# MIXED CONVECTION HEAT TRANSFER FROM A VERTICAL HEATED PLATE EMBEDDED IN A SPARSELY PACKED POROUS MEDIUM

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An improved numerical study on mixed convection from a heated vertical plate embedded in a Newtonian fluid saturated sparsely packed porous medium is undertaken by considering the variation of permeability, porosity and thermal conductivity. The boundary layer flow in the porous medium is governed by the Lapwood-Forchheimer-Brinkman extended Darcy model. Similarity transformations are employed and the resulting ordinary differential equations are solved numerically by using a shooting algorithm with the Runge-Kutta-Fehlberg integration scheme to obtain velocity and temperature distributions. Besides, the skin friction and Nusselt number are also computed for various physical parameters governing the problem under consideration. It is found that the inertial parameter has a significant influence on decreasing the flow field, whereas its influence is reversed on the rate of heat transfer for all values of permeability parameter considered. Further, the results under the limiting conditions were found to be in good agreement with the existing ones.

Key words: Newtonian fluid, heat transfer, porous medium, boundary-layer, similarity solution.

## 1. Introduction

In recent years, considerable attention has been devoted to the study of boundary layer flow behavior and heat transfer characteristics of a Newtonian fluid past a vertical plate embedded in a fluid saturated porous medium because of its extensive applications in engineering processes, especially in the enhanced recovery of petroleum resources and packed bed reactors. Considerable amount of interest has also been devoted to the study of transport properties in porous media subject to heat transfer which are characterized by highly non-linear coupled partial differential equations. The problem of free convection heat transfer from a vertical plate embedded in a fluid saturated porous medium is studied by Cheng and Minkowycz (1977), who have obtained the similarity solutions for the problem considered. Cheng (1978) has provided an extensive review of early works on free convection in porous media. Nakayama and Koyama (1987) have obtained the similarity solution for the problem of free convection in the boundary layer adjacent to a vertical plate immersed in a thermally stratified porous medium. The mixed convection boundary layer flow on an impermeable vertical surface embedded in a saturated porous medium has been treated by Merkin (1980). Hung and Chen (1997) have studied non-Darcy free convection in a thermally stratified fluid saturated porous medium along a vertical plate with variable heat flux. Hsieh et al. (1993) have obtained a non-similar solution for combined convection from vertical plates in porous media with variable surface temperatures or heat flux. Recently, Nield and Bejan (1999) have given an excellent summary of free convection flow in porous media.

Several investigators have considered the non-Darcian model in the recent past to study the convection and heat transfer rates on bodies embedded in a porous medium for Newtonian fluids. Kumari *et al.* (1990) have investigated the non-Darcian effects on forced convection heat transfer over a flat plate in a

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highly porous medium. Chen and Ho (1988) have studied the effects of flow inertia on vertical, natural convection in saturated, porous media. Hong *et al.* (1987) have studied analytically the non-Darcian effects on a vertical plate natural convection in porous media. They used a combination of Rayleigh and Darcy numbers to describe the inertia and boundary terms and obtained similar solutions. They found that these effects decrease the velocity and reduce the heat transfer rate. Hassanien *et al.* (1998) have studied the effects of thermal stratification on non-Darcy mixed convection from a vertical flat plate embedded in a porous medium. Plumb and Huenefeld (1981) have investigated non-Darcy natural convection from vertical isothermal surfaces in saturated porous media. Lai and Kulacki (1987; 1991) have used both Darcy and non-Darcy models (inertia effect only) to study mixed convection from horizontal and vertical surfaces embedded in saturated porous media. Bejan and Poulikakos (1984) have used Forchheimer's model to study vertical boundary layer natural convection in a porous medium.

Shwartz and Smith (1953), Benenati and Brosilow (1962) have shown that the permeability of a porous medium varies due to the variation of porosity from the wall to the interior of the porous medium. Chandrasekhar and Namboodiri (1985) have shown the effectiveness of variable permeability of the porous medium on velocity distribution and heat transfer. Recently, Mohammadein and El-Shaer (2004) have studied combined free and forced convective flow past a semi-infinite vertical plate embedded in a porous medium incorporating variable permeability. Nonetheless, the inertia effects become important in a sparsely packed porous medium and hence their effect on free convection problems needs to be investigated.

