional parameters related to the elastic constants by the relations

$$C_{11}=C_{11}/\mu_{12}\left[1-(E_2/E_1)\nu_{12}^2\right],$$

$$C_{22}=(E_2/E_1)C_{11}, \tag{2.3}$$

$$C_{12} = \nu_{12} C_{22} = \nu_{21} C_{11}$$

for generalized plane stress and by

$$C_{11} = (E_1/\mu_{12}\Delta)(1-\nu_{23}\nu_{32}),$$
$$C_{22} = (E_2/\mu_{12}\Delta)(1-\nu_{13}\nu_{31}), \tag{2.4}$$
$$C_{12} = (E_1/\mu_{12}\Delta)\left(\nu_{21} + \frac{E_2}{E_1}\nu_{13}\nu_{32}\right) = (E_2/\mu_{12}\Delta)\left(\nu_{12} + \frac{E_1}{E_2}\nu_{23}\nu_{31}\right),$$

for plane strain.

The stresses are related to the displacements by the relations

$$\sigma_{xx}/\mu_{12} = C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y},$$
$$\sigma_{yy}/\mu_{12} = C_{12}\frac{\partial u}{\partial x} + C_{22}\frac{\partial v}{\partial y}, \tag{2.5}$$
$$\tau_{xy}/\mu_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}.$$

In Eqs (2.1) and (2.2), the time variable may be removed by applying the Laplace transform

$$f^*(p) = \int_0^\infty f(t) e^{-pt} dt,$$
$$f(t) = \frac{1}{2\pi i}\int_{Br} f^*(p) e^{pt} dp. \tag{2.6}$$

Applying relations Eq.(2.6) to Eqs (2.1) and (2.2) and assuming zero initial conditions for the displacement and velocities, the transformed field equations become

$$C_{11}\frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} + (1+C_{12})\frac{\partial^2 v^*}{\partial x \partial y} - p^2\frac{u^*}{C_S^2} = 0, \tag{2.7}$$

$$C_{22}\frac{\partial^2 v^*}{\partial y^2} + \frac{\partial^2 v^*}{\partial x^2} + (1+C_{12})\frac{\partial^2 u^*}{\partial x \partial y} - p^2\frac{v^*}{C_S^2} = 0 \tag{2.8}$$

where the transformed displacement components, u^* and v^* are now functions of the variables x, y and p.

When the solid is subjected to a suddenly applied state of normal loading, the problem of applying stresses to the surfaces of the crack obtained by utilizing the usual principle of superposition which yields the following symmetry and boundary conditions on $y = 0$

$$\sigma_{yy}(x,0,t) = -\sigma(x)H(t), \qquad |x| \le a\,, \tag{2.9}$$

$$\tau_{xy}(x,0,t) = 0\,, \qquad |x| < \infty\,, \tag{2.10}$$

$$v(x,0,t) = 0\,, \qquad |x| > a\,, \tag{2.11}$$

in which the crack surface traction $\sigma(x)$ is a function of x and $H(t)$ is the Heaviside step function. In addition all the components of stress and displacement vanish at the remote distances from the crack region.

In the Laplace transform plane these conditions become

$$\sigma^*_{yy}(x,0,\,p) = -\,\sigma(x)/p\,, \qquad |x| \le a\,, \tag{2.12}$$

$$\tau^*_{xy}(x,0,p) = 0\,, \qquad |x| < \infty\,, \tag{2.13}$$

$$v^*(x,0,\,p) = 0\,, \qquad |x| > a\,. \tag{2.14}$$

3. Method of solution

To obtain an integral solution of differential Eqs (2.7) and (2.8) subject to the conditions (2.12)-(2.14), let

$$u^*(x,\,y,\,p) = \int_0^\infty A(s,y,p)\sin sx\,ds\,, \tag{3.1}$$

$$v^*(x,\,y,\,p) = \int_0^\infty \frac{1}{s} B(s,y,p)\cos sx\,ds \tag{3.2}$$

where A and B are arbitary functions. Substituting Eqs (3.1) and (3.2) into Eqs (2.7) and (2.8), the functions A and B are found to satisfy the simultaneous equations

$$\left(C_{11}s^2 + \frac{p^2}{C_S^2}\right)A - \frac{d^2A}{dy^2} + (1+C_{12})\frac{dB}{dy} = 0\,, \tag{3.3}$$

$$\left(s^2 + \frac{p^2}{C_S^2}\right)B - C_{22}\frac{d^2B}{dy^2} - (1+C_{12})s^2\frac{dA}{dy} = 0\,. \tag{3.4}$$