The aim of the present investigation is, therefore, to study systematically the effect of inertial terms on combined free and forced convective heat transfer past a semi-infinite vertical plate embedded in a saturated porous medium with variable permeability, porosity and thermal conductivity. In this analysis coupled non-linear partial differential equations, governing the problem, are first reduced by a similarity transformation to the ordinary differential equations and then the resultant boundary value problem is converted into the system of five simultaneous equations of first-order for five unknowns. Then these equations are solved numerically by a shooting technique with the Runge-Kutta-Fehlberg method to obtain horizontal velocity and temperature profiles for various values of physical parameters. The results obtained from the present numerical computation under limiting conditions agree well with the existing ones and thus verify the accuracy of the method used.

#### 2. Mathematical formulation

We consider a semi-infinite vertical heated plate embedded in a sparsely packed Newtonian fluid saturated porous medium of variable porosity, permeability and thermal conductivity. The x-coordinate is measured along the plate from its leading edge, and the y-coordinate normal to it. Let  $U_o$  be the velocity of the fluid in the upward direction and the gravitational field, g, is acting in the downward direction. The plate is maintained at a uniform temperature  $T_w$  which is always greater than the free stream values existing far from the plate (i.e.,  $T_w > T_\infty$ ). The boundary layer equations governing the conservation of mass, momentum and energy can be written in the following form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (2.1)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho g\beta(T-T_{\infty})+\overline{\mu}\frac{\partial^{2} u}{\partial y^{2}}+\frac{\mu\varepsilon(y)}{k(y)}(U_{o}-u)+\frac{F\varepsilon^{2}(y)}{\sqrt{k(y)}}(U_{o}^{2}-u^{2}),$$
(2.2)

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\alpha(y)\frac{\partial T}{\partial y}\right) + \frac{\overline{\mu}}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2$$
(2.3)

where, *u* and *v* are the velocity components along the *x* and *y* directions respectively, *T* is the temperature of the fluid,  $\rho$  is the fluid density,  $\overline{\mu}$  is the effective viscosity of the fluid,  $\mu$  is the fluid viscosity, k(y) is the variable permeability of the porous medium,  $\varepsilon(y)$  is the porosity of the saturated porous medium,  $\alpha(y)$  is the variable effective thermal diffusivity of the medium, *F* is the empirical constant of the second-order resistance term due to inertial effect,  $C_p$  is the specific heat at constant pressure,  $\beta$  is the coefficient of volume expansion and  $T_{\infty}$  is the ambient temperature.

The above governing equations need to be solved subject to the following boundary conditions on velocity and temperature fields

$$u = 0, \quad v = 0, \quad T = T_w \quad \text{at} \quad y = 0,$$
 (2.4)

$$u = U_o, \quad v = 0, \quad T = T_\infty \quad \text{as} \quad y \to \infty.$$
 (2.5)

We now introduce the following dimensionless variables *f* and  $\theta$  as well as the similarity variable  $\eta$  (Hady *et al.*, 1996; Mohammadein and El-Shaer, 2004)

$$\eta = \left(\frac{y}{x}\right) \left(\frac{U_o x}{v}\right)^{l/2}, \qquad \psi = \sqrt{v U_o x} f(\eta), \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$
(2.6)

where a prime represents differentiation with respect to  $\eta$  and  $T_w$  is the plate temperature.