An appropriate solution of these equations are

$$A(s,y,p) = A_1(s,p)e^{-\gamma_1 y} + A_2(s,p)e^{-\gamma_2 y}\,, \tag{3.5}$$

$$B(s,y,p) = \alpha_1 A_1(s,p)e^{-\gamma_1 y} + \alpha_2 A_2(s,p)e^{-\gamma_2 y}\,, \tag{3.6}$$

in which γ_1^2, γ_2^2 are positive roots of the equation

$$C_{22}\gamma^4 + \left[\left(C_{12}^2 + 2C_{12} - C_{11}C_{22}\right)\ s^2 - (1 + C_{22})p^2/C_S^2\right]\gamma^2 + \\ + \left(C_{11}s^2 + p^2/C_S^2\right)\left(s^2 + p^2/C_S^2\right) = 0, \tag{3.7}$$

and $A_j(s, p)(j = 1,2)$ are arbitrary functions with

$$\alpha_j(s, p) = \frac{C_{11}s^2 + \dfrac{p^2}{C_S^2} - \gamma_j^2}{(1 + C_{12})\gamma_j}, \qquad j = 1,2\,. \tag{3.8}$$

The expressions for the displacements in the Laplace transform plane become

$$u^*(x, y, p) = \int_0^\infty \left[A_1 e^{-\gamma_1 y} + A_2 e^{-\gamma_2 y}\right] \sin sx\, ds\,, \tag{3.9}$$

$$v^*(x, y, p) = -\int_0^\infty \left[\alpha_1 A_1 e^{-\gamma_1 y} + \alpha_2 A_2 e^{-\gamma_2 y}\right] \frac{\cos sx}{s} ds\,, \tag{3.10}$$

and the corresponding expression for $\tau_{xy}^*(x,\ y,\ p)$ is given by

$$\tau_{xy}^*(x, y, p) = \mu_{12}\int_0^\infty \left[\beta_1 A_1 e^{-\gamma_1 y} + \beta_2 A_2 e^{-\gamma_2 y}\right] \sin sx\, ds \tag{3.11}$$

where $\qquad \beta_j = \alpha_j + \gamma_j\,, \qquad j = 1,2\,.$

Applying condition (2.13) to the Eq.(3.11) yields

$$A_2(s,\ p) = -\beta A_1(s,\ p) \quad \text{where} \quad \beta = \beta_1/\beta_2\,. \tag{3.12}$$

The components of displacements and stresses in the transformed plane are

$$u^*(x, y, p) = \int_0^\infty \left[e^{-\gamma_1 y} - \beta e^{-\gamma_2 y}\right] A_1(s, p) \sin sx\, ds\,, \tag{3.13}$$

$$v^*(x, y, p) = \int_0^\infty \left[\alpha_1 e^{-\gamma_1 y} - \beta\alpha_2 e^{-\gamma_2 y}\right] \frac{A_1(s, p)}{s} \cos sx\, ds\,, \tag{3.14}$$

$$\sigma_{yy}^*(x, y, p) = \mu_{12}\int_0^\infty \left[\left(c_{12}s^2 - \alpha_1\gamma_1 c_{22}\right)e^{-\gamma_1 y} - \beta\left(c_{12}s^2 - \alpha_2\gamma_2 c_{22}\right)e^{-\gamma_2 y}\right] \frac{A_1(s, p)}{s} \cos sx\, ds\,, \tag{3.15}$$

$$\tau_{xy}^*(x, y, p) = \mu_{12}\int_0^\infty \beta_1 \left[e^{-\gamma_1 y} + e^{-\gamma_2 y}\right] A_1(s, p) \sin sx\, ds\,. \tag{3.16}$$

Introducing the abbreviations

$$D(s,p)=\frac{(\alpha_1-\beta\alpha_2)}{s}A_1(s,p), \tag{3.17}$$

$$F(s,p)=\frac{\left(C_{12}s^2-C_{22}\alpha_1\gamma_1\right)-\beta\left(C_{12}s^2-C_{22}\alpha_2\gamma_2\right)}{(\alpha_1-\beta\alpha_2)}. \tag{3.18}$$