In Eq.(2.6) the stream function  $\psi(x, y)$  is defined by  $u = \frac{\partial \psi}{\partial y}$ ,  $v = -\frac{\partial \psi}{\partial x}$ , such that the continuity Eq.(2.1) is satisfied automatically and the velocity components are given by

$$u = U_o f'(\eta), \qquad v = -\frac{1}{2} \sqrt{\frac{\nu U_o}{x}} (f(\eta) - \eta f'(\eta)).$$
(2.7)

Following Chandrasekhara and Namboodiri (1985), the variable permeability  $k(\eta)$ , the variable porosity  $\varepsilon(\eta)$  and variable effective thermal diffusivity  $\alpha(\eta)$  are given by

$$k(\eta) = k_o \left( l + de^{-\eta} \right), \tag{2.8}$$

$$\varepsilon(\eta) = \varepsilon_o \left( l + d^* e^{-\eta} \right), \tag{2.9}$$

$$\alpha(\eta) = \alpha_o \left[ \varepsilon_o \left( l + d^* e^{-\eta} \right) + \sigma^* \left\{ l - \varepsilon_o \left( l + d^* e^{-\eta} \right) \right\} \right]$$
(2.10)

where  $k_o$ ,  $\varepsilon_o$  and  $\alpha_o$  are the permeability, porosity and diffusivity at the edge of the boundary layer respectively,  $\sigma^*$  is the ratio of the thermal conductivity of the solid to the conductivity of the fluid, *d* and *d*<sup>\*</sup> are treated as constants having values 3.0 and 1.5 respectively.

Substituting Eqs (2.6) and (2.7) in Eqs (2.2) and (2.3), we get the following transformed equations

$$f''' + \frac{1}{2}ff'' + \frac{\mathrm{Gr}}{\mathrm{Re}^2}\theta + \frac{\alpha^*(l+d^*e^{-\eta})}{\sigma\mathrm{Re}(l+de^{-\eta})}(l-f') + \frac{\beta^*(l+d^*e^{-\eta})}{(l+de^{-\eta})^{l/2}}(l-f'^2) = 0, \qquad (2.11)$$

$$\theta'' = -\frac{(l/2)\Pr\theta' f + \Pr E f''^2 + \varepsilon_o d^* e^{-\eta} (\sigma^* - l)\theta}{\varepsilon_o + \sigma^* (l - \varepsilon_o) + \varepsilon_o d^* e^{-\eta} (l - \sigma^*)}$$
(2.12)

where,  $\beta^* = F\epsilon_o^2 x / k_o^{1/2}$  is the local inertial parameter,  $\Pr = \overline{\mu} / \rho \alpha_o$  is the Prandtl number,  $\alpha^* = \mu / \overline{\mu}$  is the ratio of viscosities,  $E = U_o^2 / C_p (T_w - T_\infty)$  is the Eckert number,  $\sigma = k_o / x^2 \epsilon_o$  is the local permeability parameter,  $\operatorname{Re} = U_o x / \nu$  is the local Reynolds number and  $\operatorname{Gr} = g\beta(T_w - T_\infty)x^3/\nu^2$  is the local Grashof number.

The transformed boundary conditions are

$$f = 0, \qquad f' = 0, \qquad \theta = 1 \qquad \text{at} \qquad \eta = 0,$$
 (2.13)

$$f' = 1, \qquad \theta = 0 \qquad \text{as} \qquad \eta \to \infty.$$
 (2.14)

Once the velocity and temperature distributions are known, the skin friction and the rate of heat transfer can be calculated respectively by

$$t = -f''(0)/\sqrt{\text{Re}}$$
, (2.15)

$$Nu = -\sqrt{Re \ \theta'(0)} \tag{2.16}$$

where  $\tau$  is the skin friction and Nu is the Nusselt number.

## 3. Numerical method

Equations (2.11) and (2.12) constitute a highly non-linear coupled boundary value problem (BVP) of third and second order respectively. An improved numerical scheme involving a shooting technique with the Runge-Kutta-Fehlberg method is developed to solve the resulting nonlinear BVP. Thus, the coupled nonlinear boundary value problem of third-order in f and second-order in  $\theta$  has been reduced to a system of five simultaneous equations of first-order for five unknowns as follows (Vajravelu, 2001)

$$f_{1}' = f_{2}, \qquad f_{2}' = f_{3},$$

$$f_{3}' = -\frac{l}{2} f_{1} f_{3} - \frac{\mathrm{Gr}}{\mathrm{Re}^{2}} f_{4} - \frac{\alpha^{*} (l + d^{*} e^{-\eta})}{\sigma \mathrm{Re} (l + d e^{-\eta})} (l - f_{2}) - \frac{\beta^{*} (l + d^{*} e^{-\eta})}{(l + d e^{-\eta})^{l/2}} (l - f_{2}^{2}), \qquad (3.1)$$

$$f_{4}' = f_{5}, \qquad f_{5}' = -\frac{(l/2) \mathrm{Pr} f_{1} f_{5} + \mathrm{Pr} \mathrm{E} f_{3}^{2} + \varepsilon_{o} d^{*} e^{-\eta} (\sigma^{*} - 1) f_{5}}{\varepsilon_{o} + \sigma^{*} (l - \varepsilon_{o}) + \varepsilon_{o} d^{*} e^{-\eta} (l - \sigma^{*})}$$

where  $f_1 = f$ ,  $f_2 = f'$ ,  $f_3 = f''$ ,  $f_4 = \theta$ ,  $f_5 = \theta'$  and a prime denotes differentiation with respect to  $\eta$ . The boundary conditions now become

$$f_1 = 0, \qquad f_2 = 0, \qquad f_4 = 1 \qquad \text{at} \qquad \eta = 0,$$
 (3.2)

$$f_2 = 1, \qquad f_4 = 0 \qquad \text{as} \qquad \eta \to \infty.$$
 (3.3)

## 4. Results and discussion

The system of first-order differential Eqs (3.1)-(3.3) is solved numerically using the shooting technique with the Runge-Kutta-Fehlberg method. In order to know the accuracy of the method used, computed values of f''(0) and  $\theta'(0)$  were obtained for  $\beta^* = 0$  and compared with those obtained by Mohammadein and El-Shaer (2004) in Tab.1 for the variable permeability (d = 3.0,  $d^* = 1.5$ ) case and good agreement has been obtained with their results. The values tabulated in Tab.1 are for  $\varepsilon_o = 0.4$ , E = 0.1, Pr = 0.71 with selected values of  $Gr/Re^2$ ,  $\sigma^*$  and  $\alpha^*/\sigma Re$ . The slight deviation in the values may be due to the use of the Runge-Kutta-Fehlberg method which has fifth order accuracy whereas, Mohammaden and El-Shaer (2004) have used the fourth-order Runge-Kutta method which has only fourth order accuracy. Thus the present results are more accurate compared to their results.

| Table 1. Results for | f <b>"(</b> 0 | ) and $-\theta'(0)$ | ) for | $Pr = 0.71, \beta$ | * = 0.0 | for variable permeability cas | e. |
|----------------------|---------------|---------------------|-------|--------------------|---------|-------------------------------|----|
|----------------------|---------------|---------------------|-------|--------------------|---------|-------------------------------|----|

| σ*  | $Gr/Re^2$ | $\alpha^*/\sigma Re$ | Presen   | nt result     | Mohammadein an El-Shaer |               |  |
|-----|-----------|----------------------|----------|---------------|-------------------------|---------------|--|
|     | Si/ Re    | a jone               | f"(0)    | $-\theta'(0)$ | f"(0)                   | $-\theta'(0)$ |  |
| 2.0 | 0.2       | 0.0                  | 0.611321 | 0.381233      | 0.61215                 | 0.38030       |  |
|     |           | 0.1                  | 0.667804 | 0.386090      | 0.64526                 | 0.38281       |  |
|     |           | 0.5                  | 0.846341 | 0.417658      | 0.75527                 | 0.38959       |  |
|     | 0.5       | 0.0                  | 0.958156 | 0.403083      | 0.95816                 | 0.40308       |  |
|     |           | 0.1                  | 0.987898 | 0.406430      | 0.97432                 | 0.40325       |  |
|     | 0.2       | 0.0                  | 2.415691 | 0.376339      | 2.31558                 | 0.40376       |  |
| 4.0 | 0.2       | 0.0                  | 0.627031 | 0.504676      | 0.62705                 | 0.50459       |  |
|     |           | 0.1                  | 0.681575 | 0.507192      | 0.65772                 | 0.50664       |  |
|     |           | 0.5                  | 0.859094 | 0.519451      | 0.76231                 | 0.51242       |  |
|     | 0.5       | 0.0                  | 0.993653 | 0.528672      | 0.99206                 | 0.52979       |  |
|     |           | 0.1                  | 1.022091 | 0.528510      | 1.00403                 | 0.52940       |  |