In view of boundary conditions (2.12) and (2.14), we get the following pair of dual integral equations for the determination of $D(s,p)$

$$\int_0^\infty F(s,p)D(s,p)\cos sx\,ds=-\frac{\sigma(x)}{\mu_{12}p}, \qquad 0\le x\le a, \tag{3.19}$$

$$\int_0^\infty D(s,p)\cos sx\,ds=0, \qquad x>a. \tag{3.20}$$

The solution of the integral Eq.(3.20) is taken in the form

$$D(s,p)=\frac{1}{s}\int_0^a f(\tau,p)\sin s\tau\,d\tau \tag{3.21}$$

where $f(\tau,p)$ is the unknown function to be determined. Substituting (3.21) in Eq.(3.19), we get

$$\frac{d}{dx}\int_0^a f(\tau,p)\log\left|\frac{\tau+x}{\tau-x}\right|d\tau=2\left[\sigma_1(x)-\frac{d}{dx}\int_0^a f(\tau,p)d\tau\int_0^a\frac{F_1(s,p)\sin s\tau\sin sx\,ds}{s}\right], \qquad 0<x<a \tag{3.22}$$

where

$$\sigma_1(x)=\frac{\sigma(x)}{\mu_{12}\theta p}, \tag{3.23}$$

$$F_1(s,p)=\frac{F(s,p)}{s\theta}-1\to 0 \quad \text{as} \quad s\to\infty, \tag{3.24}$$

$$\theta=\frac{(C_{12}-C_{22}N_1\alpha_1')(\alpha_1'+N_2)-(C_{12}-C_{22}N_2\alpha_2')(\alpha_1'+N_1)}{\alpha_1'N_2-\alpha_2'N_1},$$

$$\alpha_j'=-\frac{C_{11}-N_j^2}{(1+C_{12})N_j}, \qquad j=1,2, \tag{3.25}$$

$$N_{1,2}^2=\frac{1}{2C_{22}}\left[C_{11}C_{22}-C_{12}^2-2C_{12}\pm\sqrt{\left(C_{11}C_{22}-C_{12}^2-2C_{12}\right)^2-4C_{11}C_{22}}\right].$$

Using the relation

$$\frac{\sin sx \sin s\tau}{s^2} = \int_0^x \int_0^t \frac{vwJ_0(sw)J_0(sv)dv\,dw}{\sqrt{(x^2 - w^2)(\tau^2 - v^2)}} .$$

Equation (3.22) can now be rewritten in the form

$$\frac{d}{dx}\int_0^a f(\tau, p)\log\left|\frac{\tau + x}{\tau - x}\right|d\tau = 2\left[\sigma_1(x) - \frac{d}{dx}\int_0^a f(\tau, p)d\tau\int_0^x \int_0^t \frac{vwL(v,w)dw\,dv}{\sqrt{(x^2 - w^2)(\tau^2 - v^2)}}\right], \tag{3.26}$$

$$0 < x < a$$

where

$$L(v,w) = \int_0^\infty sF_1(s, p)J_0(sw)J_0(sv)ds , \tag{3.27}$$

and $J_0(\ )$ is the Bessel function of order zero. Applying a contour integration technique (Fig.1) the integral in $L(v,w)$ can be converted to the following finite integrals

$$L(v,w) = -\frac{2}{\pi}\frac{p^2}{C_S^2}\left[\int_0^{\frac{1}{\sqrt{C_{11}}}}[A]I_0\left(\frac{p\eta v}{C_S}\right)K_0\left(\frac{p\eta w}{C_S}\right)d\eta + \int_{\frac{1}{\sqrt{C_{11}}}}^1 [C]I_0\left(\frac{p\eta v}{C_S}\right)K_0\left(\frac{p\eta w}{C_S}\right)d\eta\right], \tag{3.28}$$

$$w > v$$

where

$$[A] = \frac{(C_{12}\eta^2 - C_{22}\bar{\alpha}_1\bar{\gamma}_1) - \bar{\beta}(C_{12}\eta^2 - C_{22}\bar{\alpha}_2\bar{\gamma}_2)}{(\bar{\alpha}_1 - \bar{\beta}\,\bar{\alpha}_2)}, \qquad [C] = \frac{\hat{\beta}(C_{12}\eta^2 - C_{22}\hat{\alpha}_2\hat{\gamma}_2)}{(\hat{\alpha}_1 + \hat{\beta}\hat{\alpha}_2)},$$