Table 2 contains the computed values of f''(0) and  $-\theta'(0)$  for the selected values of  $\alpha^*/\sigma Re$  and  $Gr/Re^2$  and  $\beta^*$  for uniform permeability (UP) and variable permeability (VP) cases. From the table, it is observed that an increase in the value of  $\beta^*$  is to increase the skin friction for all values of  $\alpha^*/\sigma Re$ ,  $\sigma^*$  and  $Gr/Re^2$  for both UP & VP. Further, it is interesting to note that the effect of  $\sigma^*$  is to increase the skin friction whereas the rate of heat transfer decreases.

| $\alpha^*/\sigma Re$ | $\sigma^{*}$ | $Gr/Re^2$ | β*  | f        | f"(0)    |          | $-\theta'(0)$ |  |
|----------------------|--------------|-----------|-----|----------|----------|----------|---------------|--|
| /                    |              | 1         | •   | UP       | VP       | UP       | VP            |  |
| 0.1                  | 2.0          | 0.0       | 0.0 | 0.451835 | 0.421933 | 0.250491 | 0.363478      |  |
|                      |              |           | 0.1 | 0.576676 | 0.564654 | 0.260377 | 0.379063      |  |
|                      |              |           | 0.5 | 0.929158 | 0.963919 | 0.357305 | 0.504579      |  |
|                      |              |           | 0.9 | 1.192470 | 1.284032 | 0.507254 | 0.796314      |  |
|                      |              | 0.1       | 0.0 | 0.584076 | 0.549309 | 0.260956 | 0.376262      |  |
|                      |              |           | 0.1 | 0.690510 | 0.672864 | 0.267553 | 0.387279      |  |
|                      |              |           | 0.5 | 1.007338 | 1.036579 | 0.288634 | 0.412960      |  |
|                      |              |           | 0.9 | 1.245152 | 1.327359 | 0.506892 | 0.795579      |  |
|                      |              | 0.2       | 0.0 | 0.707080 | 0.667804 | 0.269021 | 0.386090      |  |
|                      |              |           | 0.1 | 0.798848 | 0.776034 | 0.273416 | 0.393967      |  |
|                      |              | 2.0       | 0.1 | 2.368606 | 2.259598 | 0.291978 | 0.414071      |  |
|                      | 4.0          | 0.0       | 0.1 | 0.576676 | 0.564841 | 0.217035 | 0.538623      |  |
|                      |              |           | 0.5 | 0.929858 | 0.963919 | 0.351812 | 0.731295      |  |
|                      |              | 0.1       | 0.1 | 0.700834 | 0.671721 | 0.222808 | 0.545299      |  |
|                      |              |           | 0.5 | 1.002954 | 0.028371 | 0.352147 | 0.731303      |  |
| 0.5                  | 2.0          | 0.2       | 0.0 | 0.937729 | 0.846341 | 0.336831 | 0.417658      |  |

Table 2. Results for f''(0) and  $-\theta'(0)$  for the selected values of  $\alpha^*/\sigma Re$  and  $Gr/Re^2$  for Pr = 0.71 for Uniform Permeability (UP) and Variable Permeability (VP) cases.

Figure 1 depicts the velocity distribution for various values of second order resistance for variable permeability (VP) and uniform permeability (UP) cases. It is observed that an increase in the value of inertial parameter  $\beta^*$  leads to an increase in the velocity profile within the boundary layer, while for  $\beta^* = 0.5$ , the velocity coincides for both UP and VP cases. It is also important to note that the boundary layer decreases with an increase in the value of inertial parameter. Thus, the non-Darcian term has a very significant effect on the velocity distribution. Figure 2 exhibits the variation of velocity profiles for various values of  $\sigma^*$  for both UP and VP. It is clearly seen that the velocity profile increases with an increase in  $\sigma^*$  which is effect use only for UP but its effect diminishes for small values of  $\sigma^*$ .