$$\bar{\gamma}_1 = \left[\frac{1}{2}\left\{-B_1 + \sqrt{B_1^2 - 4B_2}\right\}\right]^{\frac{1}{2}}, \qquad \bar{\gamma}_2 = \left[\frac{1}{2}\left\{-B_1 - \sqrt{B_1^2 - 4B_2}\right\}\right]^{\frac{1}{2}},$$

$$\hat{\gamma}_1 = \left[\frac{1}{2}\left\{B_1 + \sqrt{B_1^2 - 4B_2'}\right\}\right]^{\frac{1}{2}}, \qquad \hat{\gamma}_2 = \left[\frac{1}{2}\left\{-B_1 + \sqrt{B_1^2 - 4B_2'}\right\}\right]^{\frac{1}{2}},$$

$$B_1 = \frac{1}{C_{22}}\left[(C_{12}^2 + 2C_{12} - C_{11}C_{22})\eta^2 + (1 + C_{22})\right],$$

$$B_2 = \frac{1}{C_{22}}\left(\frac{1}{C_{11}} - \eta^2\right)(1 - \eta^2), \qquad B_2' = \frac{1}{C_{22}}\left(\eta^2 - \frac{1}{C_{11}}\right)(1 - \eta^2),$$

$$\overline{\alpha}_i = \frac{C_{11}\eta^2 - 1 + \overline{\gamma}_1^2}{(1+C_{12})\overline{\gamma}_i}, \qquad \hat{\alpha}_i = \frac{C_{11}\eta^2 - 1 + (-1)^i\hat{\gamma}_i^2}{(1+C_{12})\hat{\gamma}_1}, \qquad i = 1, 2,$$

$$\overline{\beta} = \frac{\overline{\gamma}_1 - \overline{\alpha}_1}{\overline{\gamma}_2 - \overline{\alpha}_2}, \qquad \hat{\beta} = \frac{\hat{\gamma}_1 + \hat{\alpha}_1}{\hat{\gamma}_2 + \hat{\alpha}_2}.$$

The corresponding expression of $L(v, w)$ for $w < v$ is obtained by interchanging v and w in Eq.(3.28). Employing the asymptotic expression of $I_0(z)$ and $K_0(z)$ as

$$I_0(z) \approx 1,$$

$$K_0(z) \approx \log\left(\frac{2}{z}\right).$$

Equation (3.28) is found to be

$$L(v,w) = \frac{2}{\pi} P \frac{p^2}{C_S^2} \log\left(\frac{p}{C_S}\right) + O\left(\frac{p^2}{C_S^2}\right) \tag{3.29}$$

where

$$P = \frac{1}{\theta}\left[\int_0^{1/\sqrt{C_{11}}} [A]d\eta + \int_{1/\sqrt{C_{11}}}^{1} [C]d\eta\right]. \tag{3.30}$$

We now expand $f(\tau, p)$ in the form

$$f(\tau, p) = f_0(\tau, p) + \frac{p^2}{C_S^2}\log\left(\frac{p}{C_S}\right) f_1(\tau, p) + O\left(\frac{p^2}{C_S^2}\right). \tag{3.31}$$

Substituting the above Eq.(3.31) and the value of $L(v, w)$ given by Eq.(3.29) in Eq.(3.26) and equating the co-efficients of like powers of $\frac{1}{C_S}$, the following equations are derived

$$\frac{d}{dx}\int_0^a f_0(\tau, p)\log\left|\frac{\tau + x}{\tau - x}\right| d\tau = 2\sigma_1(x), \qquad 0 < x < a, \tag{3.32}$$

$$\frac{d}{dx}\int_0^a f_1(\tau, p)\log\left|\frac{\tau + x}{\tau - x}\right| d\tau = -\frac{4P}{\pi}\int_0^a \tau\, f_0(\tau, p)\, d\tau, \qquad 0 < x < a. \tag{3.33}$$

Let us rewrite Eq.(3.32) as

$$\int_0^a f_0(\tau,p)\log\left|\frac{\tau+x}{\tau-x}\right|d\tau=\pi F_1(x), \qquad 0<x<a \tag{3.34}$$

where

$$F_1(x)=\frac{2}{\pi}\int_0^x \sigma_1(y)dy\,.$$

Now for the case of concentrated loading, taking $\sigma(x)=\delta(x-x_0)$, where $\delta(.)$ is the Dirac delta function, the solution of the integral Eq.(3.34) with the help of Cooke's result (1970) is found to be

$$f_0(\tau,p)=\frac{4}{\pi^2\mu_{12}\theta\,p}\cdot\frac{\tau\sqrt{a^2-x_0^2}}{\sqrt{a^2-\tau^2}\left(x_0^2-\tau^2\right)}\,.$$

Substituting $f_0(\tau,p)$ in Eq.(3.33), $f_1(\tau,p)$ is obtained as

$$f_1(\tau,p)=\frac{8P}{\pi^3\mu_{12}\theta p}\cdot\frac{\tau\sqrt{a^2-x_0^2}}{\sqrt{a^2-\tau^2}}\,.$$

The stress intensity factor in the *p*- plane is defined as

$$K_I^*(x_0,p)=\underset{x\to a+}{Lt}\sqrt{2(x-a)}\sigma_{yy}^*(x,o), \tag{3.35}$$

and calculated as

$$K_I^*(x_0,p)=\frac{4\sqrt{a^2-x_0^2}}{\pi^2 p}\left[\frac{\pi a}{2\left(a^2-x_0^2\right)\sqrt{a}}-P\frac{p^2}{C_S^2}\log\left(\frac{p}{C_S}\right)\sqrt{a}\right]+O\left(\frac{p^2}{C_S^2}\right). \tag{3.36}$$

Now the quasi-static solution is obtained by using the final value theorem of the Laplace transform, which states that

$$K_I(\infty)=\underset{t\to\infty}{Lt}K_1(t)=\underset{p\to 0}{Lt}\,pK_I^*(x_0,p)=\frac{1}{\pi\sqrt{a}}\frac{2a}{\sqrt{a^2-x_0^2}}\,. \tag{3.37}$$

This result is in complete agrement with the result of Isida (1972).
Therefore

$$\frac{K_1(t)}{K_I(\infty)}=\frac{1}{2\pi i}\int_{Br}\left[1-\frac{2P}{\pi}\frac{p^2}{C_S^2}\log\left(\frac{p}{C_S}\right)\sqrt{a^2-x_0^2}\right]\frac{e^{pt}}{p}dp\,. \tag{3.38}$$