Fig.1. Velocity profiles for various values of second order resistance for VP and UP.



Fig.2. Velocity profiles for various values of  $\sigma^*$  for VP and UP.

Figure 3 shows the variation of velocity distribution for three values of Pr = 0.71, 3 and 10 for the case of VP. It is observed that the velocity profiles decrease as the Prandtl number increases which is very significant in the middle of the boundary layer. Further, it is clear that the boundary layer decreases with a decrease in the value of Pr.



Fig.3. Variation of velocity distribution for various values of the Prandtl number for VP.

Figure 4 depicts the temperature distribution for various values of the parameter  $Gr/Re^2$  for the cases of UP and VP. It is seen that an increase in the value of  $Gr/Re^2$  is to decrease the temperature distribution for both the cases considered. The temperature is found to be less for VP as compared to UP. It is also observed that the effect of VP is more significant on temperature distribution for higher values of  $Gr/Re^2$ .



Fig.4. Temperature distribution for various values of  $Gr/Re^2$  for VP and UP.



Fig.5. Temperature profiles for various values of second order resistance for VP and UP.

Figure 5 displays the distribution of temperature for various values of second order resistance  $\beta^*$  for UP and VP cases. From this figure it is evident that the temperature profile decreases smoothly for  $\beta^* = 0.1$  within the boundary layer whereas for higher value of  $\beta^*$  the temperature continuously decreases and this decrease is very rapid. This shows that the rate of cooling is much faster for higher values of second order resistance in both UP and VP cases.



Fig.6. Variation of temperature distribution with  $\eta$  for various values of Pr for VP.

Another interesting feature which is observed from Fig.5 is that the boundary layer decreases with an increase in the value of inertial parameter and it is more so in the case of UP as compared to VP for both the values of  $\beta^*$  considered. Figure 6 gives the variation of temperature distribution within the boundary layer

for various values of Pr in the case of VP. The temperature profiles show a typical smooth decreasing pattern for Pr = 0.71 whereas, for higher values of Pr, the temperature continuously decreases at a steeper rate in the flow region and the boundary layer decreases with the increase in Pr.

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## Nomenclature

- $C_p$  specific heat at constant pressure
- d constant defined in Eq.(2.8)
- $d^*$  constant defined in Eq.(2.9)
- E Eckert number
- f non-dimensionless stream function
- F empirical constant of the second-order resistance term due to inertial effect
- g acceleration due to gravity
- Gr Grashof number
- k(y) permeability of the porous medium
- $k_o$  permeability of the porous medium at the edge of the boundary layer
- $\kappa$  thermal conductivity
- Nu Nusselt number
- Pr Prandtl number
- Re Reynolds number
- T temperature of the fluid near the plate
- $T_w$  temperature of the plate
- $T_{\infty}$  ambient temperature  $(T_{\infty} < T_{W})$
- u, v velocity components along x and y directions
- $U_o$  free stream velocity
- x, y coordinate axes along and perpendicular to the plate
- $\alpha(y)$  thermal diffusivity
- $\alpha^*$  ratio of viscosities
- $\alpha_{o}$  thermal diffusivity at the edge of the boundary layer
- $\beta$  coefficient of volume expansion
- $\beta^*$  inertial parameter
- $\varepsilon(y)$  porosity of the saturated porous medium
- $\varepsilon_{a}$  porosity of the saturated porous medium at the edge of the boundary layer
- $\eta$  dimensionless similarity variable
- $\theta$  dimensionless temperature
- $\mu$  viscosity of the fluid
- $\overline{\mu}~-$  effective viscosity of the fluid
- $\nu$  kinematics viscosity of the fluid
- $\rho \quad \, density \, of \, fluid$
- $\sigma \quad \text{ permeability parameter}$
- $\sigma^*$  ratio of thermal conductivity of the solid to the liquid
- $\tau$  skin friction
- $\psi$  stream function

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