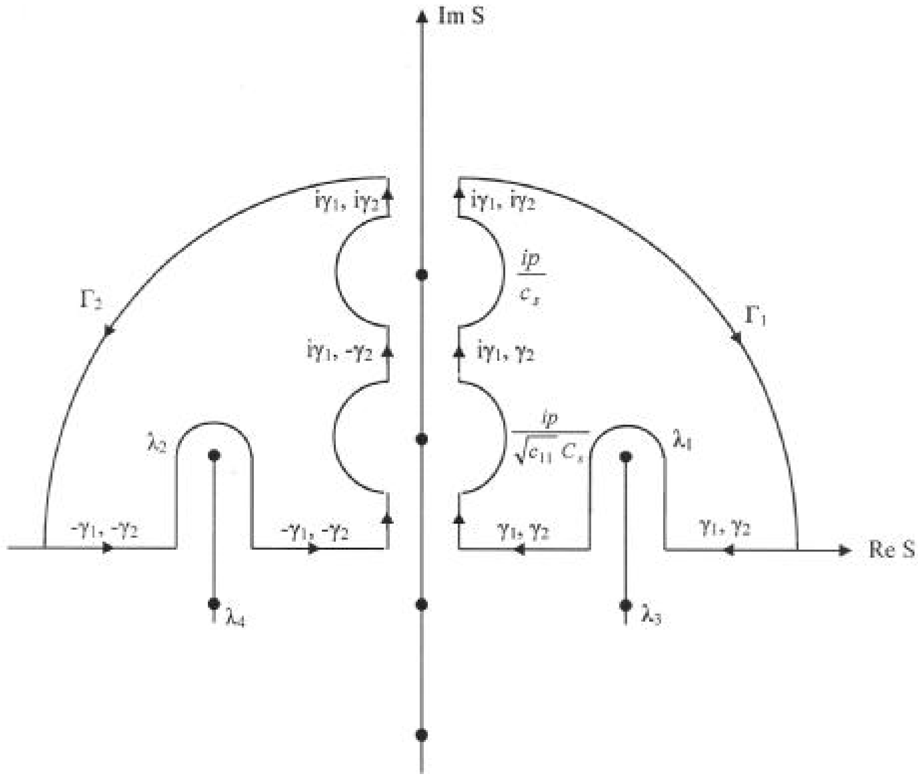

Fig.1. Contours of integration for the integral in Eq.(3.28).

4. Application of weight function to polynomial loading

We are now in a position to obtain weight function. The weight function is being the stress intensity factor at the crack tip under point force loading. The corresponding stress intensity factor for the arbitrary normal loading $\sigma(x_0)$ is

$$K_I(p) = \int_0^a K_I^*(x_0, p)\sigma(x_0)dx_0 . \tag{4.1}$$

Now for constant normal loading $\sigma(x_0) = \sigma_0$ we obtain $K_I(p)$ as

$$K_I(p) = \frac{\sigma_0\sqrt{a}}{p}\left[1 - \frac{Pa^2}{\pi}\frac{p^2}{C_S^2}\log\left(\frac{p}{C_S}\right)\right] + O\left(\frac{p^2}{C_S^2}\right). \tag{4.2}$$

This result is in complete agreement with the result of Baksi *et al.* (2004).

For linear normal loading $\sigma(x_0) = \sigma_1 \cdot \left(\frac{x}{a}\right)$

$$K_I(p)=\frac{\sigma_1\sqrt{a}}{p}\left[\frac{2}{\pi}-\frac{4Pa^2}{3\pi^2}\frac{p^2}{C_S^2}\log\left(\frac{p}{C_S}\right)\right]+O\left(\frac{p^2}{C_S^2}\right). \tag{4.3}$$

For quadratic loading $\sigma(x_0)=\sigma_2\left(\frac{x}{a}\right)^2$

$$K_I(p)=\frac{\sigma_2\sqrt{a}}{p}\left[\frac{1}{2}-\frac{Pa^2}{4\pi}\frac{p^2}{C_S^2}\log\left(\frac{p}{C_S}\right)\right]+O\left(\frac{p^2}{C_S^2}\right). \tag{4.4}$$

In all the three cases the dynamic SIF are obtained by using Eq.(2.6).

5. Numerical results and discussion

Being distinctly orthotropic in nature graphite-epoxy and glass-epoxy composite materials are selected for the numerical computation of the normalized stress intensity factor. The material constants for the selected materials are given below:

Materials	E_1	E_2	μ_{12}	ν_{12}
Graphile-Epoxy Composite (type 1)	15.3×10^9 Pa	158×10^9 Pa	5.52×10^9 Pa	$\nu_{12}=0.033$
Glass-Epoxy Composite (type II)	9.79×10^9 Pa	42.3×10^9 Pa	3.66×10^9 Pa	$\nu_{12}=0.063$

Applying the method of Papoulis (1957) the normalized stress intensity factor is calculated for different concentrated point force normal loadings and large values of normalized time variable $C_S t/a$ which are depicted through Figs 2-4.

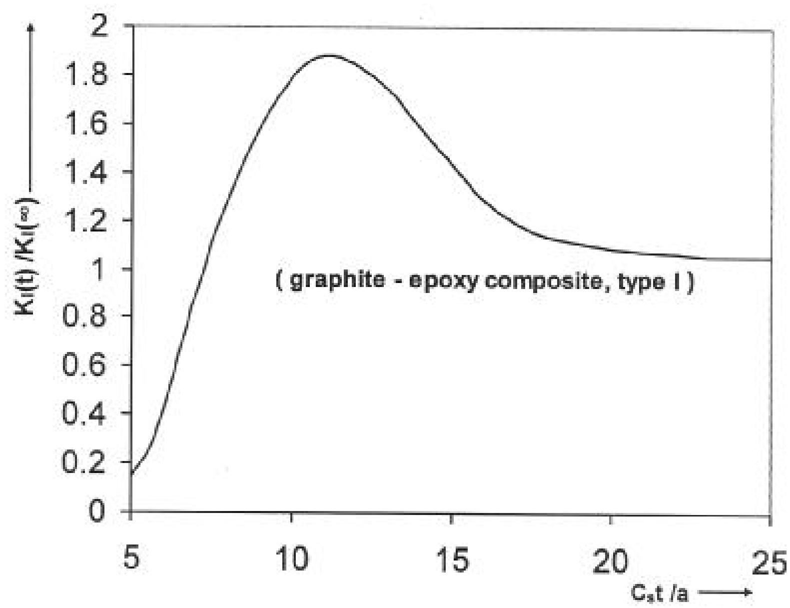

Fig.2. Plot of $K_I(t)/K_I(\infty)$ vs. $C_s t/a$ at $x_0/a=0.6$.

It is seen from Fig.2 that the dynamic stress intensity factor $K_I(t)$ at $x_0/a = 0.6$ rises very quickly with time, reaching a peak at $C_S t/a = 11.1$ and then decreases in magnitude and tends to the quasi static solution for sufficiently large normalized time. Here the overshoot is about *86%* and occurs at $C_S t/a = 11.1$. The duration of the overshoot is estimated to be approximately 1.43×10^{-4} sec. The maximum dynamic overshoot is observed due to the factor $\frac{x_0}{C_R} > \frac{a+x_0}{C_d}$ i.e., the dilatational wave from $-\frac{x_0}{a}$ arrives at $x = a$ before the Rayleigh wave from $+\frac{x_0}{a}$ does, thus their effects are added and as a result $K_1(t)$ has an overshoot. It is also observed from the figure that as time becomes larger and larger $\frac{K_I(t)}{K_I(\infty)} \to 1$. Again by using weight function we can easily show $K_I(\infty) = \sigma_0\sqrt{a}$. Hence as $t \to \infty$, $K_I(t)$ tends to a static solution $\sigma_0\sqrt{a}$, which is in complete agreement with the result of Shindo *et al.* (1986).

In Fig.3, it is seen that initially $K_I(t)$ increases with the increase of $\frac{x_0}{a} = 0.6(0.1)0.9$ but then from $C_S t/a = 7.4$ the values of $K_I(t)$ decrease with the increase in $\frac{x_0}{a}$. The variations of results of $K_I(t)$ with the normalized time are similar. However, if $\frac{x_0}{a}$ is very close to *1*, numerical difficulties arise in the solution. This is due to the discontinuity in $K_I^*(x_0, p)$ at $\frac{x_0}{a} = 1$.

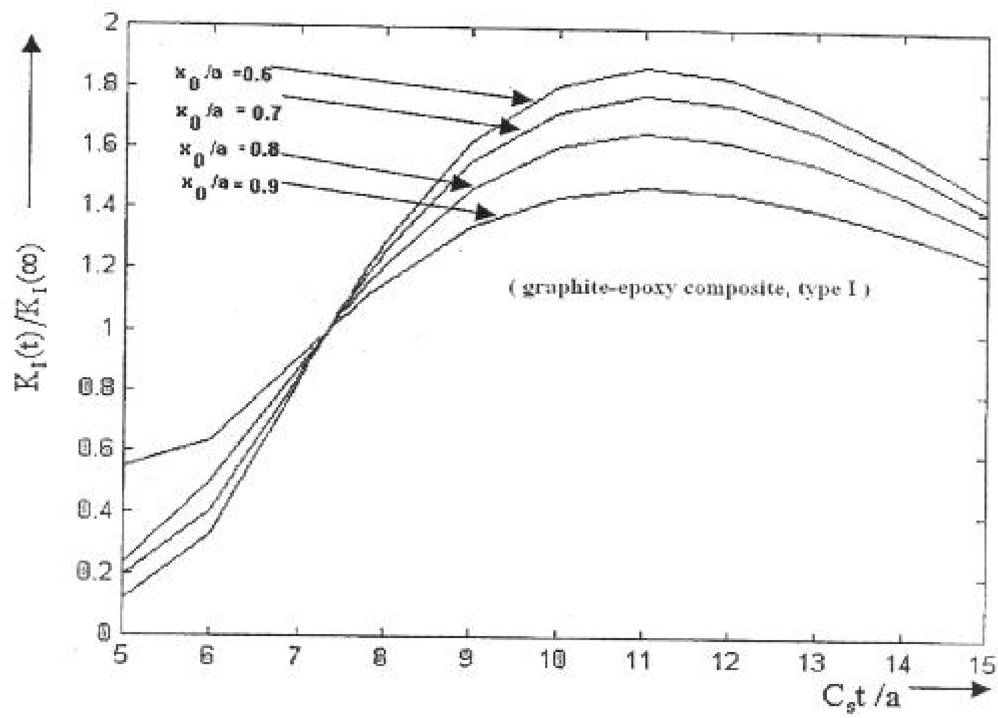

Fig.3. Plot of $K_I(t)/K_I(\infty)$ vs. $C_s t/a$ for various x_0/a.

From Fig.4, it is seen that the features for type II material are similar to that of type I material. Here the maximum overshoots are *46.7%*, *42.5%*, *35.7%* and *26%* respectively for $x_0/a = 0.6, 0.7, 0.8$ and *0.9* which are observed at $C_S t/a = 11.1$.

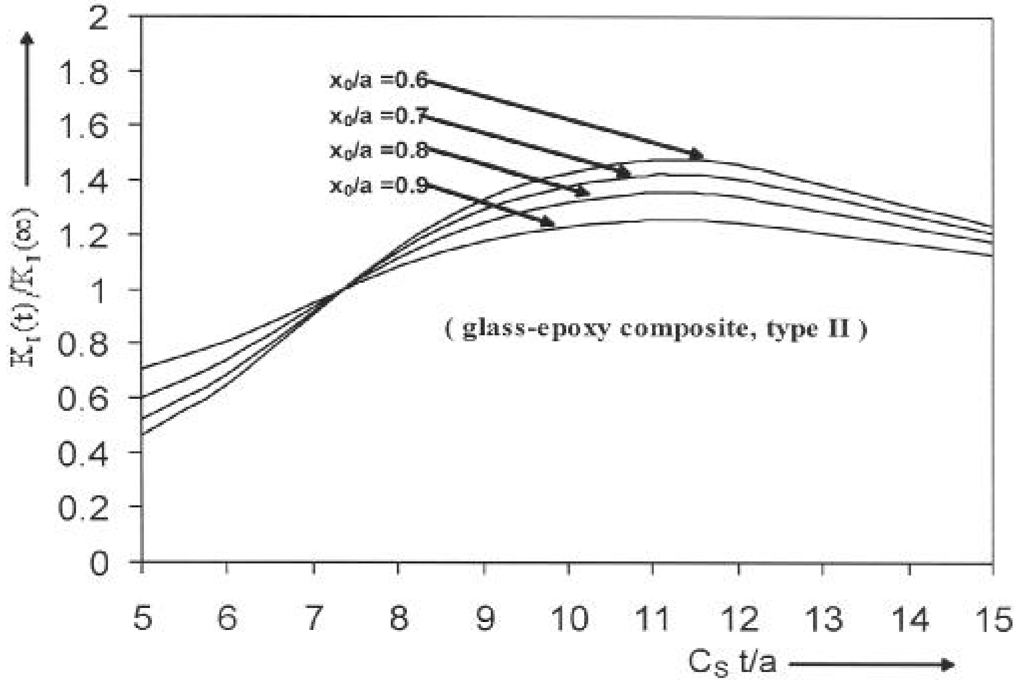

Fig.4. Plot of $K_I(t)/K_I(\infty)$ vs. $C_s t/a$ for various x_0/a .

Nomenclature

C_d	– dilatational wave speed
C_{ij}	– non-dimensional parameters
C_R	– Rayleigh wave speed
C_S	– shear wave speed
$K_I(t)$	– stress intensity factor in the *t*-plane
$K_I^*(x_0, p)$	– stress intensity factor in the *p*-plane
u, v	– *x*, *y* components of the displacement vector
u^*, v^*	– displacement components in the *p*-plane
$\delta(\cdot)$	– Dirac delta function
μ_{12}	– shear modulus
ρ	– density
σ_{xx}	– stress component along *x*-axis
σ_{yy}	– stress component along *y*-axis
σ_{xy}^*	– stress component along *x*-axis in the *p*-plane
τ_{xy}	– shearing stress in the *xy*-plane
τ_{xy}^*	– shearing stress in the *p*-plane

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